A

TREATISE

OF

NAVIGATION:

CONTAINING,

I. *The* THEORY *of* NAVIGATION *demonſtrated*; wherein it is proved, that the Nature of the thing called *Departure* is quite different from that which the Writers upon this Subject took it to be: That *Middle-Latitude Sailing* is erroneous; and that the common Method of keeping Reckonings in *Meridional Diſtance*, is groſſly falſe.

II. NAUTICAL PROBLEMS: Of a ſingle Courſe, Traverſes; of computing the Bearings and Diſtances of Places. The Uſe of *Mercator's Chart* made eaſy, and a new Method for finding the Bearings of Places upon Charts, without the Confuſion of Rhumb-Lines. Of *Currents*, and how to find them ſeveral Ways: Of turning to *Windward* in *Currents*, &c.

III. ASTRONOMICAL PROBLEMS: Improvements made in the common *Sea-Quadrant*. The Deſcription of a new *Fore-Staff*, which is much more accurate and commodious than the common one. The Errors of the *Nocturnal* ſhewed and demonſtrated.

Tables of the *Sun*'s Place, Declination, and Right Aſcenſion. A *Table* of the moſt eminent *Fixed Stars*, contrived in ſuch a manner, as to ſhew by Inſpection, which is the propereſt Star for Obſervation, in all Places, and at all Times of the Year.

IV. PRACTICAL NAVIGATION: Improvements made in the common *Sea-Compaſs*. The Deſcription of *a new Azimuth-Compaſs*, which is ſo contrived as to be kept from being diſturbed by the Motion of the Ship. How to find the *Lee-Way* to a great Exactneſs, and a new and commodious Method of keeping a *Sea-Journal*.

To which is prefixed, by way of *Introduction*, A ſuccinct *Treatiſe* of PLAIN TRIGONOMETRY.

By JOSEPH HARRIS,
Teacher of the MATHEMATICKS.

LONDON:

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M.DCC.XXX.

MVSEVM
BRITAN
NICVM

To the Right Honourable

G E O R G E

Lord Viſcount *Torrington*;

John Cokburn *Eſq*;
Sir John Norris *Knt.*
Sir Charles Wager *Knt.*
Sir Thomas Lyttleton *Bart.*
Sir William Younge, *Knt. of the Bath*; and
Lord Archibald Hamilton;

Lords Commiſſioners for executing the Office of Lord High-Admiral of *Great-Britain*:

This Treatiſe of Navigation (having by your LORDSHIPS ORDER been examined by Dr. *HALLEY*, who was pleaſed to approve of it) is humbly Dedicated, by,

MY LORDS,

Your LORDSHIPS

Moſt Dutiful,

and Obedient

Humble Servant,

JOSEPH HARRIS.

Advertiſement.

THE new AZIMUTH-COMPASS and the new FORE-STAFF, deſcribed in this Book, may be had at Mr. *Thomas Wright*'s Mathematical Inſtrument-Maker to his Majeſty, at the *Orrery and Globe* in *Fleet-ſtreet*. The new *Fore-Staff* may be alſo had at Mr. *John Coggs*'s, Mathematical Inſtrument-Maker at the *Globe* near St. *Dunſtan*'s Church in *Fleet-ſtreet*; and at Mr. *Thomas Heath*'s, next the *Fountain-Tavern* in the *Strand*. At which Places this Book is to be Sold.

THE

PREFACE.

THE great Importance of Navigation, and the vaſt Advantages that accrue from it to Mankind, are ſo well known, that it would be needleſs to ſay any thing upon that Head: Yet it is ſurprizing this uſeful Art ſhould be ſo much neglected, that palpable and groſs Errors are taught and practiſed in it even at this Day. And notwithſtanding the many conſiderable Improvements lately made in the Mathematical Sciences, yet hardly any thing has been done in Navigation, ſince our learned and ingenious Country-man Mr. Edward Wright, *about* 150 *Years ago, publiſhed his excellent Treatiſe, entituled,* Certain Errors in Navigation corrected, *&c.*

Of late Years indeed we have had a multitude of Authors upon this Subject, but moſt of them ſeem to be but indifferently qualified for ſuch Undertakings; for either they never had gained any Experience at Sea, (and therefore could be no Judges with relation to the practical Part of Navigation) or elſe had not throughly conſidered the Geometrical Principles upon which the Art of Navigation is neceſſarily built: And this muſt be the reaſon of the Errors, as well as the Intricacy and Confuſion which we meet with in our modern Books upon this Subject: And I believe the ill Succeſs of our Longitude-finders may be alſo imputed to the ſame Cauſe.

The pretended Accuracy of ſome People in their Reckonings, (which the Nature of things will not admit of) muſt certainly be one great cauſe why the Art of Navigation at this time of day, is ſo lame and imperfect; for except People are ſenſible of the Defect, it is impoſſible they ſhould know how to make any Improvement. And ſome, after they have got ſafe into their Port, are but too apt to forget the Difficulties, as well as the Dangers, they might have been in, but a little while before; and are ſo far from giving any Encouragement to any Helps which might be offered them, that they readily ſlight all things (without ever conſidering their Uſefulneſs) rather than go out of their old way, or be thought to ſtand in need of any farther Information: as if they were afraid that the owning themſelves in an Error, might be attended with ſome great Inconvenience to themſelves. Now theſe People moſt certainly judge and act wrong; for who knows not that the leaſt Improvement in Navigation, may be the means of ſaving not only the Ship and Cargoe, but alſo what is more valuable, the Lives of many People.

But although ſome (who labour under the misfortune of being but ſlightly grounded in the Nature of things) may be tenacious of the Methods they have been taught and uſed to, tho' ever ſo erroneous and abſurd; yet there is no doubt but that there are others who will as generouſly forſake any old Cuſtom when the Error of it is ſhewed, and gladly receive and make uſe of any Improvements which may be to their advantage.

In the following Treatiſe I have endeavoured to clear the Theory *of* Navigation *from ſuch Errors as from time to time had crept into it; and alſo to make ſuch Improvements in the* practical Part, *as I found by Experience would be of great help towards bringing this* Art *to a greater Certainty and Perfection. I hope what I have here done may anſwer the Expectation of my* Subſcribers, *to whom I beg leave to return thanks for their Favours: And if any thing uſeful has been herein advanced, thoſe who may more immediately reap the Benefit thereof, are no leſs obliged to them, who gave Encouragement to make it publick.*

The Particulars contained in this Book, may be readily found in the Table *of* Contents; *but that the Reader may be the better led into the Nature of the Deſign, I ſhall here give a ſhort Account of the Scheme in general.*

1. *The* INTRODUCTION *contains ſome few* Geometrical Problems, *together with a ſuccinct Treatiſe of* Plain Trigonometry. *To treat fully*

of Spherical Trigonometry, *would have exceeded my Deſign; and I thought it entirely uſeleſs to give the practical Solution of* Spherical Triangles, *conſidered ſimply as ſuch, or the* Projection *of the* Sphere in Plano; *for theſe things (tho' uſeful in themſelves, yet) without a competent Skill in the* Theory, *do rather confound than improve the Learner: And that People ſhould pretend to illuſtrate things by others, which ſtand in as much need of Illuſtration, is certainly wrong. What I have here done by way of* Introduction, *was indeed chiefly for the ſake of complying with Cuſtom; for as* Trigonometry *is applicable to ſeveral Branches of the* Mathematicks, *as well as* Navigation, *it ought to be treated of in a diſtinct Volume; and I wiſh we had a complete thing of that kind to refer to, as occaſion requires.*

The common Method among our practical Writers of crowding ſo many things together, and treating of them by halves, without either Order or Connection, has been of no ſmall Detriment to young Beginners: For except People are thoroughly grounded in what they are about, they can never form a juſt Idea of it, and the farther they go on, the more they'll be confounded; and having ſpent a great deal of time in this manner, (to little purpoſe) they are aftewards loth to begin anew, or perhaps (which is worſt of all) think they need no farther Information. Beſides, People who learn things only by rote, loſe the great advantage that accrues from being uſed to Mathematical Reaſoning; and are obliged to ſpend more time in acquiring ſome few practical Rules, than otherwiſe they need, if they were but a little grounded in the Theory: *for a Theorem, when it is well underſtood, may be eaſily applied to ſubordinate Caſes, which to a practical Reader would ſeem both tedious and intricate.*

It may be here objected, that there are ſome who have neither Capacity or Opportunity for acquiring the Theory, *and yet may be taught a practical Rule ſo as to anſwer their purpoſe ſufficiently. 'Tis true indeed, ſome are not ſo quick as others: yet I believe there are none, if they were told the reaſon of a thing, but what would gain ſomething by it; and being a little uſed to thinking, they would improve by degrees, and perhaps at laſt make a tolerable Progreſs: At leaſt this might make them ſenſible of their Deficiency, and prevent them from falling into that Obſtinacy which we meet with in ſome People. The* Mathematicks *in itſelf is not near ſo hard as what moſt People think it is, the greateſt Difficulty in it being chiefly owing to that confuſed Method in which it is commonly* taught *and treated of: And as for* Seamen, *I am ſure they have time ſufficient (if they were but put into*

a Method) for making better Proficiency in the Mathematicks *than they commonly do; neither have any greater Opportunities, as well as greater Occaſion, for putting all Parts of it into practice. And I dare ſay, if People who go to Sea were but better grounded in the Mathematicks, Navigation would have been long ſince brought to a greater Perfection than it is now in. But to return to the Subject of the enſuing Treatiſe.*

The NAVIGATION *is divided into four* Chapters: *The firſt whereof treats of the* Theoretical Part; *concerning which I have been very particular, and I hope cleared it from very material Errors, advanced by ſome of the Writers upon this Subject. I do not find, that the Nature of the thing called* Departure *or* Meridional Diſtance, (*tho' it is concerned in every Day's Work, and of the utmoſt Importance in Navigation*) *has been ever ſhewed by any Author whatſoever; but all the Writers of Navigation that I have met with, have expreſſly taken it in a wrong Senſe: and this Miſtake naturally led them into ſeveral Errors, and muſt undoubtedly be one great Cauſe of the uncertainty of* Mariners Reckonings. *It was the ſaid Miſtake of the* Departure, *I take it, firſt gave Riſe to that erroneous Method of keeping Reckonings in* Meridional Diſtance, (*or by the Sum of the ſeveral* Departures) *by which Method it is impoſſible to find the Place of the Ship in any Voyage whatſoever. Nor is it ſtrange, that People following ſuch Methods ſhould be vaſtly out in their Reckonings, when we conſider that their* Meridional Diſtance *is nothing elſe but the* Sines *or* Tangents *of different Angles added together.*

As the Rules called Middle-Latitude Sailing, *are at the beſt but a ſort of Approximations, I think they ought to be entirely laid aſide; for Approximations of any kind ought not to be uſed, but when either we have no direct Rule for ſolving the Propoſition, or that by ſuch an Approximation we ſave the Expence of ſome trouble; neither of which is the Caſe before us: And it is undoubtedly highly abſurd, that People ſhould uſe a multiplicity of approximate Rules, when a few direct ones will adequately and truly ſolve all the Caſes of the Propoſition. It were to be wiſhed that the Notion of ſailing according to the* Plain Chart, (*or upon a Suppoſition that the Earth is a Plain*) *was entirely baniſhed out of the World; but this we cannot hope for, ſo long as Youths are daily inſtructed in theſe Notions: Yet the Rules called* Plain Sailing *taken in a right Senſe may be ſafely retained, for they are ſtrictly true with relation to a ſingle Courſe, and an uſeful Expedient (ſufficiently exact) for finding the* Difference *of* Longitude *made in one Day's Run.*

The

The second Chapter *contains all the* Cases *in* Navigation *that arise in sailing upon a* Rhumb: *In the first* Section *whereof are delivered all the Rules relating to a single Course in three easy Problems; there being no longer any necessity for making the common Distinction of different Kinds of sailing. The Rules called* Middle-Latitude Sailing *I entirely rejected, for the Reasons already given; neither are the Rules called* Plain Sailing *considered according to the common Acceptations in which they have hitherto been taken.* Great-Circle Sailing *can be but of little use in* Navigation, *so long as Ships are steered by the* Compass; *and therefore it is here omitted.*

The Use of the true Sea-Chart, (*commonly called* Mercator's) *is the Subject of the second* Section; *which I have endeavoured to make as plain as possible: And I hope that the Method I have shewed for finding the* Bearings *of Places without the Confusion of* Rhumb-Lines, *will be thought an useful Improvement. Farther also, to invite People to make use of this* Projection, *I have shewed how all the Cases in Navigation may be easily wrought upon it. I thought it needless to give the Description of any other* Sea-Chart, *since that above-mentioned is the only one fit for* Mariners; *having this admirable Property, that the Bearings and Distances of Places may be exactly found upon it, which cannot be done upon any other Projection whatsoever. I have met with some People indeed who asserted, that they had found by Experience that the* Plain Chart *is truer than the* Mercator's; *which Assertion in plain* English *amounted to this, that they found by Experience, that the* Earth *is a* Plain, *and not a* Globe. *Experience indeed makes People ready in their common Practice, and is absolutely necessary to make a Man a good* Navigator, *especially with relation to the practical Management of a Ship: But with regard to keeping an Account of the Ship's Way,* Experience *alone avails but little; for excepting* Theory *be joined to it, a Man cannot tell whether he be in the right or in the wrong, for at Sea there is no other way of detecting Errors, than by examining into the Grounds of the Rules and Methods made use of. And it is a plain Case if Experience alone was sufficient, that* Mariners *would not have thus long persisted in so many Errors.*

The third Section *teaches the Method of reducing* Traverses, *or to find the Place of the Ship after she has sailed any number of Courses. And the fourth is about computing the* Bearings *and* Distances *of* Headlands, *&c.* (*which the Writers of Navigation call* Oblique Sailing) *and of which I think I have treated more full and distinct than has been done before.*

As

As the buſineſs of ſailing in Currents *is the hardeſt and moſt intricate Article in Navigation, I have been very particular upon that Head; and I think advanced ſome new Problems, which I hope may be very uſeful to Mariners.*

The third Chapter *contains the Solution of ſuch* Aſtronomical Problems *as are more immediately uſeful in* Navigation; *and particularly about finding the* Latitude, *together with a Deſcription of the* Inſtruments *now uſed at Sea for that purpoſe. And I cannot help taking notice in this place, that the common Method of ſlitting the* Horizon Vane *of the* Quadrant, *is a notable Inſtance of the great prevalency of Cuſtom even in the moſt abſurd things.*

The Places of moſt of the Fixed Stars *are now (from the Obſervations of Mr.* Flamſtead *and Dr.* Halley) *known to a very great Exactneſs; but this, how great an Improvement ſoever it might be in* Aſtronomy, *could be but of little uſe in* Navigation, *till ſuch time as we had ſome better and more certain Methods of taking the Altitudes of Stars at Sea. I hope therefore that my new* FORE-STAFF *will be thought an uſeful Improvement; and I may venture to ſay, that by this Inſtrument the Latitude may be found at Sea in the Night, to above twice the Exactneſs of what could be done by the common* Fore-Staff. *And as the Altitudes taken by theſe kind of Inſtruments muſt be under a certain Limitation, and People not ſo well skilled in* Aſtronomy, *are ſometimes at a loſs to find proper Stars for their purpoſe; I have contrived a* Table *of the Places of the moſt noted* Fixed Stars *in ſuch a manner, as to ſhew by Inſpection which is the propereſt Star for Obſervation in all Places, and at all Times of the Year: The Stars in the Heavens may be eaſily diſtinguiſhed one from the other, by my* Hemiſpheres *publiſhed by Mr.* Senex. *So that by theſe Helps the ingenious Mariner can never be at a loſs for finding his Latitude in any clear Night; which, how great an advantage that one Article alone may be in Navigation, thoſe who have had the* Sun *in their* Zenith, *or been ſailing to our Channel in dark Winter Days, may eaſily judge.*

The Table *of the* Sun's Declination *I have likewiſe ſo contrived, as to ſhew by Inſpection the allowance that ought to be made for the daily Difference, under different Meridians; which being obſerved, may ſometimes prevent a conſiderable Error in finding the Latitude.*

The Tables of Logarithms, Artificial Sines, *&c.* Meridional Parts, *and* Difference of Latitude *and* Departure, *being already in moſt Peoples hands, are here omitted. And indeed theſe* Tables *being in a manner of univerſal Uſe, ought to be either printed along with a perfect Treatiſe of* Trigonometry, *or elſe (which is more proper) to be had in a Volume by themſelves.*

The fourth and laſt Chapter *relates more immediately to the practical Rules and Methods uſed at Sea; concerning which I have been very particular, and I hope made ſeveral Improvements which will be of great advantage, if put into practice.*

As the common Sea-Compaſs *is the greateſt help for guiding the Ship in the immenſe Ocean, I have been very particular concerning it; and if People will have this Inſtrument made according to the Directions I have given, there is no doubt but That one Article alone will be of very great advantage in Navigation. What I have ſaid concerning the* Log-Line, *is no more than what had been done often enough before; tho' ſome People perſiſt to divide it erroneouſly even to this day: But I muſt acknowledge that the* Log *is at the beſt but a poor gueſs for eſtimating the Diſtance run; tho' it is undoubtedly the beſt that has hitherto been practiſed. If People will obſerve the Method I have ſhewed for finding the* Lee-Way, *I am ſure That will be one great help to them, towards keeping better Reckonings: And yet it were to be wiſhed we had ſtill ſome better Method for determining that important Article.*

The Knowledge of the Magnetick Variation *is undoubtedly of the greateſt Importance in* Navigation. *Yet it is very ſurprizing that no better Contrivance has been hitherto found, for obtaining it, than the common* Azimuth-Compaſs: *By which Inſtrument it is hardly practicable to obſerve within half a* Point, *notwithſtanding the pretended Accuracy of ſome People in obſerving the* Minutes *of a* Degree. *And whoever conſiders the Nature of the ſaid Inſtrument, and to what Exactneſs it is poſſible to obſerve by it, need not at all be ſurpized that the* Theory *of* Magnetiſm *appears ſo intricate and difficult.*

The new AZIMUTH-COMPASS *deſcribed in this Book, being ſo contrived as to be kept from being diſturbed by the yawing of the Ship, I preſume is no ſmall Improvement in Navigation: And I do not doubt but that the* Variation *may be found by this Inſtrument to a ſingle Degree, or leſs; which is more than ſufficiently exact for common Uſe, and would be of great help in* Eaſt-India Voyages *for eſtimating the* Longitude, *if the State of the* Variation *in the Southern*

Parts

Parts of the World was but better known. However, (as the Knowledge of the Variation is of that Importance, that without it the Courſe cannot be determined) if this Inſtrument be made uſe of, it will be of no ſmall help to all thoſe who go to Sea, and we may ſoon expect a better Account of the State of Magnetiſm *in all Parts of the World, than we have had hitherto.*

[But however this *Azimuth-Compaſs*, and the new *Fore-Staff* above-mentioned, may be approved of; I hope that none will offer to make or ſell them without my Privity and Conſent.]

In the laſt Section *the Rules delivered throughout the Book, are in a manner ſummed up, and applied to Practice; and I preſume that the Method I have ſhewed for keeping a* Sea-Journal, *is far more commodious, and leſs liable to Error, than any other hitherto practiſed.*

Thus have I given a ſhort Account of the enſuing Treatiſe, which I have endeavoured, according to the beſt of my Abilities, to render plain and uſeful to the ingenious Mariner. And I thought it incumbent upon me to take notice of every Particular which might be of uſe in Navigation, however trifling it might ſeem to be to others. My Deſign in the Whole being to treat of ſuch things as were of real Service, without any Oſtentation of entering into abſtruſe Matters, or treating of things barely ſpeculative; which ſome Writers are but too fond of, to the no ſmall Diſcouragement of Beginners, and People of ordinary Capacities: And as it is difficult to prevail upon People to leave any old Cuſtom, however erroneous; I have been ſometimes very plain in ſhewing the Abſurdities of ſome things; which as they are general, and do not concern any particular Perſons, I hope none have reaſon to be offended at, eſpecially as detecting of theſe Errors could be of no perſonal Advantage to me.

To conclude, I wiſh that what I have here offered may be throughly conſidered, and put into practice; and then there will be no room to doubt but that Mariners will find a much greater Certainty in their Reckonings than hitherto.

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ERRATA.

Pag. 13. Line 13. for 6 read 60. P. 20. the laſt Line but one, for *CF* read *EF*.
Pag. 54. Line 33. for *AC*, *BD* read *AD*, *BC*.
Pag. 96. Line 24, 25. read *in* 30———*to* $4\frac{1}{3}$—$34\frac{1}{3}$—$134\frac{1}{3}$ *or* 403.
Pag. 126. Line 17. for *NNW* read *NNE*. P. 127. bottom Line, for *NE* read *SE*.

A LIST of the Subſcribers Names.

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Capt. Samuel Atkins.

B.

Lewis Barlow *Eſq*;
Hugh Barlow *Eſq*;
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Mrs. Martha Blunt.
Thomas Boucher *Eſq*;
The Rev. Mr. Bowen.
Dodding Braddyll *Eſq*;
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Tracy Catchmay *Eſq*;
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Mr. Charles Chevely.
Walter Churchy *Eſq*;
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Edward Corbett *Eſq*;
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Richard Eliott *Eſq*; 2 Books.
Mrs. Elizabeth Eliott, 4 Books.
William Eliott *Eſq*;
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F.

William Forteſcue *Eſq*; *Sollicitor-General to the Prince of* Wales.

G.

The Right Honourable Francis *Earl of* Godolphin.
Thomas Goodman, *M.D.*
Mr. John Green.
Roderick Gwynne *Eſq*;
Marmaduke Gwynne *Eſq*; *of* Garth, Com. Brecon.

H.

Dr. Edmond Halley, *Aſtron. Reg. & R. S. S.*
John Hanbury, *jun. Eſq*;
Joſeph Harris *Eſq*;
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Jonathan Holloway *Eſq*;
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THE INTRODUCTION:

CONTAINING

Some few GEOMETRICAL PROPOSITIONS, chiefly relating to *Plain Triangles*.

CHAP. I.

SECT. I.

Geometrical Definitions.

THE whole Subject of *Geometry* is concerning *Magnitudes*; of which there are three Kinds, or Species, *viz.* 1. A *Line*, which is only Length, without either Breadth or Thickneſs. 2. A *Superficies*, having Length and Breadth, but no Thickneſs. 3. A *Solid*, having Length, Breadth, and Thickneſs.

[Note, *All the Figures referred to in this* Chapter, *are in Plate* 1.]

1. A RIGHT LINE is the neareſt Diſtance between two Points: Or, That which lies evenly betwixt its terminating Points, as the Line CD, *Fig.* 1.

2. A PLAIN SUPERFICIES is that which lies evenly betwixt its Extreams, (as a ſmooth Table, or poliſhed Glaſs, *&c.*)

3. A CIRCLE is a *Plain Figure* ordinarily deſcribed by a Right Line taken within a Pair of Compaſſes, one Extream thereof being kept fixed while the other is turned quite round to the Place where the Motion firſt began: The *Fixed Point* is called the CENTER; and *the Line deſcribed by the other Extream*, is called the CIRCUMFERENCE, or the PERIPHERY.

4. The RADIUS of any Circle is the *Right Line* wherewith it is conceived to be deſcribed: Or, the *Radius* is any Right Line drawn from the Center to the Circumference, as the Lines CA, CD, CS, *&c.* *Fig.* 4.

5. The DIAMETER is a *Right Line* drawn through the Center, and continued on both Sides to the Circumference; thereby dividing the Circle into two equal Parts, as the Lines AB, and DE.

6. A *Semicircle* is a *Figure* contained under the Diameter AB, and that Part of the Periphery ADB, or AEB, which is cut off by the Diameter.

7. A QUADRANT is half a Semicircle, or one fourth Part of the whole Circle, as the Fig. ACD, or DCB, *&c.*

8. An ARCH of a Circle is any *Part* or *Portion* of the *Periphery*, as ADS, AEBS, *&c.*

All Circles, however large or ſmall, are commonly underſtood to have their *Peripheries* divided into 360 equal Parts, called *Degrees*; and each Degree is either actually, or imaginarily ſubdivided into 60 equal Parts, called *Minutes*; each Minute into 60 equal Parts, called *Seconds*, and ſo on ſexageſimally into *Thirds*, *Fourths*, &c.

9. A *Plain* ANGLE is made by the Concourſe of two Right Lines lying upon the ſame Plain, ſo as not to make one Right Line: The Point where the Lines meet, is called the *Angular Point*; and the two Lines which include the Angle, are called the *Legs*, or the *Containing Sides* of the ſaid Angle.

The *Meaſure* of an *Angle* is an Arch of a Circle deſcribed from the Angular Point, and which is intercepted betwixt its *Legs*; ſo the Arch AB, is the Meaſure of the Angle made by the two Lines CA and CB; whence an Angle is eſteemed greater or leſſer, according to the Aperture of its Legs, or the quantity of the Arch intercepted between them: And if the Legs CA, CB, be made either longer or ſhorter, the Angle between them will ſtill remain the ſame; for the Arches AB, *ab*, *ab*, do all contain the ſame Number of Degrees. *Fig.* 1.

Note, *An Angle is uſually expreſſed with three Letters, whereof the middlemoſt denotes the Angle, as in* Fig. 2. ACE *denotes the Angle made by the Lines* CA, CE: *But when there are but two Lines meeting in the ſame Point, it may be expreſſed by a ſingle Letter, as the Angle* C, Fig. 1.

All Angles are either, *Right*, *Acute*, or *Obtuſe*.

10. A *Right Angle*, is that which is equal to a Quadrant, or 90 Degrees, as ACD, or DCB, *Fig.* 2.

11. An {*Acute* / *Obtuſe*} *Angle* is that which is {Leſſer / Greater} than a Right Angle, as {ACE, / ECB,} *Fig.* 2.

Note,

Note, *Both* Acute *and* Obtuſe *Angles are in general Terms called* Oblique Angles.

Note, *When a Circle, or Part of it is mentioned with regard to any Angle, there is meant only its Periphery, or an Arch thereof; as in the* 10*th* Definition *by a Quadrant is meant only* $\frac{1}{4}$ *Part of the Periphery of the Circle, or* 90 *Degrees, and not the* $\frac{1}{4}$ *Part of its Area or Superficies.*

A Line in reſpect of its Poſition with another may be either *Parallel*, *Perpendicular*, or *Oblique* to it.

12. *Parallel Lines*, are ſuch as are every where equidiſtant from each other, and if infinitely produced would neither approach to, or recede from one another, as the Lines *ab*, *cd*. *Fig.* 2.

13. A *Line* is ſaid to be *Perpendicular* to another, when it falls upon it, ſo as to make the Angles on each Side equal: Thus if the Angles A C D, D C B, are equal, the Line D C is Perpendicular to A B; alſo the Lines A C, B C are each of them perpendicular to D C. *Fig.* 2.

Lines are ſaid to be Oblique when they are neither parallel nor perpendicular to one another.

14. A *Plain Triangle* is a Figure comprehended under three Right Lines called Sides.

Triangles are denominated either by their Sides or Angles.

1. *By their Sides.*

15. If a Triangle hath { all its / only two / all three } Sides { equal / unequal } it is called { an *Equilateral* / an *Iſoſceles* / a *Scalene* } Triangle, as { A. / B. / C.D. } Fig. 3.

2. *By their Angles.*

16. If a Triangle hath { one Angle { Right / Obtuſe } / all its Angles Acute } it is called { a *Right* / an *Obtuſe* / an *Acute* } Angled Triangle, as { C. / D. / A.B. } Fig. 3.

Note, *All Triangles that are not right-angled, whether they are Acute or Obtuſe-angled, are in general Terms called* Oblique Triangles, *without any other Diſtinction.*

Note, *In a Right-Angled Triangle, as Fig.* C. *N°.* 3. *the Side* B C, *oppoſite to the Right Angle, is called the* Hypothenuſe; B A, *the* Baſe; *and* C A *the* Cathetus *or* Perpendicular.

N. B. *The following Definitions are all referred to, Fig.* 4.

17. A Chord is a Right Line joining the Extremities of an Arch: So the Right Line S F, is the *Chord* of the *Arches* S B F and S A F, alſo A S is the *Chord* of the *Arches* A D S and A E B S.

18. A Right Sine is a Right Line drawn from one Extream of an Arch perpendicular to the Diameter paſſing through the other Extream; or, it is half the Chord of twice the Arch: Thus S V is the *Right Sine* of the *Arches* S B and S D A. Hence it appears that the Sine of a Quadrant or 90 Degrees,

grees, (which is equal to the Radius) is the greateſt of all Sines; the Sine of an Arch greater than a Quadrant, being leſs than the Radius.

19. A VERSED SINE is a Part of the Diameter intercepted between one Extream of an Arch, and the Right Sine falling upon it from the other Extream; ſo V B is the *Verſed Sine* of the Arch S B, and V A the *Verſed Sine* of the Arch S D A.

20. A TANGENT of an *Arch* is a Right Line drawn perpendicular from the end of the Diameter paſſing through one Extream of the Arch, and ſo continued until it meets with the Radius produced thro' the other Extream: Thus B T is the *Tangent* of the *Arches* B S and A D S.

21. A SECANT of an *Arch* is a Right Line drawn from the Center thro' one Extream of the Arch, and continued until it meet the *Tangent* of the ſaid Arch: Thus C T is the *Secant* of the *Arches* B S, and A D S.

22. The *Difference* of an Arch from a *Quadrant*, whether it be greater or leſs than a *Quadrant*, is called its COMPLEMENT: So D S is the *Complement* of the Arches S B, S D A; whence R S is the *Sine Complement* of the ſaid Arches S B, S D A, (becauſe it is the Sine of D S) alſo D G is the *Tangent Complement*, and C G the *Secant Complement* of the ſame Arches, S B, S D A; which for Brevity's ſake are uſually expreſſed, *Co-Sine*, *Co-Tangent*, and *Co-Secant*.

23. The Difference of an Arch from a Semicircle is commonly called its *Supplement*. Hence, and from *Def.* 18, 20, and 21, every Arch and its Supplement have the ſame *Right Sine*, *Tangent*, and *Secant*, common to them both. But,

The VERSED SINE of an Arch {Greater / Leſſer} than a Quadrant, is equal to the {Sum / Differ.} of the Radius and Co-Sine of the Arch: So A C and C V (which is equal to R S the Co-Sine of S B, S A) together are equal to A V, the *Verſed Sine* of the Arch S D A; and C B leſſened by C V is equal to V B, the *Verſed Sine* of the Arch S B.

An Explanation of ſome few Symbols, which for Brevity's ſake are ſometimes uſed in the enſuing Treatiſe.

Symbols.	*Significations,*
=	Equality, *as* A=B *ſignifies that* A *and* B *are equal.*
+	Addition, *as* A+B *ſignifies that* B *muſt be added to* A.
−	Subſtraction, *as* A−B *ſignifies that* B *is to be ſubſtracted from* A.
::	Proportion; *or that commonly called the* Rule of Three, *and* : : *is always placed betwixt the two middle Terms in Proportion. Thus* A : B : : C : D, *is to be read, as* A *is to* B, *ſo is* C *to* D.

Symb.	*Significations.*
△	*A* Triangle.
∠	*An* Angle.
R.	Radius.
S.	Sine.
C. S.	Co-Sine.
T.	Tangent.
C. T.	Co-Tangent.
ſ.	Secant.
C. *ſ.*	Co-Secant.
Z ∠ˢ	Sum of the Angles.

Symb.

Symb.	*Significations.*	*Symb.*	*Significations.*
q	Square; *as* A*q* *ſignifies the Square of* A.	X∠^{s}	Difference of the Angles.
$\sqrt{^{2}}$	Square Root, *as* $\sqrt{A^{2}}$ *ſignifies the ſquare Root of* A.	⊥	A Perpendicular.

° ′ being put over any Number, do ſignify Degrees and Minutes, as 24° 30′, is 24 Degrees and 30 Minutes.

SECT. II.

Geometrical Problems; the Conſtruction of the Plain Scale, and its Uſe in protracting and meaſuring of Triangles.

Prob. I. *Upon the Middle of a given Line* AB, *to raiſe a Perpendicular* CD. *Fig.* 5.

TAKE with a Pair of Compaſſes any Extent greater than half the given Line, and from the Extreams A and B deſcribe two ſmall Arches, ſo as to interſect each other both above and below the Line, as at D and E; then a right Line drawn by the Side of a Ruler laid over the Points where the Arches interſect, from D to C, is the *Perpendicular* required. By this Problem any finite Right Line, as A B, is divided into two equal Parts in C. In like manner a Circle is divided into four equal Parts; thus, having drawn the Diameter A B through the Center C, draw the other Diameter D E, croſſing the former at Right Angles (as above) and the Circle is divided into four equal Parts, as was required. *Fig.* 4.

To *erect* a *Perpendicular* from any Point as C, in a given Line A B; take any Diſtance, and lay the ſame on each ſide the given Point C, as to *a* and *b*, and from theſe Points deſcribe two ſmall Arches interſecting each other, as at D, and draw D C, which is the *Perpendicular* required. *Fig.* 5.

Prob. II. *To raiſe a Perpendicular* C A, *upon the End of a given Line* B A.

With any extent A *a* deſcribe an Arch *a d*, then lay off the ſame extent from *a* to *b*, and from *b* to *d*; from the Points *b* and *d*, deſcribe two ſmall Arches interſecting each other at C, and through the Point of interſection draw the right Line C A, which will be the *Perpendicular* required. *Fig.* 6.

Or if you can't go ſo far as *d*, lay a Ruler over the Points *a* and *b*, and mark where it interſects the Arch which was deſcrib'd from *b*, and thro' that Interſection draw the Line C A as before.

Or take any diſtance and lay it ſomewhere from A above the given Line, as to *b*, and from the Point *b* as a Center, deſcribe an Arch greater than a Semicircle as *a* A C; then thro' the Center *b* and the Point *a* (where the *Fig.* 7.

ſaid

ſaid Arch croſſes the given Line) draw a right Line *a* C until it croſſes the Arch in C; a Line C A drawn thro' the Point of interſection C, is the Perpendicular required. This Method may be readily put in practice by a Ruler and a Pair of Compaſſes, without deſcribing any Arch; for, having taken any Extent, and laid it ſomewhere from A to *b*, keep fix'd that Foot which is in *b*, and turn the other about until it falls upon the given Line, as at *a*; the Compaſſes reſting in this Poſition, apply a Ruler cloſe to its Legs, and keep it fix'd while you turn the Foot which is in *a*, until it touches the Ruler in C; a Line drawn from C to A, will be the *Perpendicular* required, as before.

Fig. 8. Prob. III. *From a Point given* C, *to let fall a Perpendicular upon a given Line* B D.

From C deſcribe an Arch that ſhall cut the given Line in two Places, as in *a* and *b*; then from the Points *a*, *b*, deſcribe two Arches interſecting each other in *d*: a Ruler being laid from *c* to *d*, by the Edge thereof draw the right Line C D, which will be the *Perpendicular* required.

Fig. 7. Or, from the Point given *c*, draw any right Line C *a*, which biſect in *b*, then with the extent *b c* or *b a* deſcribe an Arch interſecting the given Line B D as in A; then C A is the *Perpendicular* required.

Fig. 9. Prob. IV. *Through a Point given* C, *to draw a right Line Parallel to another given Line* AB.

From the Point given C, take the neareſt Diſtance (as C *a*) to the given Line A B, then with that extent from ſome Point, as *b*, near the End of the given Line, deſcribe an Arch; a right Line C D drawn thro' the Point C, ſo as to touch that Arch, will be the *Parallel Line* required.

Or, from the given Point C, with any extent C D deſcribe an Arch; then from ſome Point, as *a*, taken in the given Line A B, lay off the ſame Extent from *a* to *b*; take the Diſtance *a* C, and with that Extent from *b*, deſcribe an Arch interſecting the former as in D; a right Line C D drawn through that Interſection and the Point given C, will be the *Parallel Line* required.

Fig. 10. Prob. V. *Having three right Lines* B C, A C, A B *given* (whereof it is neceſſary that any two be greater than the third) *to conſtitute a Triangle.*

Take either of them, as A B, and from the Extreams thereof A and B, with the other given Lines deſcribe two Arches interſecting each other in C, and join A C and B C; then is A B C the *Triangle* required.

If all the three Right Lines had been equal, the Triangle would have been *Equilateral*; or, if two only had been equal, it would have been an *Iſoſceles Triangle.*

Hence,

Hence, it is evident how an *Equilateral Triangle* may be made upon any given Line; and alſo how to make an *Iſoſceles Triangle*, whoſe *Baſe* and *Legs* ſhall be equal to two given Lines.

Prob. VI. *To make a rectangled Parallelogram* AD, *whoſe Sides ſhall be equal to two given Lines* A, B. Fig. 11.

Note, A *Parallelogram* is a Quadrilateral Figure, whoſe oppoſite Sides are Parallel, and likewiſe its oppoſite Sides and oppoſite Angles equal.

If all the Sides are equal and its Angles { *Right* / *Oblique* } it is called a { *Square* / *Rhombus* }

But if it has only two Sides equal, if the Angles are { *Right* / *Oblique* } it is called a { *Rectangle* or *Parallelogram.* / *Rhomboides.* }

Having drawn the Line AB equal to the given one B, erect the ∠ AC=A, then from the Point B with the length AB, deſcribe an Arch; alſo from C with the length AB, deſcribe another Arch, interſecting the former in D, and draw the Lines CD and BD; then is the Figure AD the *Parallelogram* required.

Prob. VII. *To conſtruct the Lines of Chords, Sines, Tangents, Secants, and Semitangents.*

1. With the *Radius* you intend for your *Scale*, deſcribe a Semicircle ADBC, and upon the Center C raiſe the Perpendicular CD (which will divide the Semicircle into two Quadrants AD, BD;) continue CD directly to S, and upon B raiſe the Perpendicular BT; then draw the right Lines BD and AD. Fig. 13.

2. Divide the Quadrant BD into 9 equal Parts, then will each of theſe Diviſions be 10 Degrees. Again, you may ſubdivide each of theſe Parts into ſingle Degrees, and theſe again, if your Radius will admit of it, into Minutes, or elſe into ſome aliquot parts of a Degree greater than Minutes.

3. Set one Foot of the Compaſſes in B, and transfer each of the Diviſions in the Quadrant BD, to the Right Line BD; then is BD a *Line* of CHORDS.

4. From the Points 10, 20, 30, *&c.* in the Quadrant BD, draw Right Lines parallel to CD till they cut the Radius CB; then is the Line CB divided into a *Line* of SINES; which muſt be numbred from C towards B.

5. If the ſame Line of *Right Sines* be numbred from B towards C, it will become a *Line* of VERSED SINES; which may be continued to 180 Degrees, if the ſame Diviſions be transferred on the other Side of the Centre C.

6. From the Center C, through the ſeveral Diviſions in the Quadrant BD draw right Lines till they cut the *Tangent* BT; ſo will the Line BT become a *Line* of TANGENTS.

7. Setting one Foot of the Compaſſes in C, extend the other to the ſeveral Diviſions 10, 20, 30, *&c.* in the *Tangent Line* BT, and transfer theſe Extents

Extents ſeverally into the Right Line C S; then will the Line C S be a Line of SECANTS.

8. *Right Lines* drawn from A to the ſeveral Diviſions 10, 20, 30, *&c.* in the Quadrant B D, will divide the Radius C D, into a *Line* of SEMI-TANGENTS.

9. Divide the Quadrant A D into 8 equal Parts, and from A tranſfer theſe Diviſions ſeverally into the Line A D; then is A D a *Line* of *Rhumbs*; each Diviſion therein anſwering to 11° 15′ upon the *Line* of *Chords*. The uſe of this Line is for protracting and meaſuring of Angles, according to the common Diviſion of the *Mariners Compaſs*.

If the Radius A C be divided into 100, or 1000, *&c.* equal Parts, and the Lengths of the ſeveral *Sines*, *Tangents* and *Secants*, correſponding to the ſeveral Arches of the Quadrant, be meaſured thereby; and theſe Numbers be ſet down in a Table, each in its proper Column; you will by this means have a *Triangular Canon* of Numbers, by which the ſeveral Caſes in *Trigonometry* may be reſolved.

The Right Lines graduated as above, being placed ſeverally upon a Ruler, as in *Fig.* 14. do form the *Inſtrument* called the PLAIN-SCALE; by which the Sides and Angles of all Triangles may be readily meaſured.

All Right Lines (as the Sides of Plain Triangles, *&c.* when they are conſidered ſimply as ſuch, without having any relation to a Circle) are meaſured by *Scales* of equal Parts; one of which Parts is ſubdivided equally into 10, and this ſerves as a common Diviſion for all the reſt. In moſt Scales an Inch is taken for a common Meaſure to determine their Largeneſs; and the Number of Parts that an Inch is divided into, is generally ſet at the End of the Scale: As in the *Scales* A, B, and C, the Numbers 12, 20, 30, do ſhew that ſo many Parts of the Scales A, B, C, are contained in an Inch. *Fig.* 15.

By any Scale of equal Parts, divided as above, any Number, leſs than 100 may be readily taken; but if the Number ſhould conſiſt of three Places of Figures, the Value of the third Figure can be only gueſs'd at; wherefore in theſe Caſes, it is beſt to uſe ſuch a Scale as D, called a *Diagonal Scale*, by which any Number of three Figures may be exactly found.

Having prepared a Ruler of a convenient Breadth for your Scale, (which may be an Inch more or leſs) firſt, near the Edges thereof, draw two Right Lines *a f*, *c g*, parallel to each other; then divide one of theſe Lines, as *a f*, into equal Parts, according to the Largeneſs you intend your Scale, and thro' each of theſe Diviſions draw perpendicular Right Lines, as far as the Line *c g*; next divide the Breadth *a c* into 10 equal Parts, and thro' each of theſe Diviſions draw Right Lines, parallel to the former *a f*, *c g*. Again, divide the Lengths *a b*, *c d*, each into 10 equal Parts, and from the Point *d*, to the firſt Diviſion in the Line *a b*, draw a Right Line; then parallel to that Line, draw Right Lines thro' all the other Diviſions, and the *Scale* is done.

Of the Lines upon the Plain Scale we need only the *Line* of *Chords*, and that of *Equal Parts*, for Meaſuring the Sides and Angles of Plain Tri-

angles: How to uſe theſe ſhall be ſhewed in the two following Problems.

Prob. VIII. *To lay off, or to meaſure an Angle by the Line of* Chords.

1. Let it be required to make an ∠B A C, with a given Line A B, of any number of Degrees; ſuppoſe 40 Deg. With the Chord of 60 Degrees (A *a*) from the Angular Point A, deſcribe an Arch A *c*; then from the ſame Line of Chords take 40 Degrees (the number given) and lay it from *a* to *b*, and through the Point *b* draw the Right Line A C; then is B A C the *Angle* required. *Fig.* 12.

2. If the Angle B A C be given to be meaſured; With the Chord of 60 Degrees from the Angular Point A deſcribe an Arch as before; then take the Length of the Arch *a b*, intercepted betwixt the Legs A B, A C, and apply it to the ſame Line of *Chords*, ſetting one Foot of the Compaſſes in the beginning; and the Point where the other falls, gives the number of Degrees the Angle contains.

Note, *The Reaſon why the Arch muſt be always deſcribed with the* Chord *of 60 Degrees, is, becauſe that is the* Radius *of the Circle which the* Line *of* Chords *will divide into Degrees*, &c.

When the propoſed Angle is *Obtuſe*, if your Line of Chords reaches no farther than 90 Degrees, having deſcribed an Arch *a d*, as before, (large enough) the Angle muſt be laid off or meaſured at twice; let the Angle be 130 Degrees, ſet off the Chord of 60 from *a* to *b*, then take the Chord of 70 (the Remainder from 60 to 130) and lay it from *b* to *c*; then is B A C the Angle required.

In like manner you muſt have proceeded, if the ∠B A C had been given to be meaſured; thus, take ſome even number of Degrees, (large enough) *viz.* 60 or 90, and lay it from *a* upon the Arch *a d*, and from that Point whereon the Compaſſes falls, take the neareſt diſtance to the Point *c*, (where the Arch interſects the Line A C) and apply that extent to the *Line* of Chords, and ſee what number of Degrees it reaches to; then that Number added to the Number firſt laid off, gives the quantity of the ∠B A C required.

All Angles are moſt readily laid down, and meaſured by an Inſtrument called a *Protractor*: But the Deſcription and Uſe of a particular *Protractor* contrived for the Uſe of *Mariners*, ſhall be ſhewn in Chap. II. Sect. 2. of *Navigation*.

Prob. IX. *To lay down or to meaſure a Side (or a Right Line) of any number of Parts.*

1. Let there be an *Indefinite* Right Line A *b*, and it be required to lay off 456 Parts from A to *b*: Having fixed upon the *Scale*, by which you intend *Fig.* 12.

intend to protract the Figure, (for there is no matter which Scale it is, ſo that you uſe the ſame in meaſuring all the Sides of that Figure:) for Example, we'll here take the Scale 20; ſet one Foot of the Compaſſes in 40, (which in this Caſe ſtand for 400, and conſequently the ſmall Diviſions at the end are each of them 10) and extend the other (as near as you can eſtimate) to $\frac{6}{10}$ of the Diſtance between 5 and 6, and that Point is 56; and therefore the whole Extent between the Compaſſes is 456, which lay off from A to B; then is A B the Line required.

If the *Diagonal Scale* be uſed, this Number may be taken off to the greateſt Exactneſs; thus, ſet one Foot of the Compaſſes in 4 on the Side of the Scale, and carry it along that Line until it comes to the 6th parallel Line; then extend the Compaſſes from that Point, to the Point of interſection of that Parallel, and the Diagonal marked 5 at the top, (ſtanding in this Caſe for 50) and that Extent is 456, the Length required; which may be ſet off from A to B, as before.

If the Line A B be given, to find the number of Parts it contains, according to any given Scale; take the Length thereof between the Compaſſes, and apply that Extent to the Scale, ſo that one Foot being placed in one of the large Diviſions, the other may fall among the ſmall Diviſions: The Number contained between the Feet of the Compaſſes, ſhews the Length of the Line in parts of that Scale.

Or, if you uſe the *Diagonal Scale*, ſet one Foot of the Compaſſes in ſuch a one of the large Diviſions, that the other may fall upon the Diagonals at top; then carry both Feet parallel to the ſide of the Scale, until that which is at top interſects one of the Diagonals: And that Extent ſhews how many Parts (of that Scale) the Line contains (being rightly counted according to the above Directions.)

SECT. III.

The Uſe of the Logarithms, *of* Natural Numbers; *of the* Artificial Sines, Tangents, *and* Secants; *of the* Sliding Rule, *and* Gunter's Scale.

Logarithms are Numbers that differ *Arithmetically*, as the Numbers anſwering them differ *Geometrically*; and ſo perform Multiplication by Addition, and Diviſion by Subſtraction, *&c.*

It would be too tedious here, and alſo foreign to the Deſign of this Book, to ſhew the various Uſes of Logarithms, or the Methods of conſtructing them; I ſhall therefore only ſhew their Uſe in working a Proportion in the Rule of Three, which is ſufficient for our preſent Purpoſe.

1. *Of*

Plate 1.

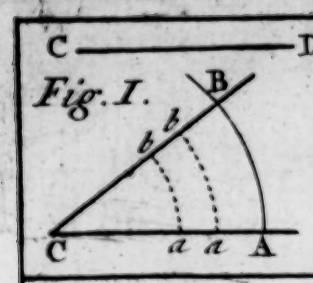


Fig. I.

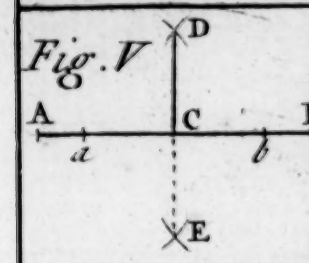


Fig. V

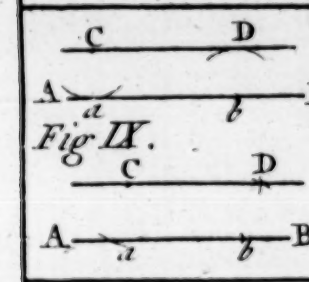
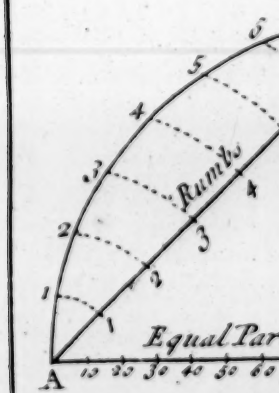
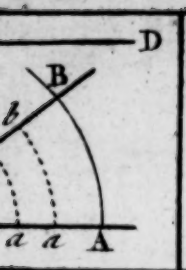
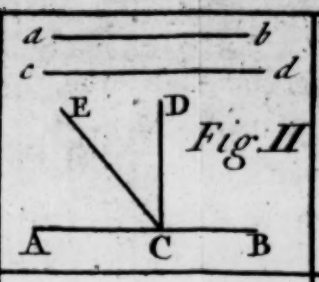


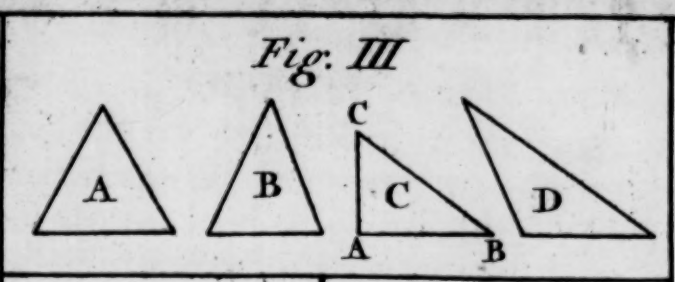
Fig. IX.

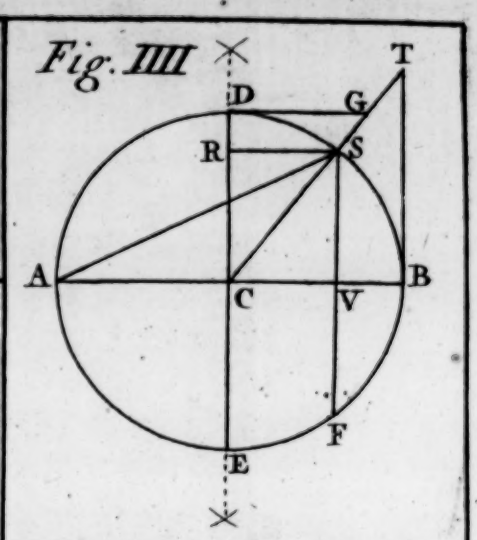
Fig. XIII

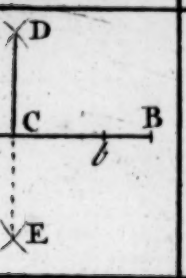


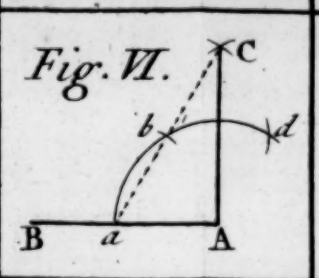

D
B
b
a a A

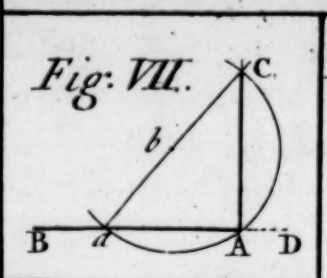

a
b
c
d
E
D
Fig. II
A
C
B

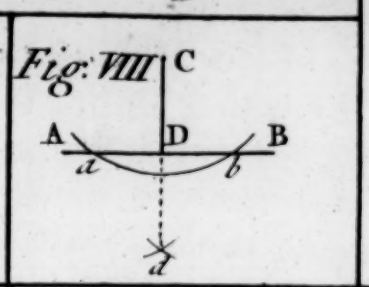

Fig. III
A
B
C
D
A
B

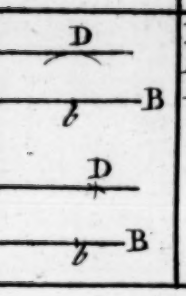

Fig. IIII
T
D
G
R
S
A
C
V
B
F
E

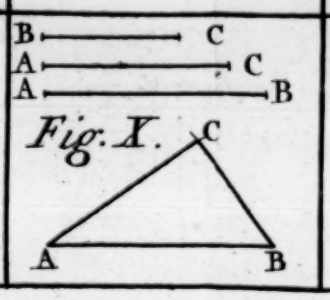

D
C
B
b
E

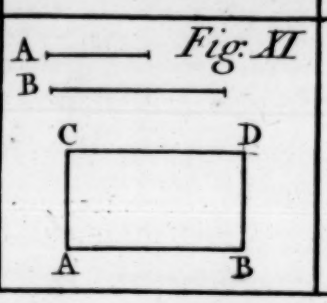

Fig. VI.
C
b
d
B
a
A

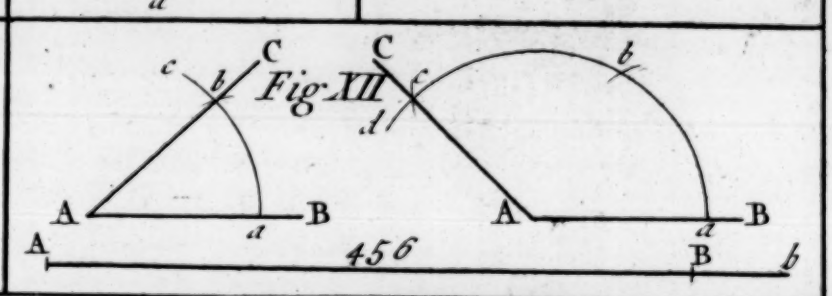

Fig. VII.
C
b
B
a
A
D


Fig. VIII
C
A
D
B
a
b
d


D
B
b
D
B
b


B
C
A
C
A
B
Fig. X.
C
A
B


A
Fig. XI
B
C
D
A
B


C
C
c
b
Fig. XII
c
b
d
A
B
a
A
B
a
A
456
B
b

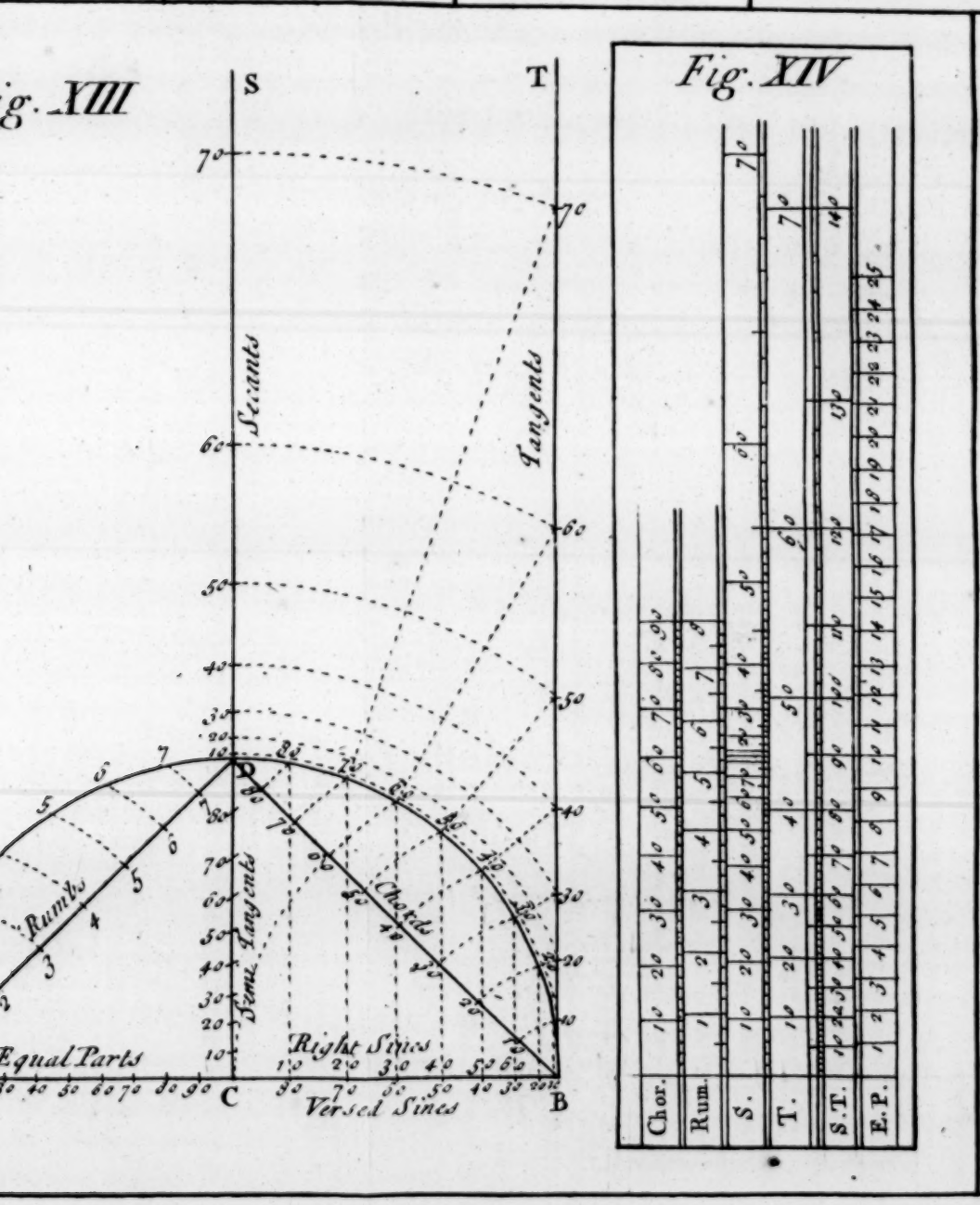
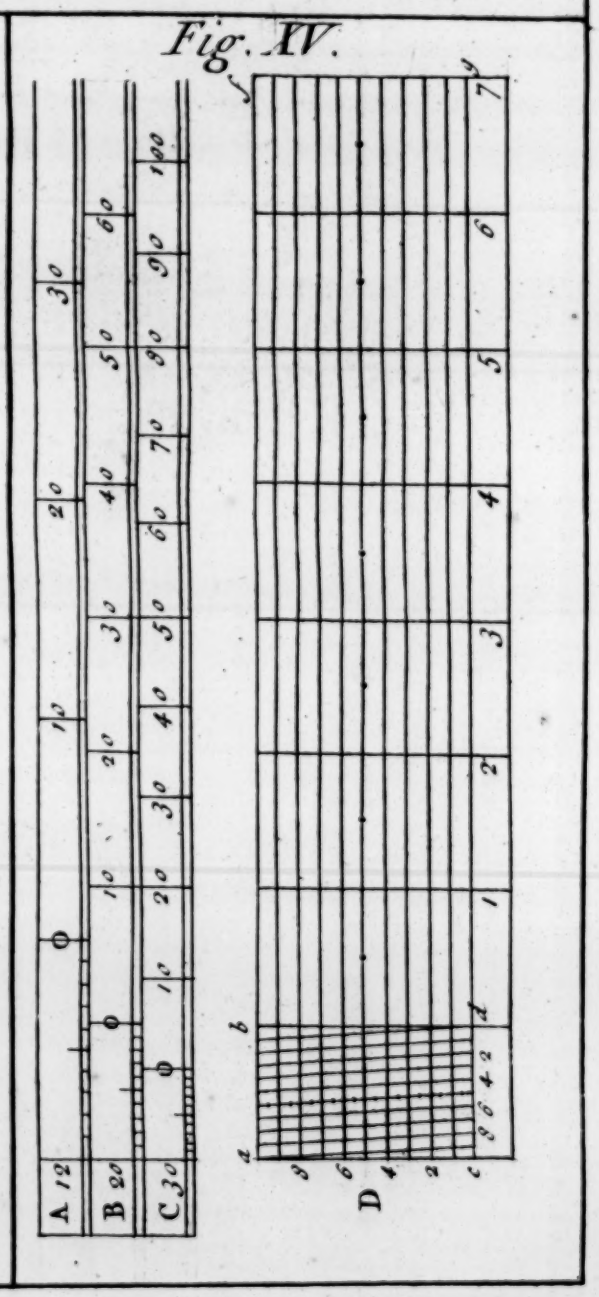

ig. XIII
S
T
7°
7°
Secants
Tangents
60
50
40
30
20
10
Rumbs
Semi Tangents
Chords
Equal Parts
Right Sines
Versed Sines
C
B

Fig. XIV
Chor.
Rum.
S.
T.
S.T.
E.P.


Fig. XV.
A 12
B 20
C 30
a
b
c
D

I. *Of the Logarithms of Natural Numbers, or Numbers increasing in their natural Order.*

Every *Logarithm* hath a *Characteristick*, or an *Index* belonging to it, and which is always an Unit less than the Number of Places in its correspondent absolute Number, thus;

The *Index* of { Units, Tens, Hundreds, Thousands, &c. } is { 0, 1, 2, 3, &c. }

As in this Examp.						
Indices	5	4	3	2	1	0
	|	|	|	|	|	|
Numbers	8	7	6	0	5	2

Here the Index of the first Figure 2 is 0, of 5 is 1, of 0 is 2, of 6 is 3, &c.

To find the Logarithm answering to any given Number.

The *Logarithmetick Tables* that are now mostly used, are made to reach from 1 to 10,000: The first Page generally contains the Logarithms of all whole Numbers less than 100; wherefore, if the given Number does not exceed two Places of Figures, you have the Logarithm placed right against it in the first Page of the Table, thus:

The Logarithm of { 6, 26 } is { 0.778151, 1.414973 } If the Number consists of three Places of Figures, look for it in the left-hand Column of the Table, under *Numb.* and the Logarithm is found right against it in the Column under 0; so the Logarithm of 246 is 2.390935, of 864 is 2.936514. If the Number consists of four Places, seek the three first Figures under *Numb.* as before, and the last Figure on the *Top*, under which lineally against the three first Figures, you have the Logarithm required. For Example, the Logarithm of 3456 is 3.538574; finding 345 under *Numb.* against it in the Column under 6, is the Logarithm above; so the Logarithm of 5672 is 3.753736; of 8608 is 3.934902, &c.

Note, *You must not regard the* Index *in the Table, but you are to consider the number of Places the given Number consists of, and make your* Index *accordingly; which must be always separated from the Logarithm by a Point, as in the Examples above.*

The Logarithm of a *Fraction* being a *Decimal*, is the same with that of an *Integer*, only the *Index* is *Negative*, and is so much less than 0, as the place of the Decimal is removed from Unity; and these *Indices* may be distinguished from the absolute Ones, by setting a *Negative Sign* over them:

As for Examp.

Indices	2	1	0	$\bar{9}$	$\bar{8}$	$\bar{7}$	$\bar{6}$	$\bar{5}$
	|	|	|	|	|	|	|	|
Numb.	6	7	6.	2	5	6	7	8

Here you see the *Indices* of the *absolute Numbers* are the same as was shew'd before; of the first place of Decimals the *Index* is $\bar{9}$, of the second place $\bar{8}$, of the third $\bar{7}$, &c.

Another Examp.

4568	3.659726
456.8	2.659726
45.68	1.659726
4.568	$0.\overline{6}59726$
.4568	$9.\overline{6}59726$
.04568	$8.\overline{6}59726$
.004568	$7.\overline{6}59726$

Here you see the Logarithm is the same, only the Index differs according to the Value of the left hand Figure of the given Number.

By the above Directions the *Logarithm* of any Number not exceeding 4 Places of Figures, may be readily found, to which must be subjoined its proper *Index*; which is sufficient for common Use.

II. *Of the Artificial Sines, Tangents, and Secants.*

The *Artificial Sines*, &c. are only the *Logarithms* of the *Natural Sines*, &c. What the *Natural Sines*, *Tangents* and *Secants* are, may be understood from what has been said of the Construction of the *Plain Scale*.

The *Artificial Sines*, &c. have their proper *Indices* prefixed to them in the Table, and which never alters, because Degrees and Minutes always retain the same Value. In the common Tables, every Page is divided into 8 Columns, the first and last of which contain the Minutes of every Degree, and the other six the Logarithmetical *Sines*, *Tangents*, and *Secants* of those Degrees and Minutes.

The {Left / Right} hand Column of each Page {under / over} *min.* at the {Head / Bottom} of the Table, contains the Minutes of those Degrees which are {less / greater} than 45 Degrees, and increase in their {Descent / Ascent} from 0 to 60 Minutes.

To find the Sine, Tangent, and Secant of any given Degree and Minute.

If the Degrees be {less / more} than 45°, they are found in those Columns which have the Words *Sine*, *Tangent*, *Secant*, at the {Head / Bottom} of the Table.

Examp. Let it be required to find the *Sine*, *Tangent* and *Secant* of 27° 35′. Look for 27 Degrees on the Top of the Table, and in the first Column of the right-hand Page, you'll find 35 under *min.* against which, and under *Sine* you'll find 9.665617, under *Tang.* 9.718017, and under *Secant* 10.052400; which are the *Log. Sine*, *Tangent* and *Secant* required. How to find the *Log. Sine*, &c. of any other Deg. and Min. is so plain and easy, that we need no other Example.

You may observe, that the Columns mark'd *Sine*, *Tangent*, *Secant*, at the Head and Bottom of the Table, are Complements to each other: Where-

fore, to find the *Co-Sine*, &c. of any Number of Degrees and Minutes, ſeek for the Place of the *Right Sine*, &c. as above, and right againſt it in the next *Col.* you have the *Co-Sine*, &c. required.

To Work a Proportion by the Logarithms.

Add the Logarithms of the ſecond and third Terms together, and from the Sum ſubſtract the Logar. of the firſt Term; the Remainder is the Logarithm of the fourth Term required.

Examp. 1. If 36 Yards, coſt 3 *l.* what will 60 Yards coſt?

		Logarithms.	
1ſt Term	36	1.556302	
2d Term	3	0.477121	
3d Term	6	1.778151	
		2.255272	The Sum of the Logarithms of the 2*d* and 3*d* Terms.
4th Term reqd.	5	0.698970	Logar. of the 4th Term. *Being the diff. of the Log. of the firſt, from the Sum of the Log. of the 2d and 3d Terms.*

Examp. 2. *As the Radius* —— 10.000000 *Here you ſee when the Radius is the 1ſt Term, you need only add the Log. of the 2d and 3d Terms together, and from the Sum cancel the left hand Fig. (which is the Rad.) and that is the Log. required.*

To the Sine of 36° 52′ — 9.778119
So is any Side of 90 *pts.* 1.954242
To a Side of 53.99 — 1.732361

Examp. 3. *As the Tangent of* 51° 10′ — 10.094215 *Here you may alſo ſet down the 4th Log. at once, by ſubſtracting the 1ſt Log. from the Log. of the 3d Term, (the Radius being underſtood to be added to it.)*

Is to the Radius —— 10.
So is 76.4 —— 1.883093
To 61.5 —— 1.788878

Examp. 4. *As the Co-Sine of* 38° 50′ — 9.891523

To the Sine of 38° 50′ — 9.797307
So is 76.4 —— 1.883093
11.680400
To 61.5 —— 1.788877

N. B. When

N. B. When the *Radius* is not concerned in the Operation, (as in the 1ſt and 4th Example) you may abbreviate the Work, by taking the *Arithmetical Complement* of the *Logarithm* of the firſt Term, or what it wants of 10.000000) and adding it to the Logarithms of the 2d and 3d Terms; the *Sum* abating the Radius, is the Logarithm of the 4th Term required.

Examp. 1.	36—$\bar{2}$.443698 *Arith. Comp.*	Exam. 4.	C.S. 38° 50′—$\bar{0}$.108477 *Ar. Co.*
	3—0.477121		S. 38 50—9.797307
	60—1.778151		76.4—1.883093
	5—0.698970 *the ſame as before.*		61.5—1.788877

The Arithmetical Complement of any Number is readily ſet down, by taking the remainder of each Figure to 9, only that of the right hand to 10; the reaſon is evident, becauſe you muſt borrow 1, after the firſt Subſtraction every time from 10.

III. *Of* Gunter's *Scale, and the Sliding Rule.*

The *Artificial Lines* placed upon theſe *Inſtruments*, are the ſame with the *Logarithms* above explained; and are laid down upon the Ruler, by a Scale of equal parts.

1. The *Line* of *Numbers*, commonly called *Gunter*'s *Line*, is divided, firſt into nine unequal Parts, called *Primes*, marked 1, 2, 3, *&c.* each of theſe Primes is ſubdivided into 10 unequal Parts, called *Tenths*; and each of theſe Tenths (either actually or imaginarily) into 10 other Parts, called *Centeſms*; and theſe Centeſms may be ſuppoſed to be divided into 10 other Parts, *&c.* but moſt commonly the actual Diviſions upon this Line are; *viz.* From {1, 2} to {2, 5} every Tenth is ſubdivided into {5, 2} Parts, each denoting {2, 5} *Centeſms.*

N. B. *The* {Tenths, Centeſms} *are always in Value* {10, 100} *times leſs than their* Primes.

2. The *Line* of *artificial Sines* is numbered unequally to 90°, and is commonly ſubdivided, thus; From {1, 10, 30} to {10, 30, 50} every *Degree* is divided into {6, 3, 2} Parts, each ſignifying {10, 20, 30} *Minutes.* And from {50, 70, 80} to {70, 80, 90} every Diviſion is {1, 2, 5} *Deg.*

3. The

3. The *Line* of *Tangents* is numbered unequally to 45°, and back again to 90°, thus; From {1, 10} to {10, 45} every *Degree* is ſubdivided into {6, 3} *Parts*, each ſignifying {10, 20} *Minutes*.

There is alſo uſually placed upon *Gunter's Scale*, a *Line* of *Verſed Sines*; but that we ſhall make no uſe of.

Note, *The Diviſions upon the Lines of* artificial Sines, *and* Tangents, *always retain the ſame Value; but thoſe upon the* Line *of* Numbers *do alter according to the Value of the firſt Figure, which being known, the proper Value of the reſt is eaſily known; thus, when* 1 *upon the beginning of the Line ſtands for* {1, 10, 100} *then the* 2, 3, 4, &c. *ſtand for* {2, 3, 4, &c. / 20, 30, 40, &c. / 200, 300, &c.} Units, *and* 1 *upon the middle ſtands for* {10, 100, 1000} *and the Numbers* 2, 3, &c. *upon the ſecond Line, ſtand for* {20, 30, / 200, 300, / 2000, 3000,} &c. *and the* 1 *upon the end ſtands for* {100, 1000, 10000}

Thus knowing the *Value* of any *Prime* upon the Line, the *Value* of the others is alſo known, and conſequently the *Value* of their *Tenths* and *Centeſms*.

To Work a Proportion by the Artificial Lines of Numbers, Sines, and Tangents.

Let it be required to work the four Examples before given, in the Uſe of the Logarithms.

1. *By Extenſion of the Compaſſes.*

Rule. Extend the Compaſſes either from the *firſt Term*, to the *ſecond*, and the ſame Extent will reach from the *third*, to the *fourth Term* required: Or, you may extend the Compaſſes from the *firſt Term* to the *third*, and the ſame Extent will reach from the *ſecond* to the *fourth Term* required.

As in Examp. 1. If you extend the Compaſſes from 36 to {3, 60} the ſame Extent will reach from {60, 3} to 5; which is the fourth *proportional Term* required.

Note, When *Numbers* and *Sines*, or *Numbers* and *Tangents* are concerned in the ſame Proportion; (as in the 2d, 3d, and 4th Examples) you muſt make

make that the *ſecond Term*, which falls upon the ſame Line with the *firſt*; then will the *third* and *fourth Terms* likewiſe fall upon the ſame Line.

Note, The *Radius* upon the *Line* of {*Sines* / *Tangents*} is {90 / 45} Degrees.

Examp. 2. Extend the Compaſſes from the *Radius* to 36° 52′ upon the *Line* of *Sines*; the ſame Extent will reach from 90 to 53.99 upon the Line of *Numbers*.

Examp. 3. Becauſe the *Tangent* of 51° 10′ is greater than the *Radius*, you muſt make the *Radius* the *firſt Term* in the Proportion, and extend the Compaſſes therefrom to the *Co-Tangent* of 51° 10′ (or which is the ſame, to the Tangent of 38°50′) the ſame Extent will reach from 76.4 to 61.5 upon the *Line* of *Numbers*, which is the 4th proportional *Term* required.

Note, (In the laſt Examp.) making the Radius the firſt Term, *&c.* was in effect changing the Proportion, by making another Side Radius; which how to do, ſhall be ſhewed in *Sect.* 2 of the next *Chapter*.

Note. If your ſecond Term be {upwards / downwards} from, or is {greater / leſs} than the firſt; then the fourth Term muſt be {greater / leſs} than, or lie {upwards / downwards} from the firſt.

Examp. 4. Extend the Compaſſes from the *Sine* of 51° 10′ (becauſe that is the C. S. of 38° 50′) to the *Sine* of 38° 50′, and that Extent will reach upon the *Line* of *Numbers*, from 76.4 to 61.5 the 4th Term required.

2. *By the Sliding Rule.*

Note, The *firſt* and *third Terms*, muſt be always taken upon the ſame *Plane*, then the *ſecond* and *fourth* will fall upon the other: that is, if you take the *firſt Term* upon the {*Slider* / *Ruler*} you muſt alſo take the *third* upon the {*Slider* / *Ruler*} then the *ſecond* and *fourth* will fall upon the {*Ruler* / *Slider*} right againſt the *firſt* and *third* Terms, upon the adverſe Planes.

Note, When *Numbers* and *Sines*, or *Numbers* and *Tangents* are concerned in the ſame Proportion, you muſt make that the *third Term*, which falls under the ſame Denomination with the *firſt*.

Note, When a Proportion is wholly in *Numbers*, or wholly in *Sines*; or partly in *Numbers* and partly in *Sines*, &c. the *Sliders* muſt be ſo diſpoſed that *Numbers* and *Numbers*, &c. or *Numbers* and *Sines*, &c. may come together.

Examp. 1. If you bring 36 upon the {Slider / Ruler} againſt 3 upon the {Ruler / Slider} then

then right againſt 60 upon the {Slider / Ruler} you'll find 5 upon the {Ruler / Slider} being the fourth proportional Term required.

Or, If you bring 36 upon the {Slider / Ruler} againſt 60 upon the {Ruler / Slider} againſt 3 upon the {Slider / Ruler} you'll find 5 upon the {Ruler / Slider} being the 4th proportional Term required.

Examp. 2. Bring the *Radius* upon the *Line* of *Sines*, againſt 90 upon the *Line* of *Numbers*; and againſt 36° 52′ upon the *Line* of *Sines*, you'll find 53.99 upon the *Line* of *Numbers*; being the fourth Term required.

Examp. 3. Here (for the Reaſons given before) you muſt alter the Proportion, by making the Radius the firſt Term, and taking the Co-Tang. of 51° 10′: Then, if you bring the *Radius* upon the *Line* of *Tangents*, againſt 76.4 upon the *Line* of *Numbers*; right againſt the *Tang.* of 38° 50′ (which is the C. T. of 51° 10′) you'll have 61.5 upon the *Line* of *Numbers*; the fourth proportional Term required.

Examp. 4. Bring the *Sine* of 51° 10′ (becauſe that is the *Co-Sine* of 38° 50′) to 76.4 upon the *Line* of *Numbers*; and right againſt the *Sine* of 38° 50′, you'll have 61.5 the *fourth Term* required.

The Precepts delivered in this Section, being underſtood; you'll be able to work any Proportion, that may happen in *Trigonometry*, both by the *Logar.* and the *Artificial Lines* above deſcribed.

CHAP. II.

Plain Trigonometry.

Plain Trigonometry *teaches how from any three given, of the Sides and Angles of a Plain Triangle (whereof one at leaſt being a Side,) to find the reſt.*

SECT. I.

Theorems, *Of the Primary Affections and Properties of Plain Triangles.*

[Note, *All the Figures referred to in this Chapter, are in Plate 2.*]

I. *IN every Triangle, the greateſt Side is oppoſite to the greateſt Angle: And on the contrary, the greateſt Angle is oppoſite to the greateſt Side.* Euclid. Lib. I. Prop. 18. and 19.

II. *In every Triangle, equal Sides ſubtend equal Angles; and on the contrary: whence all the Angles of an Equilateral Triangle, and alſo the Angles at the Baſe of an Iſoſceles Triangle, are equal.* Euclid. Lib. I. Prop. 5. and 6.

III. *In every Triangle, the three Angles taken together, are equal to two Right Angles, or* 180 *Degrees.* Euclid. Lib. I. Prop. 32. *Hence,*

4. If in a Triangle one Angle be Right, or Obtuſe, the reſt are Acute.

5. If in a Triangle one Angle be Right, the other two together make one Right Angle; alſo, that ∠ of a △ which is equal to the other two ∠ˢ, is itſelf a Right Angle.

6. In

6. In any Plain Triangle, if one Angle be known or given, the Sum of the other two Angles is alſo known by taking the Supplement, or the Remainder to 180 Degrees.

7. If in any Plain Triangle, two Angles be known (either ſeparately or conjunctly) the remaining Angle is alſo known.

8. If in a *Right-Angled* △, one of the Acute ∠ˢ be known or given, the other is alſo known, by taking the Complement (of the former to 90°.)

9. If in an *Iſoſceles* △ one ∠ be known, the reſt are alſo known; for if the ∠ contained by the equal Legs be given, then half the Supplement (or Remainder to 180°) is equal to each of the other two: Again, if one of the adjacent ∠ˢ be given, the other is alſo given, (as being equal to it) and the remaining ∠ is equal to the Supplement of the Sum (to 180°.)

X. *In any Right-Angled Triangle* B A C, *the Square of the Hypothenuſe* B C, *is equal to the Sum of the Squares of the two Sides* B A *and* A C. Euclid. 47. 1. *Fig.* 1.

Hence, Any two Sides of a Right-Angled Plain △ being known, the other may be found by Extraction of the Square Root: As for Example, let the two Sides B A=4, C A=3 be given, to find B C; firſt ſquare the two given Sides, and it will be B Aq=16, and C Aq=9; then 16 + 9=25, the Square Root whereof =5, is the Side B C required. Again, let the Hypothenuſe B C=5, and one of the Sides (as) C A=3, be given, to find the other Side B A; from the Square of the Hypothenuſe, *viz.* B Cq=25, ſubſtract C Aq=9, and the Remainder 16 is the Square of B A, the Square Root whereof =4, is B A required.

XI. *If to one Side* B C *of a* △ B A C, *be drawn a parallel right Line* D E, *the ſame ſhall cut the Sides of the* △ *porportionally: that is,* *Fig.* 2.

A D : D B :: A E : E C. Euclid 2. VI.

XII. *Equiangular* △ˢ *b c d*, B C D, (*that is, ſuch as have the three* ∠ˢ *of the one equal to the three* ∠ˢ *of the other, each to its reſpective* ∠; viz. *the* ∠ *b*=B, *d*=D, *and c*=C,) *have the Sides ſubtended by the equal* ∠ˢ *porportional: that is,* *Fig.* 3.

d c : D C :: *b c* : B C :: *b d* : B D. Euclid 4. VI.

XIII. *If a* △ B C D *be inſcribed in a Circle, every* ∠ *is half of the Arch oppoſite to it; as if the Arch* C D *is* 80 *Deg. then the* ∠ *at* B *is* 40 *Degrees.* Euclid 20. III. *Fig.* 4.

XIV. *If from the Vertex* D *of an Iſoſceles* △ B D C, *be let fall a* ⊥ D A *upon the Baſe* B C, *it ſhall divide the ſaid Baſe* B C *into two equal Parts*; viz. B A=A C. Euclid 26. I. *Fig.* 5.

Fig. 6. XV. *If from ſome Point* B *without a Circle, two right Lines* BD, BZ, *be drawn to cut the Circle; it will be* $\left\{\begin{matrix} BD : BZ :: BX : BO. \\ BO : BZ :: BX : BD. \end{matrix}\right\}$ *This is deduced from the* 36 *of the* IIId, *and the* 16 *of the* VIth Book *of* Euclid.

XVI. *Half the Sum of any two Quantities* $\left\{\begin{matrix} + \\ - \end{matrix}\right\}$ *half their difference, gives the* $\left\{\begin{matrix} greater \\ leſſer \end{matrix}\right\}$ *Quantity. Let the two Quantities be* 10 *and* 6, *then is their Sum* $= 16$, *and Difference* $= 4$, *and conſequently* $\frac{1}{2}$ *their Sum is* 8, *and* $\frac{1}{2}$ *their Difference* 2 : *Now* $8 \left\{\begin{matrix} + \\ - \end{matrix}\right\} 2$ *is* $= \left\{\begin{matrix} 10 \\ 6 \end{matrix}\right\}$ *the* $\left\{\begin{matrix} greater \\ leſſer \end{matrix}\right\}$ *Quantity.*

The following Theorems (by ſome called *Axioms*) being not expreſsly contained in *Euclid*, ſhall be here demonſtrated; and admitting the aforegoing, their Demonſtrations will be eaſily underſtood.

Fig. 7. XVII. *In every Right-Angled* $\triangle$ BAC, *any Side thereof may be made the* Radius *of a Circle, and then the other Sides will become either* Sines, Tangents, *or* Secants.

1. If the *Hypothenuſe* BC is made *Radius*, then are the *Sides* CA and BA the *Sines* of their oppoſite $\angle^s$; that is, CA is the *Sine* of the Arch CD, the Meaſure of the $\angle$ at B, (by *Def.* 18.) alſo BA is the *Sine* of the Arch BE, the Meaſure of the $\angle$ C. (By the ſame *Def.* 18.)

2. If one of the *Legs* (BA or CA) be made *Radius*, then ſhall the other *Leg* be the *Tangent* of the $\angle$ oppoſite to it, and the *Hypothenuſe* the *Secant* of the ſame $\angle$; that is, if BA be made *Radius*, then will CA be the *Tangent* of the $\angle$ B, and BC its *Secant*, (by *Def.* 20, 21.) Again, if CA is made *Radius*, then is BA the *Tangent* of the $\angle$ C, and BC the *Secant* of the ſame $\angle$, (by *Def.* 20, 21.) and conſequently (when CA is Rad.) BA is the *Co-Tang.* and BC the *Co-Secant* of the $\angle$ B.

Fig. 8. XVIII. *In all Plain Triangles, the Sides are proportional to the Sines of their oppoſite Angles: that is,*

In the $\triangle$ BCD, it will be BC : CD :: S.$\angle$D : S.$\angle$B.

This is evident from *Def.* 18. and *Theorem* 13. becauſe the Sines are half the Chords of the double Arches, and the $\angle^s$ ſubtended by the Sides of a $\triangle$ inſcribed in a Circle, are equal to half thoſe Arches. But this Theorem may be otherwiſe demonſtrated, thus;

Let the Side DC be produced to E, until DE be $=$BC, and let fall
[a] 18 *Def.* the $\perp^s$ EF, CA, [a] which will be the Sines of their oppoſite $\angle^s$ D and B, to the Radius DE$=$BC. Wherefore,

[b] 11 *Th.* DC : DE :: [b] CA : CF, that is
DC : BC :: S.$\angle$B : S.$\angle$D; which was to be proved.

XIX.

XIX. In all plain Triangles, it will be

As the Sum of any two Sides,
Is to their Difference;
So is the Tangent of half the Sum of the ∠s oppoſite to thoſe Sides,
To the Tangent of half their Difference: That is,

In the △ B C D, it will be, D C + C B : D C − C B :: T. ½ Z ∠s B + D : T. ½ their difference. *Fig.* 9.

Make C E = C B and draw B E, upon which let fall the ⊥ C A, [a] which will biſect B E in A: Draw A F parallel to B D, and it will [b] biſect E D (the Difference of the Legs) in F. Now becauſe in the △s B C E, B C D (having the ∠ C common) the Z ∠s C B E + C E B = [c] C B D + C D B, the ∠ C B E (= [d] C E B) = ½ Z ∠s C B D + C D B, and conſequently E B D is [e] = ½ their difference. Let C A be continued to G, then if B A be made Radius, C A will be the Tangent of [f] ½ Z ∠s oppoſite (*viz.* B + D) and A G the Tangent of [f] half their difference (*viz.* ∠ E B D.) Now becauſe A F is parallel to G D, it will be

[a] 14. *Th.* [b] 11. *Th.* [c] 3. *Th.* [d] 2. *Th.* [e] 16. *Th.* [f] 20. *Def.*

C F : F D :: [b] C A : A G; or,
2 C F : 2 F D :: C A : A G; that is,

As the Sum of the two Sides, is to their difference; ſo is the Tangent of ½ the Sum of the ∠s oppoſite to thoſe Sides, to the Tangent of ½ their difference. Which was to be proved.

Note, If a Side B D of a △ B D C, having the adjacent ∠s B and D acute, be made the Baſe; a ⊥ C A let fall from the Vertex C, will fall within the △, and divide the Baſe B D into two parts B A, A D, which are called the *Segments* of the *Baſe*; and if A O be made = A D, then will B O be the *Difference* of the *Segments*: But when the ⊥ C A falls without the △, the Line B O is called the *Alternate Baſe.* *Fig.* 10.

XX. In all plain Triangles, it will be

As the Baſe,
Is to the Sum of the other two Sides;
So is their Difference,
To the Difference of the Segments of the Baſe, when the ⊥ falls within:
Or, *To the Alternate Baſe,* when it falls without.

That is, In the △s B C D, it will be, B D : B C + C D :: B C − C D : B O. *Fig.* 10.

From C, with the Radius C D deſcribe the Semicircle X D Z, produce B C to Z, let fall the ⊥ C A, and join C O; then is B Z = B C + C D, B X = B C − C D, and B O = [a] $\left\{\begin{matrix} B\,A - A\,D \\ B\,A + A\,D \end{matrix}\right\}$ in the $\left\{\begin{matrix} 1. \\ 2. \end{matrix}\right\}$ Caſe; [a] 14. *Th.*

And becauſe the Point B is without the Circle, it will be,

B D

[b] 15. *Th.* BD : BZ (= BC + CD) [b] : : BX (= BC — CD) : BO. That is, *As the Base* BD, *is to the Sum of the two Sides* BC, CD; *so is their Difference* BX, *to* BO *the* {*Diff. of the Segments of the Base* / *Alternate Base*} *in the* {1. / 2.} *Case.*

Which was to be proved.

Now, $BD \pm BO = 2 \left\{\begin{matrix} B & A \\ A & D \end{matrix}\right\}$ in the first Case.

And $BO \pm BD = 2 \left\{\begin{matrix} B & A \\ A & D \end{matrix}\right\}$ in the second Case.

SECT. II.

Containing the Solutions of all the Cases in Plain Trigonometry.

THIS Section shall be divided into eight Problems, according to the several *Data* that may be proposed in the Question; and withal into twelve Cases, being all the Varieties of the things required.

Note, 1. In any plain Triangle, Three things, whereof one (at least) being a Side, is sufficient to find the rest. Wherefore,

2. In a Right-angled Triangle, Two things besides the right Angle (which is always known) is sufficient to find the rest.

3. To find an Angle, you must begin the Proportion with a Side; and to find a Side, you must begin with an Angle.

1. *Of Right-Angled Triangles.*

The Parts of a Right-angled Plain Triangle, (being the Subject of the four first Problems) are computed by help of Theorem 17. from whence you may observe, that as different Sides are made *Radius*, the other Sides acquire different Denominations, which are either *Sines*, *Tangents*, or *Secants*.

1. To find a Side, any Side may be made Radius; then,

As the Denomination of the Side given,

Is to the Denomination of the Side required;

So is the Side given, to the Side required.

2. But to find an Angle, one of the given Sides must be made Radius; then,

As the Side made Radius, is to the other Side;

So is the Denomination of the first Side (which is Radius)

To the Denomination of the second Side, which gives the Quantity of the Angle required.

PROBLEM I.

The Angles, and one of the Legs of a Right-angled Plain Triangle being given; to find the other Leg, and the Hypothenuſe.

CASE 1. *The Baſe and the ∠ˢ given, to find the Perpendicular.*

Examp. Given { B A = 76.4 ; ∠ B = 38° 50′ } required C A. Fig. 11.

Conſtruction. Having made the Side B A = 76.4, erect the ⊥ A C, and make the ∠ A B C = 38° 50′, and produce B C until it meets A C in C; then is the Triangle conſtructed, and the Side required A C (as alſo B C) may be meaſured by the ſame Scale of equal Parts, that B A was plotted by. *N. B.* This, and the following Problems, are conſtructed by *Prob.* 8. and 9. of *Chap.* I.

To find C A *by Calculation.*

1. If B A be made Radius, then will C A be the Tangent of the ∠ B. Wherefore,

R : T. B :: B A : A C; that is,

As the Radius	10.
To the Tangent of the ∠ B = 38° 50′	9.905784
So is the Side B A = 76.4	1.883093
To the Side A C = 61.5	11.788877

2. If B C be made Radius, then will B A be the Co-Sine of B, and C A the Sine of B. Wherefore,

C. S. B : S. B :: B A : A C; that is,

As the Co-Sine of B 38° 50′	0.108477 *Arith. Compl.*
Is to the Sine of B 38.50	9.797307
So is the Side B A 76.4	1.883093
To the Side A C 61.5	1.788877

3. If A C be made Radius, then will B A be the Co-Tang. of B. Wherefore,

C. T. B : R :: B A : A C; that is,

As the Co-Tang. of B 38° 50′	10.094215
Is to the Radius,	10.
So is the Side B A 76.4	1.883093
To the Side A C 61.5	1.788878

Caſe

CASE 2. *The Baſe and the ∠ᵉ given, to find the Hypothenuſe.*

Examp. Given $\left\{\begin{matrix}\text{B A } 76.4\\ \text{B } 38^\circ\ 50'\end{matrix}\right\}$ required B C.

1. If B A be made Radius, then will B C be the Secant of the ∠ B. Wherefore,

R : *ſ.* B :: B A : B C; that is,

As the Radius	10.
Is to the Secant of B 38° 50′	10.108477
So is the Side B A 76.4	1.883093
To the Side B C 98.08	1.991570

2. If B C be made Radius, then will B A be the Co-Sine of the ∠ B. Wherefore,

C.S. B : R :: B A : B C; that is,

As the Co-Sine of the ∠ B 38° 50′	9.891523
Is to the Radius,	10.
So is the Side B A 76.4	1.883093
To the Side B C 98.08	1.991570

3. When A C is Radius, then B A is the Co-Tangent of B, and B C its Co-Secant. Wherefore,

As the Co-Tang. of B 38° 50′	$\bar{9}$.905785	Here you muſt cancel 20 from the Index of the 4th Log. becauſe in ſubſtracting the Index of the Log. of the 1ſt Term, you were obliged to borrow 10 beſides the Radius, which in all is 20.
To the Co-Secant of B 38 50	10.202692	
So is the Side A B 76.4	1.883093	
To the Side required B C 98.08	21.991570	

You may obſerve, that in the Solutions of the ſix Examples aforegoing, the Logarithms are managed juſt according to the Directions given in *Sect.* 3. *Chap.* I. according to which they are likewiſe ordered in all the following Caſes: But of theſe ſix Examples, only three; *viz.* 1 and 2 of *Caſe* 1, and the 2 of *Caſe* 2, can be worked by the *Sliding Rule*, and *Gunter*'s *Scale*; for whenever a Secant is concerned in the Operation, or a Tangent is greater than the Radius (or 45°) you muſt change the Proportion.

PROB. II. CASE 3.

The Hypothenuſe, and the ∠ᵉ of a Right-angled Plain Triangle being given; to find either of the Legs.

Fig. 12. *Examp.* Given $\left\{\begin{matrix}\text{BC } 98.08\\ \text{B } 38^\circ\ 50'\end{matrix}\right\}$ required A B.

Con-

Conſtruction. Draw B A at pleaſure, and make the ∠ B=38° 50′, continue B C until it becomes =98.08, and let fall the ⊥ C A; then is the △ conſtructed, and the Sides A B, A C may be meaſured by the *Scale* of *Equal Parts*, (wherewith B C was plotted.) But,

To find B A *by Calculation.*

1. If B C be made Radius, then will B A be the Co-Sine of B. Wherefore,

R : C.S. B :: B C : B A; that is,

As the Radius	10.
Is to the Co-Sine of B 38° 50′	9.891523
So is the Hypothenuſe B C 98.08	1.991570
To the Side required B A 76.4	1.883093

2. If B A be made Rad. it will be *ſ.* B : R :: B C : B A.

3. If C A be made Rad. it will be C. *ſ.* B : C.T. B :: B C : B A.

Note, In the Examples here given, it is ſufficiently exact that a Side be expreſſed only to four Places of Figures, and an ∠ to a Minute: But when a Side or an ∠, whoſe *Logarithm* had been found before, is given; we ſhall take the *Logar.* ſo found for the ſaid Side or ∠, though it may ſomewhat differ from the *Logar.* of the Quantity expreſſed: as in the laſt Example, we took the *Log.* found before for the Hypothenuſe B C, which is ſomewhat leſs than the *Log.* of 98.08; becauſe the ſaid *Numb.* 98.08 is ſomewhat greater than the true Quantity of B C.

PROB. III.

The Legs of a Right-Angled Plain Triangle being given; to find the Acute Angles, and the Hypothenuſe. Fig. 13.

CASE 4. *The Legs given, to find the Angles.*

Examp. Given $\left\{\begin{matrix} \text{B A} & 76.4 \\ \text{C A} & 61.5 \end{matrix}\right\}$ required B.

Conſtruction. Make B A=76.4, and from A erect the ⊥ A C=61.5, join B C; then the ∠ˢ B and C may be meaſured by the *Line* of *Chords*, and the Side B C by the *Scale* of *equal Parts*. But,

To find the ∠ B *by Calculation.*

1. B A being made Radius, C A is the Tangent of B. Wherefore,

B A : C A :: R : T. B; that is,

As the Side BA 76.4	1.883093
Is to the Side CA 61.5	1.788877
So is the Radius	10.
To the Tang. of the ∠B 38° 50′	9.905784

The Complement of B (=38° 50′) is the ∠C=51° 10′.

2. If CA be made Radius, then will BA be the Tangent of the ∠C. Wherefore,

CA : BA :: R : T.C.

CASE 5. *Given* {BA 76.4 / CA 61.5} *required* BC.

1. Find the ∠ˢ by Caſe 4, then the Hypothenuſe may be found ſeveral ways by Caſe 2d, thus;

If BC is Radius, CA is the Sine of B, therefore S.B : R :: AC : BC, that is,

As the Sine of the ∠B 38° 50′	9.797307
Is to the Radius,	10.
So is the Side CA 61.5	1.788877
To the Hypothenuſe BC 98.08	1.991570

It is evident from Caſe 2, (becauſe here are two known Sides) that the Side BC may be found by Calculation, after five more different Proportions: And alſo by *Theor.* X. without knowing the Angles.

PROB. IV.

The Hypothenuſe, and one of the Legs of a Right-Angled Plain Triangle being given; to find the Acute Angles, and the other Leg.

CASE 6. *Given* {BC 98.08 / BA 76.4} *required* B.

Fig. 14. *Conſtruction.* Make BA=76.4, and erect the ⊥ AC, then with 98.08 betwixt the Compaſſes, ſetting one foot in B, with the other interſect the ⊥ AC in C, and join BC; then is the △ conſtructed, and the Parts required may be meaſured by the *Line* of *Chords*, and the *Scale* of *equal Parts*.

To

To find the ∠B by Calculation.

If BC be made Radius, then will BA be the Co-Sine of B. Wherefore,

BC : BA :: R : C.S.B; that is,

As the Hypothenuſe BC 98.08	1.991570
Is to the Side BA 76.4	1.883093
So is the Radius	10.
To the Co-Sine of B 38° 50′	9.891523

Or, If BA be made Radius, BC will be the Secant of B. Wherefore,

BA : BC :: R : *ſ.* B; that is,

As the Side BA 76.4	1.883093
Is to the Hypothenuſe BC 98.08	1.991570
So is the Radius	10.
To the Secant of the ∠B 38° 50′	10.108477

The ∠B 38° 50′ being ſubſtracted from 90°, leaves the ∠C=51° 10′.

CASE 7. *Given* $\left\{\begin{matrix}\text{BC } 98.08\\ \text{BA } 76.4\end{matrix}\right\}$ *required* CA.

Firſt, Find the Acute ∠ˢ by the laſt Caſe; then the Side CA may be found ſix different ways; *viz.* three ways by the help of the ∠ˢ and the Baſe BA, by Caſe 1. and alſo three ways by the ∠ˢ and the Hypothenuſe, as in the 3d Caſe; all which, ſince they have been illuſtrated before, may be here omitted. This Caſe may be alſo ſolved by *Theor.* X. without knowing the Angles.

PROB. V. CASE 8.

In any Plain Triangle, whether Right-Angled or Oblique, having the ∠ˢ and one Side given; to find either of the other Sides.

The Analogy for reſolving this Problem, and the next following, is deduced out of Theorem XVIII. which is as followeth,

1. *As the Sine of any one Angle,*
Is to the Sine of any other Angle;
So is the Side oppoſite to the firſt Angle,
To the Side oppoſite to the ſecond Angle.

2. *As any one Side,*
Is to any other Side;
So is the Sine of the ∠ oppoſite to the firſt Side,
To the Sine of the ∠ oppoſite to the ſecond Side.

Therefore, If you would find a Side, begin the Proportion with the Sine of the ∠ oppoſite to the given Side: but to find an ∠, begin the Proportion with the Side which is oppoſite to the given Angle.

Fig. 15. *Examp.* Given { BD 76, B 28° 29′, D 114.21, C 37.10 } required BC.

Conſtruction. Having made the Side BD=76, make the ∠B=28° 29′, and the ∠D=114° 21′, and continue the Sides BC, DC till they interſect each other in the Point C; then the △ is conſtructed, and the Sides BC, DC may be meaſured by the *Scale* of *equal Parts.*

1. *To find* BC *by Calculation.*

Note, The Sine, *&c.* of any ∠, and its Supplement, is the ſame, (as was ſhewed before) therefore (becauſe the Tables reach no farther than 90°) ſubſtract the Obtuſe ∠D=114° 21′ from 180°, and note the Remainder 65° 39′. Then it will be,

As the Sine of the ∠ C 37° 10′	9̄.218866 *Arith. Co.*
To the Sine of the ∠ D 114° 21′ (65° 39′)	9.959539
So is the Side BD 76	1.880813
To the Side BC 114.6	2.059218

2. *To find* DC.

As the Sine of the ∠ C 37° 10′	9̄.218866
Is to the Sine of the ∠ B 28° 29′	9.678430
So is the Side BD 76	1.880813
To the Side DC 60	1.778109

In right ∠d △s the proportion is the very ſame, (as in Prob. I. and II.) becauſe the Radius is the Sine of 90°.

PROB. VI.

In any Plain Triangle, whether Right-Angled or Oblique, having two Sides and an Angle oppoſite to one of them given; to find the remaining Angles, and the other Side.

Fig. 16. CASE 9. *Two Sides and an ∠ oppoſite to one of them, given; to find the remaining ∠s.*

Examp. 1. Given { BD 60, DC 76, B 37° 10′ } required C and D.

Con-

Conſtruction. Having drawn the indefinite right Line B C, make the ∠ B=37° 10′, and draw B D=60; then with 76 betwixt the Compaſſes, ſetting one foot in D, with the other interſect the Line B C in C, and join the Points D and C; then the △ is conſtructed, and the ∠ˢ D and C may be meaſured by the *Line* of *Chords*; and alſo the Side B C by the *Scale* of *Equal Parts*.

Note, That if the given ∠ be Obtuſe, the ∠ required is *Acute*; and if Acute and oppoſite to the {greater / leſſer} Side, the ∠ required is {*Acute* / *Doubtful*} *i. e.* whether Acute or Obtuſe, as ſhall be ſhewed in Examp. 2.

To find the ∠ C by Calculation.

As the Side D C 76	$\bar{8}.119187$ *Ar. Co.*
Is to the Side B D 60	1.778109
So is the Sine of B 37° 10′	9.781134
To the Sine of C 28° 29′	9.678430

The ∠ˢ B and C being known, if you ſubſtract their Sum from 180°, the Remainder will be the ∠ D, thus;

180°—65° 39′ (=Z ∠ˢ B, C) =114° 21′ the ∠ D.

CASE 9. EX. 2. *Given* {D C 76 / B D 60 / C 28° 29′} *required* ∠ˢ B, D.

The Ambiguity of this Caſe appears from the Conſtruction; for having formed the ∠ C=28° 29′, and made the Side B D=76; if with the Length of D B=60, you deſcribe an Arch from D, it will interſect the Side C D in two Points B and B, on the ſame ſide of the Point C, and conſequently the ∠ D, as well as B, will have two different Values: And to find either it will be, *Fig.* 17.

As the Side D B 60	$\bar{8}.221892$ *Ar. Co.*
To the Side C D 76	1.880813
So is the Sine of C 28° 29′	9.678430
To the Sine of B 37° 10′	9.781135

The Meaſure of the ∠ B in the greater △ C D B, but taken from 180°, (becauſe the △ B D B is an Iſoſceles) leaves 142° 50′ for the Meaſure of the ∠ B in the leſſer △ C B D; whence, and from Th. III. the ∠ D of the {greater / leſſer} Triangle, will be {114° 21′ / 8.41}

If inſtead of B D in the firſt Example of this Caſe, B C had been given, the Caſe would have been likewiſe doubtful; as will appear by the following

Examp.

Examp. 3. Given $\left\{\begin{matrix}BC\ 114.6\\ BD\ 76\\ B\ 37^\circ\ 10'\end{matrix}\right\}$ required the ∠ D.

Fig. 18. *Conſtruction.* Make B C=114.6, and the ∠ B=37° 10′, and draw the indefinite right Line B D; then taking C D=76 betwixt the Compaſſes, ſet one foot in C, and with the other deſcribe the Arch D D, which will cut the Side B D, in the two Points D and D, on the ſame Side of the Point B; and the different Values of the ∠ D may be found by the *Line* of *Chords*: But to determine the ſame by *Calculation*, it will be,

As the Side D C 76	8.119187 *Ar. Co.*
Is to the Side B C 114.6	2.059217
So is the Sine of B 37° 10′	9.781135
To the Sine of D 65° 39′	9.959539

The Meaſure of the ∠ D in the greater △ B D C, but taken from 180°, (becauſe the △ D C D is an *Iſoſceles*) leaves 114° 21′, for the ∠ D in the leſſer △ B D C; whence the ∠ C of the $\left\{\begin{matrix}\text{greater}\\ \text{leſſer}\end{matrix}\right\}$ Triangle, is $\left\{\begin{matrix}77^\circ\ 11'\\ 28\ 29.\end{matrix}\right.$

Fig. 16. CASE 10. *Ex.* 1. *Given* $\left\{\begin{matrix}BD\ 60\\ DC\ 76\\ B\ 37^\circ\ 10'\end{matrix}\right\}$ *required* B C.

The *Data* being here the ſame as in *Examp.* 1. of *Caſe* 9. the *Conſtruction* is ſo likewiſe; but before the Side B C can be found by *Calculation*, the ∠ D oppoſite thereto muſt be determined by *Caſe* 9. which being obtained, (as in *Examp.* 1. *viz.*=114° 21′) it will be,

As the Sine of the ∠ B 37° 10′	0.218865 *Ar. Co.*
To the Sine of the ∠ D 114° 21′ (*its Sup.* 65° 39′)	9.959539
So is the Side D C 76	1.880813
To the Side B C 114.6	2.059217

Fig. 17. CASE 10. *Ex.* 2. *Given* $\left\{\begin{matrix}DC\ 76\\ BD\ 60\\ C\ 28^\circ\ 29'\end{matrix}\right\}$ *required* C B.

This Caſe is conſtructed as in *Examp.* 1. of *Caſe* 9. where the ∠ D oppoſite to the required Side B C, has been determined of two Values; *viz.* when $\left\{\begin{matrix}\text{Obtuſe}=114^\circ\ 21'\\ \text{Acute}=\ 8\ 41\end{matrix}\right\}$ wherefore, to find the Value of the Side B C in both Caſes,

1. In the greater Δ B C D.		2. In the leſſer Δ B C D.	
As the Sine of C 28° 29′	9.321570	*As the Sine of* C 28° 29′	9.321570
To the Sine of D 114° 21′	9.959539	*To the Sine of* D 8.41	9.178900
So is the Side B D 60	1.778108	*So is the Side* B D 60	1.778108
To the Side C B 114.6	2.059217	*To the Side* C B 18.99	1.278578

In like manner, if the Data were the ſame, as in Examp. 3. of Caſe 9 and the Side B D required; in the {greater / leſſer} Triangle B D C, B D will be found = {125.6 / 76.} *Fig.* 18.

PROB. VII. CASE 11.

In any Plain Triangle, whether Right-Angled or Oblique, having two Sides and the Angle included; to find the other two Angles.

Examp. In the Oblique ∠d Δ B C D are given {B D 114.6 / B C 76 / B 28° 29′} reqd. *Fig.* 19. the ∠s C and D.

Conſtruction. Make the Side B D = 114.6, and the ∠ B = 28° 29′, and draw B C = 76; join the Points C and D, and the Δ is conſtructed; then the ∠s C and D may be meaſured by the *Line* of *Chords.*

By *Calculation:* This Problem is ſolved by the following Analogy, demonſtrated in *Theor.* XIX.

PREPARATION.

B D = 114.6	∠s B + D + C = 180° 00′
B C = 76	∠ B = 28 29
Z = 190.6	Z ∠s C + D = 151 31
X = 38.6	½ Z ∠s C + D = 75 45½

As the Sum of the two Sides B D + B C = 190.6	2.280123
Is to their Difference = 38.6	1.586587
So is the Tang. of ½ Z *opp.* ∠s = 75° 45′ ½	10.595486
	12 181073
To the Tang. of ½ *their Diff.* = 38° 35′ ½	9.901950

Now

Now

To the $\frac{1}{2}$ *Sum* $=75^{\circ}\ 45'\ \frac{1}{2}$	*From the* $\frac{1}{2}$ *Sum* $=75^{\circ}\ 45'\ \frac{1}{2}$
Add the $\frac{1}{2}$ *Diff.* $=38\ 35\ \frac{1}{2}$	*Take the* $\frac{1}{2}$ *Diff.* $=38\ 35\ \frac{1}{2}$
Sum=greater $\angle$ C $114\ 21$	*Rem. leſſer* $\angle$ D. $=37\ 10$

This Caſe may be ſolved by reducing the Δ into two right-angled Δ^{s}; thus, if from the Extream of one of the given Sides, you let fall a $\perp$ upon the other given Side, (produced if need be) the Δ (B C D) will be reduced into two right-angled Δ^{s}, (as B A C, C A D,) in one of which (as B A C) will be given the Hypothenuſe (B C) and all the $\angle^{s}$, to find the Legs, (B A and A C;) then in the other right-angled Δ (C A D) will be given the Legs (C A, A D) to find the Acute $\angle^{s}$ (C, D.) If the $\perp$ ſhould fall without the Δ, the things required may be found with as much eaſe as before. [*Vide* 2 Δ^{s} *Fig.* X.]

If the Side C D ſhould be required, it may be found either by the Vth or VIth Prob. aforegoing.

PROB. VII. CASE 12.

The three Sides of any Plain Triangle being given; to find the Angles.

Fig. 20. *Examp.* Given $\left\{\begin{matrix} \text{B D}=114.6 \\ \text{B C}=76 \\ \text{D C}=60 \end{matrix}\right\}$ required the $\angle^{s}$ B, D and B C D.

This Problem is conſtructed like *Prob.* V. in *Sect.* 2. of *Chap.* I. and the ſeveral $\angle^{s}$ may be meaſured by the *Line* of *Chords*: but to determine their Quantities by *Calculation*,

From C let fall the $\perp$ C A, then will the Δ B C D be reduced into two right-angled Δ^{s} B A C, D A C; in each of which are given the *Hypothenuſes* B C, C D, and the *Right Angle* at A; and the *Baſes* B A, A D, or the *Segments* of the *Baſe* B D, may be found by *Theor.* XX.

B C $=76$	*As the Baſe* B D $=114.6$	2.059184
D C $=60$		
Z. 136	*To the Sum of the two Sides* B C, C D $=136$	2.134177
	So is their Difference $=16$	1.204120
X. 16		3.338297
	To B O *the Diff. of the Segm.* B A, A D $=19$	1.279113

Which

Plate 2.

Fig. I

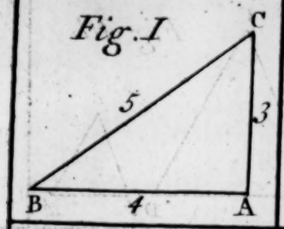


Fig. II

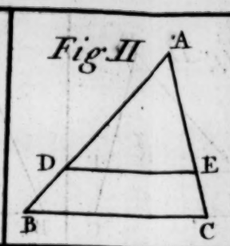


Fig. III



Fig. IV

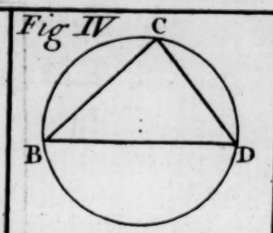


Fig. V

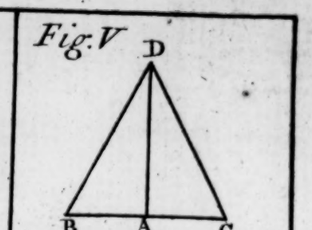


Fig. VI

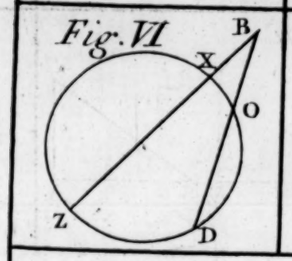


Fig. VII

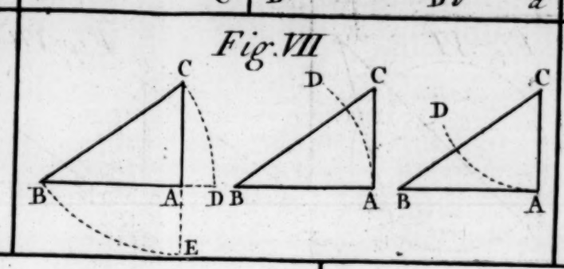


Fig. VIII



Fig. IX

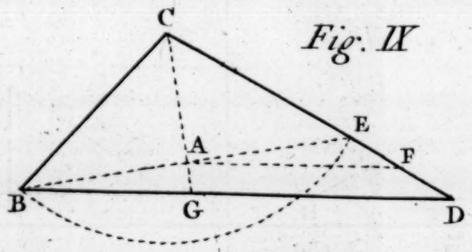


Fig. X

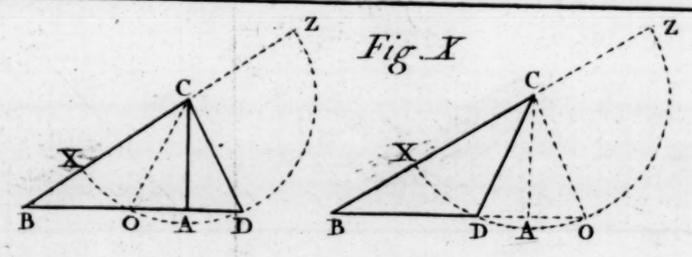


Fig. XI

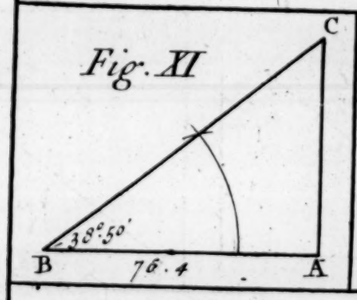


Fig. XII

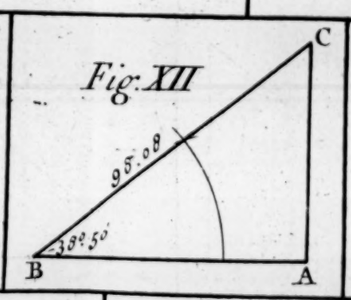


Fig. XIII

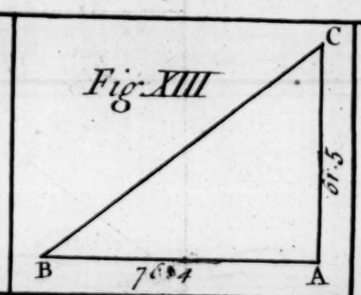


Fig. XIV

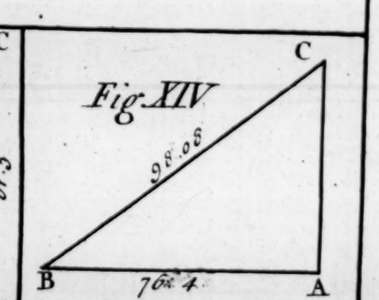


Fig. XV

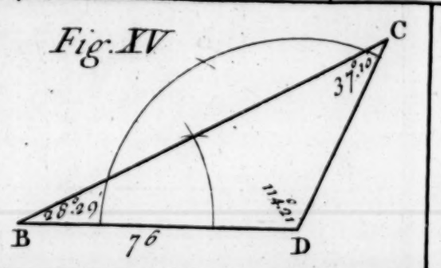


Fig. XVI

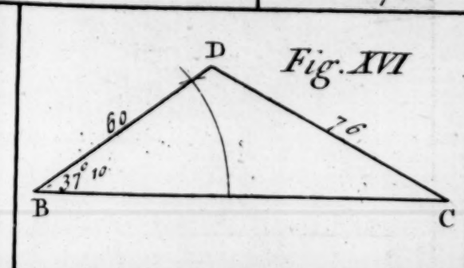


Fig. XVII

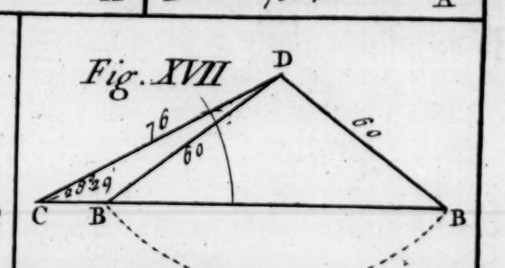


Fig. XVIII

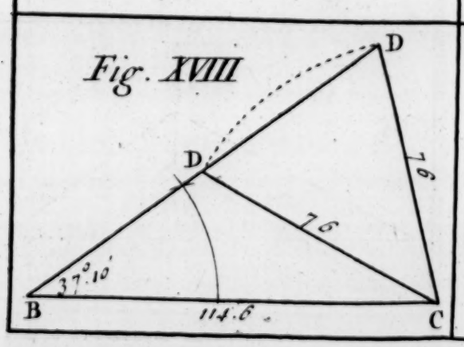


Fig. XIX

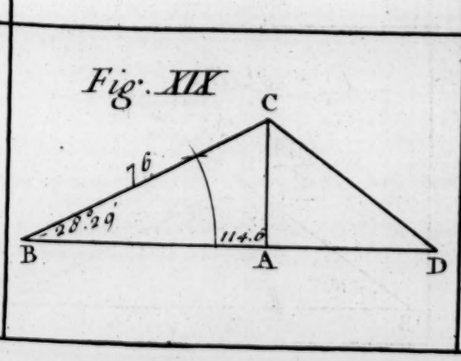
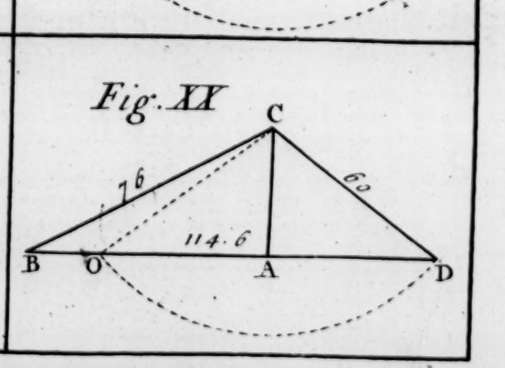


Fig. XX



Which taken from B D=114.6, leaves O D=95.6, the half of which is 47.8=A D the leſſer Segment; then A D taken from B D the whole Baſe =114.6, leaves 66.8=A B the greater Segment.

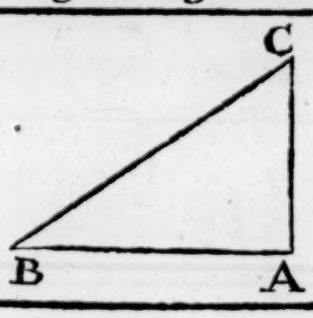
The *Segments* B A, A D, or the *Baſes* of the right ∠ᵈ △ˢ B A C, D A C, being thus found, the ∠ˢ may be had by *Prob.* IV. Thus,

As the Side B C=76	1.880813
To the Side B A=66.8	1.824776
So is the Radius	10.
To the Co-Sine of B 28° 29′	9.943963

After the ſame Manner may the ∠ D be found, and conſequently the ∠ B C D.

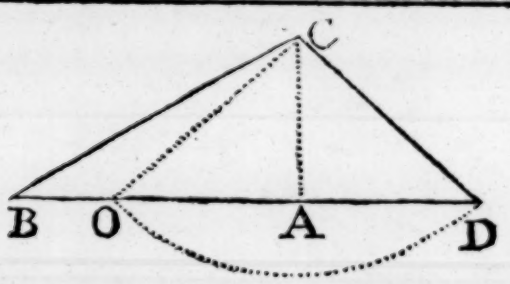
A Table of Proportions for the Solutions of the ſeveral Caſes in Plain Trigonometry.

1. Of Right-Angled Triangles.



Given	Reqd.	Rad.	Proportions.	Caſe	Prob.
B A		B A	R : T. B :: B A : A C.		
B	C A	C A	C.T.B : R :: B A : A C.	1	
C		B C	C. S. B : S.B :: B A : A C.		I.
B A		B A	R : ſ. B :: B A : B C.		
B	B C	B C	C. S. B : R :: B A : B C.	2	
C		C A	C.T.B : C. ſ. B :: B A : B C.		
B C		B C	R : C. S. B :: B C : B A.		
B	B A	B A	ſ. B : R :: B C : B A.	3	II.
C		C A	C. ſ. B : C.T.B :: B C : B A.		
B A	B &	B A	B A : C A :: R : T.B, Comp. is C		
C A	C	C A	C A : B A :: R : T.C. Comp. is B	4	
B A			1. B A : C A :: R : T.B; then		III.
C A	B C	B C	C.S.B : R :: B A : B C; or	5	
			$\sqrt{\text{B A q} + \text{C A q}} = \text{B C}$.		
B C	B &	B C	B C : B A :: R : C. S. B.		
B A	C	B A	B A : B C :: R : ſ. B.	6	
B C			1. B C : B A :: R : C.S.B, then		IV.
B A	A C	B C	R : S.B :: B C : A C; or	7	
			$\sqrt{\text{B C q} - \text{A C q}} = \text{A C}$.		

2. Of Oblique Triangles.



Given	Reqd.	Proportions.	Caſe	Prob.
B C				
B	C D	S. D : S.B :: B C : C D.	8	V.
D				
B C	B &	B C : C D :: S.B : S.D, then		
C D	C	180° — Z ∠ˢ B + D = ∠ C.	9	
D				
B C		1. Find the ∠ C, as in Caſe 9.		VI.
C D	B D	Then	10	
D		S.D : S. C :: B C : B D.		
B C	D &	B D + B C : B D — B C ::		
B D	C	T. ½ Z ∠ˢ op. : T. ½ X ∠ˢ op.	11	VII.
B		Then		
		½ Z ∠ˢ + ½ X ∠ˢ = ∠ C.		
		½ Z ∠ˢ — ½ X ∠ˢ = ∠ D.		
		B D : B C + C D :: B C — C D : B O.		
B D	B	Then ½ B D + ½ B O = B A.		
B C	C	And ½ B D — ½ B O = A D; now	12	VI I.
D C	D.	B C : B A :: R : C.S.B.		
		D C : D A :: R : C.S.D; then		
		180° — Z ∠ˢ B + D = ∠ C.		

The End of the Introduction.

NAVIGATION.

NAVIGATION is an Art that teacheth how a Ship may be conducted the ſafeſt way, and in the ſhorteſt time, between any Places aſſigned in Paſſages navigable.

When the Places propoſed are at no great diſtance, but ſo that a Ship may ſail in ſight of Land, or within Soundings, as between *England*, *Holland*, *France*, &c. it is then called *Coaſting*; and for the ſafe conducting of the Ship, the Pilot muſt be well acquainted with the Coaſts, ſo as to know the proper Channels, the Setting and Drift of the *Tides*, alſo Places of imminent danger, ſuch as *Rocks*, *Sands*, &c.

But the *Proper* Buſineſs of *Navigation*, is how to conduct a Ship through the wide and pathleſs *Ocean*, where nothing is viſible but Sky and Water for a conſiderable time: For the due performing of which, the Mariner muſt know, 1. The true Situation both of the Place he departed from, and of the Place he is bound to; *viz.* their *Latitudes*, *Difference* of *Longitude*, and conſequently their mutual *Bearing* and *Diſtance*: Then for finding the Place of the Ship at any time, and conſequently the Bearing and Diſtance of the Place intended for, he has theſe helps; *viz.* the Courſe ſteered, the Diſtance ſailed, and the Latitude by Obſervation; which laſt indeed is the only certain *Data* Mariners (at preſent) have at Sea: The Deſcription and Uſe of the ſeveral *Inſtruments* neceſſary for obtaining theſe things, and how from them to find the neceſſary *Quæſita* (as near as poſſible) ſhall be ſhewed at large in the enſuing Treatiſe.

CHAP. I.

The Theory of Navigation.

THE firſt thing neceſſary to be known, in order to ſettle the *Theory* of *Navigation*, is the true Figure and Magnitude of the Earth; wherefore in *Sect.* 1. I ſhall ſhow how that has been obtained: but firſt, it is convenient to premiſe the following *Definitions* of a *Sphere*, &c.

A SPHERE or GLOBE is a Geometrical Figure, conceived to be generated by the Revolution of a Semicircle round its Diameter; whence it follows, that all Points upon the *Surface* of a *Sphere*, are every where equidiſtant from a Point within it, called its *Center*. Any Circle dividing the Sphere into two equal Parts or Segments, is called a GREAT CIRCLE; and the Segments of the Sphere ſo divided, are called HEMISPHERES.

A *Right Line* paſſing through the *Center* of the Sphere, and continued on both Sides to the *Superficies*, is called the AXIS of that *Great Circle* to which it is Perpendicular; and the two Points in the Superficies of the Sphere, which are the Extreams of the *Axis*, are called the POLES of the ſaid *Great Circle*.

All Great Circles cut each other into two equal Parts, for they all paſs through the Center of the Sphere: Alſo any Circle paſſing through the Poles of another, is a *Great Circle*.

All Great Circles paſſing through the Poles of any Great Circle, interſect it in two Places diametrically oppoſite, and alſo at right Angles; and with reſpect to the ſaid great Circle, they may be called its *Secundaries*.

Any Circle dividing the Sphere into two unequal Parts, is called a *Leſſer Circle*, or elſe a *Parallel Circle*; and theſe Circles upon the *Artificial Globes*, (made to repreſent the Ball of the Earth, and the Sphere of the Heavens) are generally denominated by the Great Circle, to which they are parallel.

SECT.

SECT. I.

Of the Figure and Magnitude of the Earth.

THE Earth is endued with a wonderful Principle of Gravitation, whereby all its Parts are ſtrictly united together; and all Bodies that are looſe upon it, cloſely adhere to its Surface, tending directly towards its Center. Hence it is, that Ships are able to ſail with the ſame Facility every where (void of Impediments) upon the Surface of the Sea, quite round the Terraqueous Globe; and that (as to Senſe) there is no ſuch thing as an upper or lower Part of the Earth; for let the Inhabitant be in what part ſoever, he'll there gravitate towards the Earth's Center, and imagine himſelf to be on the higheſt Point of its Surface; from whence he'll obſerve the Heavens like a large Vault over his Head, and his Antipodes he'll imagine to be directly under him, as they will alſo theirs, for the like reaſon.

Of the Figure of the Earth.

According to this Law of Gravity, if the Earth were at reſt (and not acted upon by any other Power) and its Parts looſe, or its Surface all over covered with a deep Fluid; it would naturally form itſelf into a true *Sphere* or *Globe*. But admitting the Earth revolves about its own Axis, with a rapid Motion (from Weſt to Eaſt) in 24 Hours, the Gravity towards its Center will be thereby diſturbed, and all the Parts endeavour to fly off from the Axis of the Motion; and this Inclination is greateſt, in that part of the Surface which is at the greateſt Diſtance from the Axis, and conſequently the Gravity towards the Center is there the leaſt: whence it will follow, that thoſe Parts which gravitate the leaſt, muſt yield or give way to thoſe that have a greater Gravitation, to reſtore an Equilibrium. And conſequently here will be formed a *Spheroid*, (or a Figure ſomewhat like an Onion) whoſe greateſt Diameter will be perpendicular to the *Axis* of Motion, (commonly called the *Earth's Axis*) and the ſhorteſt Diameter will be the *Axis* itſelf.

It is demonſtrated by the Writers of *Mechanicks*, that the Times of the Periodical Vibrations of all Pendulums of equal Lengths, are in a certain Proportion to the Gravity by which they are acted upon: And it has alſo been demonſtrated, that *Gravity* acts in a certain Proportion to the Diſtance from its Center. Hence, by the help of Pendulums, we may find the Proportion of Gravity upon any Part of the Earth; and conſequently the proportional Diſtance of that Part, to the Diſtance of any other Part, from the Earth's Center.

Now,

Now, it has been found by Experience, that the Degree of Gravitation upon the Earth's Surface under the Equinoctial, is to the ſame in any Parallel of Latitude; in the ſame Proportion (as near as Obſervations could be made) that it would be, if the whole Body of the Earth was compoſed of a Fluid Subſtance, and ſo formed itſelf into ſuch a Figure, as above-mentioned. Hence, we may infer, that the Earth is a *Spheroid*; and its greateſt Diameter (which is under the Equinoctial) is computed to be to the leſſer Diameter (which is under the Poles, or the Earth's Axis) as 289 to 288; and conſequently the Space upon the Earth's Surface, anſwering to a Degree of a great Circle where it is greateſt, (or under the Equin.) is to the Space anſwering to a Degree near the Poles, (where it is leaſt,) as 289 to 288; or, as 1000 to 996.5 nearly. But this difference is ſo ſmall, that in all Aſtronomical and Geographical Caſes, the Figure of the Earth may be eſteemed truly *Spherical*, though the ſmall Difference from it does ſenſibly affect the Motions of *Pendulums*.

The *Sphericity* of the Earth is farther confirmed by Vulgar Experience, for in going directly *North* or *South*, we have the Stars either Elevated or Depreſſed, exactly in proportion to the Diſtances we go: Again, it is manifeſt by ſeveral reaſons, that the Heavenly Bodies appear ſooner upon the Meridian the more *Eaſterly*, and the contrary the more *Weſterly*, and that in proportion to the Diſtances; This is demonſtrated by the *Eclipſes* of the Moon, for theſe Eclipſes happen at the ſame inſtant of Time without reſpect of Places; but they are obſerved to begin in the more *Eaſterly* Places ſooner than in the *Weſtern*, agreeable to the Hypotheſis of the Earth's Sphericity: all which could not happen if the Earth was of any other Figure. To theſe we might add Reaſons from the *Eclipſes* of the Sun, and alſo from the different Appearances of Ships at Sea; but it would be needleſs to inſiſt upon any further Arguments in this Place.

Of the Magnitude of the Earth.

Now it being proved, that the Earth is *Spherical*, a Line drawn directly betwixt any two Places upon its Surface, muſt be an *Arch* of a Circle. Wherefore, if we have the Meridian Altitude of any Heavenly Object, in two different Places lying under the ſame Meridian, exactly at the ſame time; we may from thence find the difference of Latitude of theſe two Places: Or, the Meridian Altitudes may be taken at different times, ſo the Declination at thoſe times be known, and the Difference of Latitude will be found as before; then the Diſtance between theſe two Places being meaſured, will give the Quantity (in known Meaſure) upon the Earth's Surface, anſwering to an *Arch* of a Great Circle, which is equal to the Difference of Latitude of the two Places; which being known, the Quantity anſwering to any other Arch may be eaſily found. Or, if the Places lie not under the ſame Meridian, their Bearing and Diſtance may be found (allowing for the Windings and Turnings, Aſcents and Deſcents of the Road) and from thence, their Difference of Latitude in the ſame Meaſure, and conſequently the Quantity anſwering to an Arch equal to their Difference of Latitude by Obſervation; which being known, the whole

whole *Circumference* of the Earth may from thence be found, and conſequently its *Diameter*, *Superficies*, &c.

This Method has been put in Practice in *England* by Mr. *Norwood*, *Anno* 1635, by obſerving the Latitudes of *London* and *York*, and meaſuring the Diſtances between them; by which Experiment, (after he had made allowance for the Windings, *&c.* of the Road) he found 367200 *Engliſh* Feet to anſwer one Degree of a Great Circle upon the Surface of the Earth. This has been farther confirmed by Experiments made ſince in our own Country; and alſo by the ſame ſort of Experiment made in *France*, by which a Degree of a Great Circle upon the Earth's Superficies was found to conſiſt of 365184 *Engliſh* Feet; differing from Mr. *Norwood*'s Experiment but 2016 Feet, which in ſuch a Diſtance is not very conſiderable. Hence, a Degree of a Great Circle upon the Earth contains about 69 $\frac{1}{2}$ *Engliſh* Miles; (Mr. *Norwood*'s Experiment making it 69.545, and the *French* Experiment about 69 $\frac{1}{6}$) and conſequently a *Geographical* Mile is to an *Engliſh* Mile as 69.5 to 60; and from this Suppoſition, the whole *Circumference* of the Earth will be found to be 25020, and its *Diameter* 7964 *Engliſh* Miles.

There are other Methods for finding the Magnitude of the Earth, but none ſo accurate, or ſo eaſily put in Practice as the aforegoing.

SECT. II.

Geographical Definitions.

THE *Aſtronomers* in order to determine the Places of *Phænomenas* in the Heavens; and likewiſe the *Geographers* for aſſerting the Poſitions of Places upon the Earth; ſuppoſe the Sphere of the Heavens to be divided by ſeveral *Imaginary* Circles: Of theſe Circles ſome are *Fixed*, and always obtain the ſame Poſition in the Heavens; others again are *Moveable*, and depend only upon the Poſition of the Obſerver.

Thoſe Circles that are *Fixed*, owe their Origin to the two-fold Motion of the Earth; ſuch are, the *Equinoctial*, the *Ecliptick*, and their *Secundaries* and *Parallels*. The *Moveable Circles* are only the *Horizon*, its *Parallels*, and *Secundaries*.

Of theſe Circles we ſhall here conſider only thoſe that are more immediately uſeful in *Geography* and *Navigation*; viz. the *Equator*, its *Parallels*, the *Horizon*, and the *Meridian*.

I. The

I. The EQUATOR or EQUINOCTIAL, is that Great Circle in the Heavens, in whoſe Plane the *Earth* performs its *Diurnal Motion* round its *Axis*: Or, it is that Great Circle, parallel to which the whole *Heavens* ſeem to move round the Earth, from *Eaſt* to *Weſt* in 24 Hours.

Note, The *Equator* and the *Equinoctial* are generally ſynonimous Terms; but ſometimes the *Equator* particularly ſignifies that Great Circle upon the Surface of the Earth, which coincides with the *Equinoctial* in the Heavens: This Circle is alſo by *Mariners* often called the LINE.

The Equinoctial divides the Globe of the Earth, and alſo the whole Heavens, into two equal Parts, *North* and *South*; which are called the *Northern* and *Southern Hemiſpheres*. The *Axis* of this Circle, is called the EARTH'S AXIS, or the AXIS of the *World*; becauſe the Earth revolves about it (from Weſt to Eaſt) in 24 Hours. The *Extreams* of this *Axis* are called the POLES of the *World*; whereof that which lies in the *Northern Hemiſphere* is called the *North Pole*, and the other is called the *South Pole*.

Note, The {Northern / Southern} Hemiſphere is that wherein the Sun is in our {Summer / Winter.}

The *Equinoctial* is divided into Degrees and Minutes, and alſo into 24 Hours of Time; ſo that one *Hour* is equal to 15 *Degrees*, one Minute of *Time* equal to 15 Minutes of a *Degree*, &c.

All Circles parallel to the Equator, are with reſpect to Places upon the Earth, called *Parallels* of *Latitude*. Whence,

II. The LATITUDE of any Place (upon the Earth) is its neareſt Diſtance from the Equator, either North or South; and if the Place be in the {Northern / Southern} Hemiſphere, it is called {*North* / *South*} Latitude. Hence, all Places that lie on the ſame Side, and at the ſame Diſtance from the Equator, are ſaid to be in the ſame *Parallel of Latitude*.

From this Definition ariſe the following Corollaries.

1. *That no Place can have above 90 Degrees of Latitude, either North or South.*

2. *Thoſe Places lying under the Equinoctial have no Latitude; it being from thence that the Calculation of Latitude is counted.*

3. *Thoſe Places lying under the Poles have the greateſt Latitude; thoſe Points being at the greateſt Diſtance from the Equator.*

4. *The Latitude of any Place is always equal to the Elevation of the Pole in the ſame Place; and the contrary.*

5. *The Elevation of the Equator in any Place, is always equal to the Complement of the Latitude of that Place; and the contrary.*

6. *A Ship ſailing directly* {*towards* / *from*} *the Equator,* {*Depreſſes* / *Raiſes*} *the Pole, or* {*Leſſens* / *Augments*} *her Latitude, juſt ſo much as is her Diſtance ſailed.*

III. *Difference* of *Latitude* is the neareſt Diſtance of any two Parallels of Latitude, ſhewing how far the one is to the Northward or to the Southward of the other; which can never exceed 180 Degrees.

IV. The HORIZON is that Great Circle which (in any Place) divides the *Upper* or *Viſible* Hemiſphere of the World, from the *Lower* or *Inviſible*: This Circle is diſtinguiſhed into two ſorts; *viz.* the *Rational* and the *Senſible*.

The *Rational Horizon* is that Circle paſſing through the Center of the Earth, in which all the Heavenly Bodies appear to *Riſe* and *Set*. This Circle divides the Earth, and alſo the Heavens, into two equal Parts; it is alſo called the *Real* or *True* Horizon.

The *Senſible* or *Apparent Horizon*, is that Circle which bounds the Proſpect of any Perſon, who being placed either in a large Plain, or in the midſt of the Sea, looks round about; and by which the Earth and the Heavens ſeem to be joined together.

The *Senſible Horizon* is that which is conſidered by *Mariners*, and in a ſtrict Senſe its Semidiameter is infinitely ſmall; for the Eye ought to be conſidered as being cloſe to the Earth's Superficies: but the *Apparent* Semidiameter of this Circle varieth according to the height of the Eye of the Obſerver.

This *Circle*, and alſo the *Rational Horizon*, is divided into four Quadrants, by four Points called the *Cardinal Points*; theſe Points are termed the *Eaſt*, *Weſt*, *North*, and *South*. The *Eaſt* is that Point of the Horizon where the Heavenly Bodies appear to Riſe and Set when they are in the Equinoctial; and the *Weſt* is that Point of the Horizon which is directly oppoſite to the *Eaſt*, or where they appear to ſet on the ſame Day. The *South* is 90 Degrees diſtant from the *Eaſt* and the *Weſt*, and is in that Part of the Horizon where the Sun always appears to us at Noon; and the *North* is that Point of the Horizon which is directly oppoſite to the *South*: Or, the *North* and *South* Points of the Horizon may be found, by turning yourſelf directly towards the *Eaſt* or *Weſt*; if you look directly towards the {*Eaſt*, / *Weſt*,} the {*South* / *North*} will be to the right Hand, and the {*North* / *South*} to the left.

Each of theſe Quadrants are ſubdivided into 90 *Degrees*, each Degree into 60 *Minutes*, *&c.* But by *Mariners* the ſenſible Horizon is commonly divided into 32 equal Parts, which they call the *Points* of the *Compaſs*; each of theſe Points are again ſubdivided into Halfs and Quarters. [*Vide Fig.* 1. *Plate* 3.]

The Poles of the Horizon are two Points, one exactly over our Head, called the *Zenith*; and the other exactly under our Feet, called the *Nadir*. Whence it follows, that which way ſoever we move, we continually change our Horizon.

V. All Circles paſſing through the Poles of the World, (and conſequently interſecting the Equinoctial at Right Angles) are called *Meridians*, and ſometimes *Hour-Circles*.

VI. The MERIDIAN of any Place, is a Great Circle paſſing through the *Poles* of the World, and the *Zenith* and *Nadir* of that Place; and conſequently paſſing through the North and South Points of the Horizon.

The Difference of Meridians betwixt any two Places, is called their *Difference* of *Longitude*. So that,

VII. The LONGITUDE of any Place (upon the Earth) is an *Arch* of the *Equator*, contained betwixt the Meridian of the given Place, and ſome *fixed* or known Meridian: Or, it is equal to the Angle formed by the two Meridians. Whence it follows,

That no Place can have above 180 Degrees of Longitude from any other Place; yet ſometimes the Longitude is counted Eaſterly quite round the Globe; wherefore in theſe Caſes, if the Longitude exceeds 180 Deg. it muſt be ſubſtracted from 360°, and the Remainder will be the Difference of Longitude of the two Places, of a contrary Denomination to what it was at firſt: that is, if the Longitude of any Place be counted *Eaſterly* from any other Place, and it ſhould exceed 180°, the Complement of it to 360 Deg. muſt be reckoned the Longitude of the ſaid Place from the firſt, *Weſterly*.

Since the Meridians are all Moveable, and not one that can be Fixed in the Heavens, (as the Equinoctial Circle is Fixed, from whence the *Latitudes* of all Places are determined to be ſo much either North or South,) the *Longitudes* of Places cannot ſo well be fixed from any one Meridian; but every *Geographer* is at his liberty to fix which he pleaſes for his *firſt Meridian*, from whence to calculate the *Longitudes* of other Places: Hence it is, that the *Geographers* of all Nations generally chuſe the Meridian paſſing through the *Metropolis* of their own Country, for their firſt Meridian: thus, the *Engliſh* Geographers generally make the Meridian of *London* to be their firſt; the *French* that of *Paris*; and the *Dutch* that of *Amſterdam*, &c. But *Mariners* generally reckon their *Longitude* from the laſt known Land they ſaw; ſo that when the Longitude is expreſſed, it is neceſſary (before it can be determined) that the Place from whence it is counted, be alſo expreſſed.

From the preceding Definitions naturally ariſe the following Corollaries.

1. *If a Body ſhould ſteer directly North, or directly South, quite round the Globe, he will continually change his Latitude, without deviating the leaſt from the*

the Meridian of the Place he departed from; and conſequently at his return will not differ in his account of Time from the People reſiding in the ſaid Place.

2. *If a Body ſhould ſteer round the Globe, either directly Eaſt or Weſt, he'll continually change his Longitude, but will go quite round without altering his Latitude: And if his Courſe ſhould be due Eaſt, he'll gain one Day compleatly in his Reckoning, or reckon one Day more than the Inhabitants of the Place from whence he departed; and if his Courſe had been Weſt, he would have loſt one Day, or reckon'd one leſs. Whence it follows, if two Bodies ſhould ſet out from the ſame Place, one ſteering Eaſt, and the other Weſt, and ſo continue their Courſes quite round, till they arrive at the Place from whence they ſet out, they will differ two Days in their Reckoning at the time of their Return.*

3. *If a Body ſhould ſteer in any other Direction, he'll continually change both Latitude and Longitude, and that more or leſs, according to the Courſe he ſteers; and if he ſhould go quite round the Globe, he will differ in his Account of Time, as by the laſt* Cor.

4. *The People reſiding in the Eaſtermoſt of any two Places, will reckon their Time ſo much the ſooner than in the other Place, according to the Difference of Longitude betwixt the two Places, allowing* 1 *Hour for every* 15 *Degrees, &c. and the contrary.*

SECT. III.

Nautical Definitions and Theorems.

THE only Guide that *Mariners* have for directing the Ship in the immenſe Ocean (where nothing is viſible but Sky and Water) is their *Compaſs:* A particular Deſcription of which Inſtrument ſhall be given in *Chap.* IV. *Sect.* 1. but here we ſhall conſider only the *Card.*

I. The *Card* is a Circle quartered and divided into *Degrees*, and alſo into 32 equal Parts, called the *Points* of the *Compaſs*, and by virtue of a Needle fixed underneath, theſe Points always inſpect the like Points of the ſenſible Horizon, (the Variation not here conſidered.) The Points of the Compaſs are denominated as in *Plate* 3. *Fig.* 1. where you may note, that the Capital Letters do each of them ſignify the four *Cardinal Points*; thus, N ſignifies the *North*, E the *Eaſt*, S the *South*, and W the *Weſt*; and the ſmall Letter *b* ſignifies the Word *by*, thus for Example, N E *b* N, is to be read *North Eaſt* by *North*, &c.

The ſeveral Points may be eaſily known by Inſpection, without having the Letters annexed to them, (which indeed are ſeldom placed upon the Cards,) thus; The four great Bars are the four Cardinal Points, whereof

that which has the *Flower-de-luce* at the top of it, is always the *North*, and that at right Angles to it having the little Croſs at the end of it, is the *Eaſt*; and conſequently the two oppoſite Points muſt be the *South* and the *Weſt*. Again, the four next leſſer Bars, are the middle Points betwixt the four Cardinals, and are always expreſſed by the two Cardinals terminating that Quarter, that which denotes the Meridian having always the Precedence, as N E, S E, S W, N W. The eight leſſer Bars, are the middle Points between the N E, S E, *&c.* and are expreſſed by the neareſt Cardinal twice repeated, (that alſo having the Precedence) and the 2d in order the Meridian; as N N E, E N E, *&c.* The ſmall Δ's are the Points between theſe laſt; whereof the neareſt to every Cardinal is expreſſed by the two Points terminating that Quarter, and the Word *by* between them, as N *b* E, E *b* N, *&c.* and the 3d and 5th Points (from the Meridian) are always expreſſed by the two Letters denominating that Quarter, the Word *by* immediately following them, then the next Cardinal; as N E *b* N, N E *b* E, *&c.* And all Cards (if they are not made as above deſcribed) have always ſome diſtinction, for the ready expreſſing the Points.

If a Ship ſail any where betwixt the Eaſt and Weſt in the {Northern / Southern} Semicircle, then with reſpect to the Meridian, ſhe is ſaid to ſail {*Northerly*; / *Southerly*;} And when ſhe ſails betwixt the North and South any where in the {Eaſtern / Weſtern} Semicircle, then with reſpect to that Semicircle ſhe is ſaid to ſail {*Eaſterly*: / *Weſterly*:} And when any particular Quarter is intended to be expreſſed, it muſt be ſaid in the N E, or S E, *&c.* Quarter; or N Ely or S Wly, *&c.*

If a Ship ſails upon any of the four Cardinal Points, ſhe is then ſaid to ſail *due North*, or *due Eaſt*, &c.

II. A *Right Line* drawn from the *Center* to the *Circumference* of the *Compaſs*, (or paſſing from you to any Point of the *Apparent Horizon*,) is called A Rumb Line; and this *Line* is denominated by that Point of the Compaſs which it interſects, as if it ſhould paſs through the N N E Point, then it is called a N N E *Rumb*.

Or, *A Rumb* is a *Line* making the ſame or equal Angles, with all the Meridians through which it paſſeth, (for the Senſible Horizon is to be conſidered as a Circle indefinitely ſmall, as has been ſhewed before) and conſequently the Direction of every Rumb, excepting the four Cardinals, muſt be continually changed.

A TABLE of the ∠s in Deg. and Min. which every ¼ Point of the Compaſs makes with the Meridian.

North.	*South.*	Points.	o ′	*South.*	*North.*
		¼	2 48¾		
		½	5 37½		
		¾	8 26¼		
N *b* E.	S *b* E.	1	11 15	S *b* W.	N *b* W.
		¼	14 03¾		
		½	16 52½		
		¾	19 41¼		
N N E.	S S E.	2	22 30	S S W.	N N W.
		¼	25 18¾		
		½	28 07½		
		¾	30 56¼		
N E *b* N.	S E *b* S.	3	33 45	S W *b* S.	N W *b* N.
		¼	36 33¾		
		½	39 22½		
		¾	42 11¼		
N E.	S E.	4	45 00	S W.	N W.
		¼	47 48¾		
		½	50 37½		
		¾	53 26¼		
N E *b* E.	S E *b* E.	5	56 15	S W *b* W.	N W *b* W.
		¼	59 03¾		
		½	61 52½		
		¾	64 41¼		
E N E.	E S E.	6	67 30	W S W.	W N W.
		¼	70 18¾		
		½	73 07½		
		¾	75 56¼		
E *b* N.	E *b* S.	7	78 45	W *b* S.	W *b* N.
		¼	81 33¾		
East.		½	84 22½	*West.*	
		¾	87 11¼		
		8	90 00		

The COURSE is the *Angle* which any *Rumb Line* makes with the Meridian: ſo that if a Ship ſhould ſail upon the third *Rumb*, as N E *b* N, &c. the Courſe is an ∠ of 33° 45′; and ſo for any other, as in the annexed Table.

That

That Point in the Horizon which any Rumb Line interſects, is called the *Bearing* of the ſaid Point from the Obſerver: Alſo the Denomination of the Rumb paſſing between any two Places, is called the Bearing of the ſaid Places. Hence, the *Rumb*, *Courſe*, and *Bearing*, do all ſignify the ſame thing; *viz.* the ∠ which they make with the Meridian.

III. The *Length* of the *Rumb* intercepted between any two Places, is called their DISTANCE (upon that Rumb;) but this Diſtance is never the ſhorteſt, but when the two Places are either under the Equinoctial or the ſame Meridian.

IV. The *Length* of any *Parallel* of Latitude intercepted between any two Meridians, is called the MERIDIONAL DISTANCE, (in that Parallel.)

From *Defin.* 2. it follows, (ſince none but the Equator and its Parallels cut the Meridians at equal Angles) that all *Rumbs*, (excepting the four Cardinals) muſt be *Curves* or *Heliſpherical Lines*, always tending towards the Pole, and ſo approaching it by infinite Gyrations or Turnings, but never falling into it: Thus, in *Fig.* 2. *Plate* 3. where P repreſents the Pole of the World, A Q an Arch of the Equinoctial; the Curves A B P, A C P are *Rumb Lines*; the firſt whereof (A B P) makes an ∠ of 4 Points, and the other an ∠ of 6 Points with the Meridian. Now from the Definition of a *Rumb Line*, the ſeveral ∠ˢ at the Points A, *a*, *a*, B, made by the Rumb A B P, and the Meridians paſſing through the ſaid Points, are all equal: alſo the ſeveral ∠ˢ at the Points A, *b*, *b*, C made by the Rumb A C P, and the reſpective Meridians, are likewiſe equal. In like manner, if you conceive Meridians to be drawn through any other Points in the ſaid Rumbs, the ∠ˢ made thereby will be all equal; whence it follows, that a Rumb can never fall into the Pole, though it may approach it infinitely near. And you may obſerve, that the nearer any two Places are to the Equator, the nearer does the Rumb, paſſing between them, approach to the Arch of a great Circle; and the contrary.

Before we begin to examine into the Nature and Properties of the ſeveral Parts of the △ˢ formed by the *Rumb Lines*, *Meridians*, and the *Parallels* of *Latitude*, I ſhall ſhew how to project the Sphere *in Plano*, in ſuch a manner, that they may be truly repreſented by right Lines; the Degrees of Latitude and Longitude bearing the ſame proportion to each other, as they do upon the Globe. In order to which, it is neceſſary to premiſe the following

LEMMA.

The Degrees of Longitude, (or the Diſtance Eaſt and Weſt intercepted between any two Meridians) continually decreaſe towards the Pole, in the ſame Proportion that the Co-Sines of the Latitudes grow leſs than the Radius.

Let

Let P repreſent the Pole of the World, P Q a Quadrant of the Meridian, C Q the Semidiameter of the Equator, and A B (=C D) the Semidiameter of any Parallel (for Examp. of 60°;) then is A B the Co-Sine of the Latitude of the Arch Q B, (C Q being Radius;) whence it is evident, that the Radius, (=C Q,) is to the Co-Sine of the Latitude (=A B;) as the Semidiameter of the Equator (=C Q,) to the Semidiameter of the Parallel. *Fig.* 3.

Whence (becauſe the Peripheries of all Circles are directly as their Radij) it follows;

1. *The Circumference of the Equator, Is to the Circumference of any Parallel; As the Radius, To the Co-Sine of the Latitude of the ſaid Parallel.*

And ſince ſimilar Parts have the ſame Ratio with their Wholes, it follows,

2. *The Length of a Degree, or any Portion of the Equator, Is to the Length of a Degree, or the like Portion of any Parallel; As the Radius, To the Co-Sine of the Latitude, or Diſtance of that Parallel from the Equator.*

Again, becauſe the $\triangle^s$ C D B, C Q E are Similar, it will be,

C D : C B :: C Q : C E; that is,

As the Co-Sine of the Latitude of any Parallel, Is to the Radius; So is the Radius, To the Secant of that Parallel. Whence it follows,

3. *As the Secant of any Parallel, Is to the Radius; So is the Length of a Degree, or ſmall Portion of the Meridian,* (which is always equal to the like Portion of the Equator) *To the Length of a Degree, or the like Portion in that Parallel.*

From the Conſideration of the laſt Corollary, Mr. *Edward Wright* (the famous Inventor of the following Projection) made the Meridians parallel right Lines, and ſo protracted or lengthened the Degrees of Latitude in the Projection, in like proportion to thoſe of Longitude, as they are upon the Globe: that is, By making a Minute (or ſmall Portion) of Latitude ſo protracted, To a Minute of Longitude; As the Secant of the Latitude, Is to the Radius. Hence it follows;

1. *The Length of an Infiniteſimal Arch upon the inlarged Meridian, is every where equal to the Secant of the Arch contained between it and the Equator; the like infiniteſimal Arch of the Equator being made Radius.*

2. *The Diſtance of any Parallel from the Equator, is equal to the Sum of all the Secants contained between it and the Equator.*

3. *The Diſtance between any two Parallels on the ſame Side of the Equator, is equal to the Difference of the Sum of the Secants,* (of all the Arches,) *contained between each of the Parallels and the Equator.*

4. *The Diſtance between any two Parallels on contrary Sides of the Equator, is equal to the Sum of the Sum of the Secants,* (of all the Arches) *contained between each of the Parallels and the Equator.*

Note, In the three laſt Corollaries, the ſame infiniteſimal Arch mentioned in the 1 *Cor.* is underſtood to be made Radius.

The

The eſſential Property of this incomparable Projection, being thus found out and demonſtrated; the next thing requiſite was to form a Table for the ready dividing of the Meridian.

Now, it being demonſtrated, that the Diſtance of each Point of Latitude from the Equator, (in this Projection) is equal to the Sum of the Secants of all the intermediate Arches contained between any Parallel and the Equator; Mr. *Wright* ſet about forming ſuch a Table by the continual Addition of the Secants of all the Parallels of Latitude; beginning with the Parallel of 1 Min. (which he made Radius) and thereto adding the Secant of the 2d Parallel, or of 2′ and to the Sum of theſe two adjoining the Secant of 3 Min. *&c.*

As for Example, the Secant of 1 *Min.* which is 10.000000, ſheweth the Diſtance of 1 *Min.* of the Meridian from the Equator in the *Nautical Chart*; to which, if the Secant of 2′ be added, which is 10.000002, the Sum =20.000002 gives the Diſtance of 2 *Min.* from the Equator; to this Sum add the Secant of 3′, which is 10.000004, the Sum =30.000006 is the Diſtance of 3 *Min.* from the Equator, *&c.* and the Table thus formed is that which is commonly called, *A Table of Meridional Parts.*

A Table conſtructed as above, is ſufficiently exact for all Uſes in Navigation, though it may differ ſomewhat from the Truth, becauſe the ſeveral Arches taken ſhould have been infiniteſimals; but the Error in all Caſes (at leaſt ſo far as Navigation is practicable) is ſo ſmall, that it need not be regarded.

From a Table of Meridional Parts ready calculated, it will be eaſy to make a true *Nautical Chart* (commonly called *Mercator's*) that ſhall contain any Number of Degrees of Latitude and Longitude. For Example, let it be required to make a *Chart* to reach from the Equator to the Parallel of 70 Degrees of Latitude, and to contain 90 Degrees of Longitude.

Plate 4. 1. Draw an Eaſt and Weſt-Line Æ Q, for to repreſent the Equator, and at right Angles to it, draw the Meridian Line Æ V.

2. From a convenient Scale of equal Parts lay 5400 (the Number of *Min.* in 90 Deg.) from Æ to Q; and draw Q W parallel to Æ V.

3. Lay the Meridional Parts anſwering to 70 Deg. of Lat. =5966 from Æ to V, and from Q to W, and draw the Line V W, and the Chart is limited.

4. With 600 betwixt the Compaſſes divide the Lines Æ Q, V W into 9 equal Parts, or to every 10 Degrees, and draw the *Meridians* 10 10, 20 20, *&c.* then ſubdivide each of theſe Diviſions into 10 equal Parts for Degrees, through which, if your Scale is pretty large, you may draw Meridians as before; again, you may ſubdivide each of theſe Degrees into ſo many aliquot Parts as the Length of a Degree will admit of.

5. When the Chart commences from the Equator (as in this Example,) take the Meridional Parts out of the Table anſwering to each Degree, *&c.* of Latitude, and (from the ſame Scale wherewith the Equator was divided) lay

Fig. I.

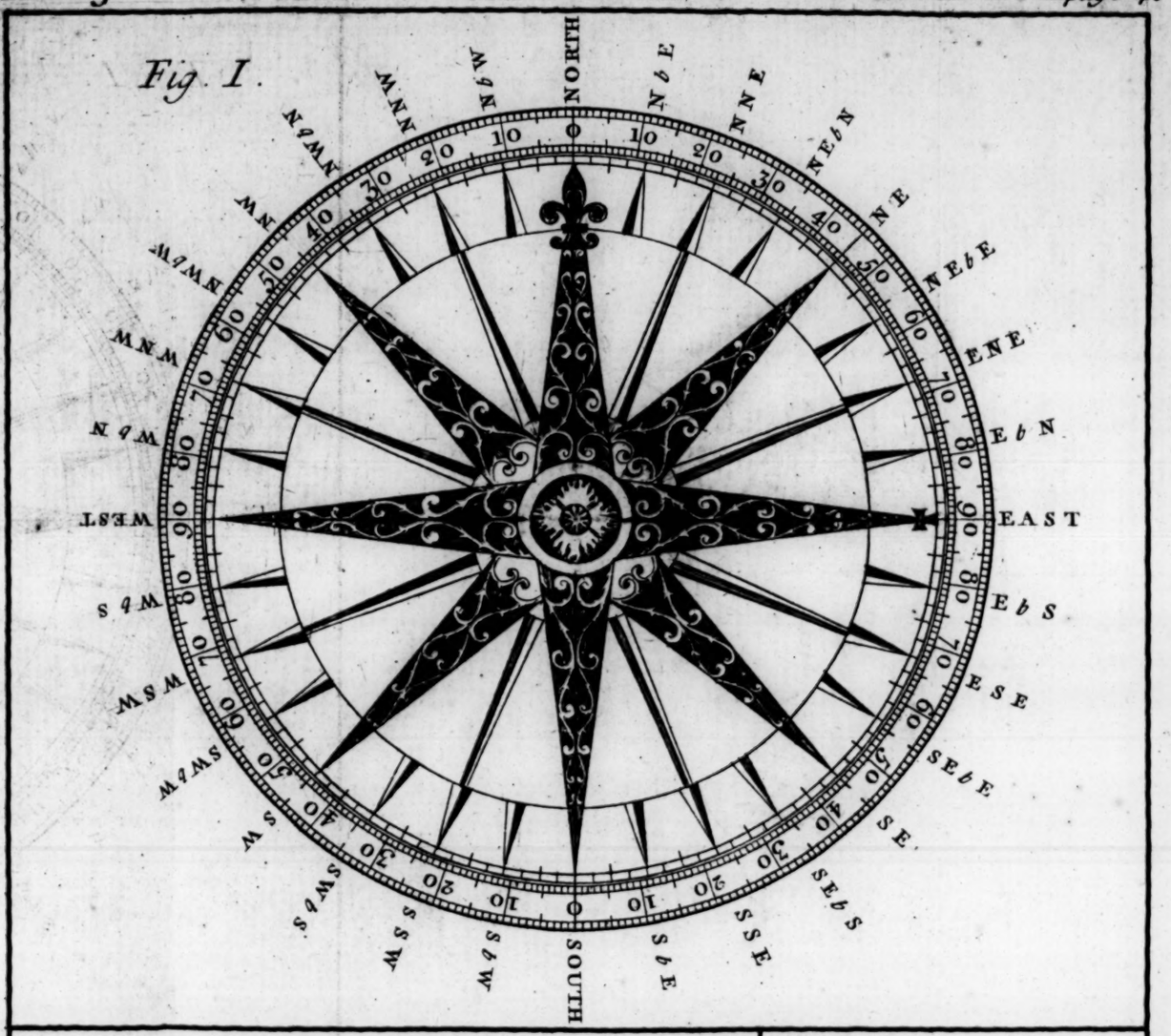


Fig. II

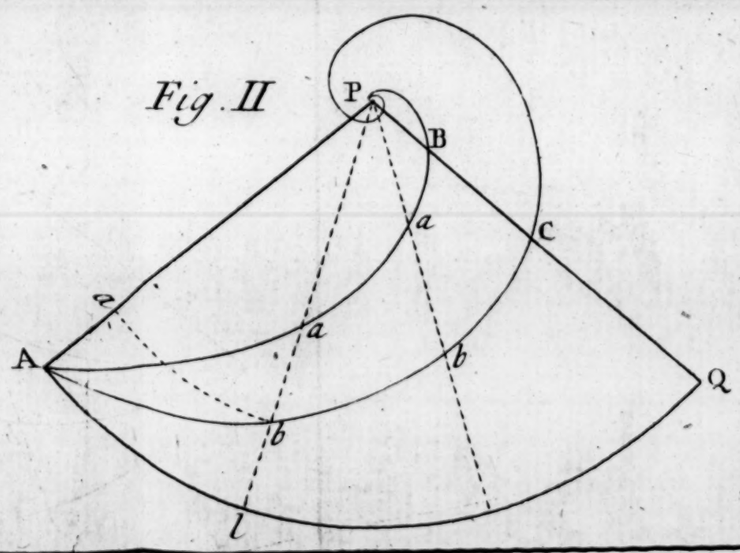
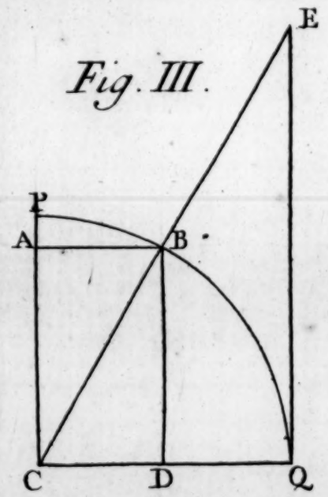


Fig. III.



lay the ſaid Meridional Parts ſeverally in the Lines Æ V, Q W, from Æ and Q; then through every 10 Degrees (or as you judge moſt convenient) draw the *Parallels* of *Latitude* 10 10, 20 20, *&c.* and the *Chart* is finiſhed.

N. B. It is cuſtomary, whether the Chart be for a Northern or Southern Latitude, to make the Top or Fore-part the *North:* But the *Meridians* may be numbered either Eaſterly or Weſterly, as you judge convenient.

If the Chart does not commence from the Equator, the *Meridians* may be drawn after the ſame manner as in the aforegoing Example: But in order to draw the *Parallels* of *Latitude*, ſubſtract from the Meridional Parts anſwering to each Point of Latitude the Meridional Parts anſwering to the leaſt Latitude in the Chart, and lay the Differences ſeverally upon the two extreme Meridians, from the Parallel of the leaſt Latitude, and through the Diviſions of every 10 Degrees, or as you judge moſt convenient, draw right Lines repreſenting the *Parallels of Latitude*, as before.

Or, the *Meridian* may be divided without any previous Subſtraction of the Meridional Parts; for Example, let it be required to divide the Meridian from 20° to 50°. 1. Set 225 the Exceſs above 1000, of the Meridional Parts anſwering to 20 Deg. downwards from 20° in the Meridian to *p*; from the Point *p* lay 289, the Exceſs of the Meridional Parts anſwering to 21 Deg. above 1000 upwards, and that will give the Point anſwering to the Parallel of 21°. In like manner ſet the ſeveral Exceſſes (above 1000) of each Point of Latitude upwards from *p*, until you come to the Parallel of 32°, where the Meridional Parts exceed 2000; then lay 28 the Exceſs of the Meridional Parts anſwering to 32 Deg. of Latitude (above 2000) from the Point of 32° downwards to *q*, and from the Point *q* lay 99 the Exceſs (above 2000) of the Meridional Parts anſwering to 33 Deg. upwards to 33°, *&c.* Thus, the *Meridian Line* may be divided with as much Eaſe and Expedition, as if it were a *Line* of *equal Parts.*

After the ſame manner may the *Minutes*, (or any Point of Latitude) be ſet off, by taking the Exceſſes of their Meridional Parts, as above directed.

N. B. For Diſtinction ſake, the equal Degrees, *&c.* upon the Meridian, contained between any two Parallels, (which are every where equal to the Degrees, *&c.* upon the Equator,) are called the *Proper Difference of Latitude*; and the Difference of the Meridional Parts anſwering to any two Latitudes, is called the *Meridional Difference of Latitude.*

Upon a Chart thus made, all Places may be laid down true, according to their Latitudes and Longitudes; the Degrees of Latitude here increaſing, in the ſame Proportion that the Degrees of Longitude decreaſe upon the Globe; as for Example, the Diſtance between the Parallel of 60° and the Parallel of 1 *Min.* diſtant from it, is twice as great as the Length of 1 *Min.* of Longitude; the Diſtance between the Parallel of 70° and the Parallel of 1 *Min.* next adjacent to it, is near three times as great; and the Diſtance between the Parallel of 80° and the next Parallel to it, is

above 5 times as great as a Minute of Longitude in this Projection; exactly anſwering to the Proportion a Minute of Latitude has to a Minute of Longitude, in theſe Parallels upon the Globe; and ſo of all the reſt.

Becauſe the Meridians here are all parallel right Lines, it follows, that all the Rhumbs (or the Paths deſcribed by a Ship, in ſailing according to the Direction of the Compaſs) are perfect right Lines; which Property renders this *Projection* far more uſeful and commodious for the *Mariners* Uſe, than any other whatſoever.

The ſeveral Uſes of this Projection, and how to ſolve all the Caſes relating to the Ship's Motion upon a Rhumb thereby, ſhall be ſhewed in *Sect.* 2. of the next *Chapter*. I ſhall next proceed to examine into the Qualities and Properties of the Triangles (already mentioned to be) formed upon the Globe, by the Meridians, Parallels of Latitudes, and the Rhumb-Lines.

Let there be drawn a *Rhumb-Line* A H (for example here, of 3 Points from the Meridian) quite athwart the Chart; and from the ſeveral Points B, C, D, *&c.* where the ſaid Rhumb interſects the Parallels of every 10 Degrees of Latitude, let there be drawn the *Meridians* B *b*, C *c*, D *d*, *&c.* from each Parallel to the next Parallel of 10 Degrees; then make A *a*, B *a*, C *a*, D *a*, *&c.* equal to the *Proper Difference* of *Latitude* of 10 Degrees, which of neceſſity muſt be every where equal.

Again, from the ſeveral Points *a*, draw the ſeveral Lines *a b* all at *Departure.* right Angles to the Meridian; which Lines may be called the DEPARTURE.

Now in the ſeveral $\triangle^s$ A *a* B, B *b* C, C *c* D, *&c.* A *a*, B *b*, C *c*, D *d*, *&c.* are the *Meridional Differences* of *Latitude* (or the Diff. of Latitude according to this Projection) A *a*, B *a*, C *a*, D *a*, *&c.* are the *proper Differences of Latitude*; the Segments of the Parallels *a* B, *b* C, *d* E, *e* F, *&c.* are the *Differences* of *Longitudes*; The ſeveral $\angle^s$ at A, B, C, D, *&c.* are the *Courſe*; And the ſeveral Lines *a b* the *Departures*; The Segments of the Rhumb-Line A *b*, B *b*, C *b*, *&c.* are the *Diſtances* upon the Rhumb in equal Parts; and the Segments A B, B C, C D, *&c.* intercepted between the Parallels, are the Diſtances according to this Projection.

Now, becauſe in the ſeveral $\triangle^s$ A *a* B, B *b* C, C *c* D, *&c.* the Meridians A *a*, B *b*, C *c*, D *d*, *&c.* are all parallel to each other, and the Rhumbs A B, B C, C D, *&c.* are all Segments of the ſame right Line; it follows, that the Courſes, or the ſeveral $\angle^s$ at A, B, C, *&c.* are all equal, in this Example = 3 Points. Again, becauſe the right Lines *a* B, *b* C, *c* D, *d* E, *&c.* and alſo the ſeveral Lines *a b*, are all parallel, all theſe $\triangle^s$ are *ſimilar*. Whence ariſe the following Corollaries;

Corol. 1. *The Segments or Portions of the ſame or like Rhumb, contained between any two Parallels having the ſame Difference of Latitude, are equal to each other; that is, the Diſtance* A *b contained between the Equator and the Parallel of* 10 *Degrees of*

of Latitude, is equal to G*b the Diſtance upon the ſame Rhumb, contained between the Parallels of* 60 *and* 70 *Degrees; for the proper Differences of Latitude* A*a*, G*a in both Caſes is the ſame, and the* Δ^s A*a b*, G*ab are ſimilar, therefore,* &c. *and ſo betwixt any other Parallels whoſe Difference of Latitude is the ſame.*

Corol. 2. *After the ſame manner we prove, that to a like Diſtance upon the ſame Rhumb, or to the ſame Courſe and Diſtance, there will be always the ſame Difference of Latitude and Departure; &* vice verſa. *And to the ſame Difference of Latitude and Diſtance, there will be always the ſame Courſe and Departure; &* vice verſa. *And to the ſame Courſe and Difference of Latitude, there will be always the ſame Departure and Diſtance;* vice verſa, *&c.*

Corol. 3. *The Differences of Longitude a*B, *b*C, *c*D, *d*E, &c. *correſponding to the equal Departures* a b, *between different Parallels are unequal; increaſing the farther they are from the Equator, in proportion, as the proper Difference of Latitude to the Meridional Difference of Latitude; for Inſtance, in the* Δ G g H *lying between the Parallels of* 60 *and* 70 *Degrees, becauſe the Departure* a b *is parallel to the Difference of Longitude* g H; *it will be,*

G*a* : G*g* :: *ab* : *g*H; *that is,*

As the Proper Difference of Latitude,
Is to the Meridional Difference of Latitude;
So is the Departure,
To the Difference of Longitude.

Hence it appears, that the *Departure* and the *Meridional Diſtance*, (which all the Writers of Navigation took to be the ſame) are different things; for by the preceding *Lemma*, the Proportion of the *Meridional Diſtance* (in *Miles*) To the Difference of Longitude (in *Deg.* and *Min.*) in any Parallel, is, As the Co-Sine of the Latitude of that Parallel, To the Radius; or, As the Radius, To the Secant of the Latitude of that Parallel. But it has been demonſtrated above, that the Proportion of the *Departure* made by ſailing upon any Rhumb, between any two Parallels, Is to the Difference of Longitude made upon that Rhumb; As the Proper Difference of Latitude between theſe Parallels, To their Meridional Difference of Latitude; or, which is the ſame thing, As the Proper Difference of Latitude, To the Difference of the Sum of the Secants contained between each of the ſaid Parallels and the Equator.

That all the Writers of Navigation miſtook the *Departure* and *Meridional Diſtance* for the ſame, may be eaſily made appear; for if they underſtood them to be different, they ought to have diſtinguiſhed between them, but this Diſtinction was never made by any Author that I met with: and ſeeing that moſt Books that have been wrote upon this Subject, are, as it were, Copies of one another, both as to Subſtance and Method, I may venture to ſay, that the above-mentioned Diſtinction was never made by any Author whatſoever; but on the contrary, ſome of the moſt noted of them expreſs themſelves in this Manner, or to this Effect; viz. "*The "Departure or the Meridional Diſtance, is the Diſtance of any two Places lying "under the ſame Parallel, or the Diſtance of the Meridians paſſing through two "Places lying under different Parallels, numbered in Degrees or Miles of the "Equator.*" And leſt the Reader ſhould think, that our Diſpute is only about Words, I muſt take notice further, that the Quantity which they call the Departure or Meridional Diſtance, is ſometimes one, and ſometimes the other; for when a Ship ſails directly Eaſt or Weſt under any Parallel, the Quantity which they call promiſcuouſly as above, is then the *Meridional Diſtance*, and anſwers truly to their Definition; but when a Ship ſails upon an Oblique Rhumb, what they call the Departure or Meridional Diſtance, then anſwers only to our Definition of *Departure*; (as appears from the *Data* for finding it) and conſequently is quite another thing from what they took it to be; and ſince we never had but one Definition of theſe two different things, I preſume it is ſufficiently plain, that all the Writers upon this Subject took them to be the ſame. And though it may be alledged that the Rules called *Mercator's Sailing* are true, (which indeed are demonſtrated ſo to be, in the laſt *Cor.* foregoing) yet as I have made it appear, that the *Departure*, (or the Quantity ſo called) concerned in theſe Rules, is a different thing from that which the Writers of Navigation took it to be; it appears farther that theſe Rules are true, when theſe Authors differ from themſelves, and conſequently their Writings are ſo much the more abſurd.

Here, I find myſelf under a neceſſity of taking notice of a Paragraph in N°. 219. of ſome Philoſophical Writings, wherein the Author of an ingenious Tract in the ſaid *Numb.* made no diſtinction betwixt the Departure and the Meridional Diſtance; *viz.* "Becauſe that in every point of "any *Rhumb-Line*, the difference of Latitude is to the *Departure*, as the "*Radius* to the *Tangent* of the angle that Rhumb makes with the Meri-"dian; and thoſe equal *Departures* are every where to the differences of "Longitude, as the Radius to the Secant of the Latitude; it follows, *&c.*" The Proportions expreſſed in the afore-cited Paragraph, I have already demonſtrated to be true, the one of the Departure only, and the other of the Meridional Diſtance only; and I have likewiſe demonſtrated, that the Departure and Meridional Diſtance are things vaſtly different, and conſequently ought to have been diſtinguiſhed under different Denominations.

The

The Author of the Treatiſe above-mentioned, being a Perſon who by ſeveral learned and ingenious Tracts, has given the World a ſufficient Proof of his eminent Skill in all Parts of the Mathematicks; I was very unwilling to take notice of the above-cited Paragraph, as a Miſtake: but becauſe what I have advanced relating to the Departure and Meridional Diſtance, is, I think, entirely new; I was obliged to be the more particular, in order to obviate the Objections perhaps ſome might make againſt me, upon our Author's Authority, as well as that of the Place where the aforeſaid Tract is inſerted; and the more ſo, becauſe I find the ſame Expreſſion verbally uſed by one of our moſt eminent Writers of Navigation.

Some of the Writers of Navigation ſay, that the Departure or Meridional Diſtance, is the Difference of Longitude according to the *Plain Chart*, which ſuppoſes the Earth to be one continued Plain, or Flat; as to thoſe Gentlemen, I do not think it worth while to diſpute with them, any more than if they had ſuppoſed the Earth to be a Pyramid, or of any other Figure; but under the next Corollary I ſhall ſhew, that the Rules which they call Plain Sailing are ſtrictly true, though undoubtedly falſe according to their Notions of the Earth being a Plain.

That middle Latitude ſailing is erroneous and abſurd.

From the laſt *Corollary* and the preceding *Lemma*, the Error as well as the Abſurdity of what the Writers of Navigation call *middle Latitude Sailing*, may be eaſily made appear. The Rules called middle Latitude Sailing are grounded upon this Suppoſition; *viz.* That *the Co-Sine of the middle Latitude*, *the Radius*, the *Departure or Meridional Diſtance*, and the *Difference of Longitude*, are directly proportional.

What firſt introduced this abſurd Notion of *middle Latitude Sailing*, I can't conceive; but here it ſeems the Authors imagined, that the *Departure* or *Meridional Diſtance* (as they call it) made upon an Oblique Rhumb, is a Mean betwixt the *Meridional Diſtance* intercepted in each of the extreme Parallels betwixt the Meridian departed from, and that arrived to; but it has been already demonſtrated that there is no manner of relation betwixt the *Departure* and *Meridional Diſtance*, therefore their Notion muſt be falſe: but granting the ſaid Notion to be true, becauſe the Co-Sines of the Latitudes do not decreaſe in an arithmetical Ratio, the aforeſaid Proportion will even then be falſe.

Again, If we take the *Departure* to be what it really is, I do not conceive how we can make any manner of Compariſon betwixt the true Proportion, and that which they make uſe of; the true Proportion is, *As the Proper Difference of Latitude, To the Meridional Difference of Latitude; So the Departure, To the Difference of Longitude.* The two laſt Terms in this Proportion, being the ſame with the two laſt Terms in the fore-mentioned Proportion of *middle Latitude Sailing*; it follows, that there ſhould be ſome Similitude betwixt the two firſt in each;

viz. { *As the Proper Difference of Latitude, To the Meridional Diff. of Latitude.*
{ *As the Co-Sine of the middle Latitude, To the Radius.*

But

But how theſe two *Ratio's* may be compared together, is what I can't conceive.

Here perhaps ſome may ſay, that notwithſtanding theſe Rules are falſe, yet in Voyages near the Equator, and alſo in very ſhort Runs, the Error is but ſmall; I grant, that near the Equator, the Difference betwixt the Length of an Arch of a Parallel, and a correſpondent Arch of the Equator, is not very great, and conſequently there the Error cannot be ſo conſiderable; and undoubtedly in very ſhort Runs the Error cannot be ſo great as in long ones: But for as much as theſe Rules are every where more or leſs erroneous, it is certainly very abſurd to uſe them; and the more ſo, becauſe there is neither time nor trouble ſaved. The Abſurdity of uſing theſe Rules appears farther from their Multiplicity, their making more Variety of Caſes than all the true ones put together. It is very odd, that People muſt thus burden their Memories with a heap of Falſities! and I am very apt to believe, that the Multiplicity of Rules heaped together in our Books of Navigation, has often deterred *Mariners* from learning what they really ought, and more eſpecially, becauſe they ſee all theſe Rules different, and do not know which to chuſe; and I often admired how deliberately the Writers of Navigation (and I ſuppoſe the Teachers the ſame) teach the Geometrical Conſtruction of theſe erroneous Caſes, tho' at the ſame time they are grounded upon no one Principle in *Geometry*, no not even ſo much as upon any *Hypotheſis* of their own.

Corol. 4. *If a Ship, either from the Equator, or from ſome Parallel of Latitude, ſuppoſe from* A *in the Parallel of* 60 *Deg. ſails upon any Rhumb towards the Pole, until ſhe arrives, ſuppoſe at* B *in the Parallel of* 70 *Deg. If ſhe returns upon the oppoſite Rhumb* B A, *ſhe'll make the ſame Departure and Difference of Longitude homewards, as ſhe did in her Voyage outward.*

This is evident of itſelf, becauſe the *Differences of Latitude* A *a*, B *a*, and alſo the *Courſes* C A B, A B D are equal, the *Departures a b*, *b a*, (as alſo the *Diſtances* A *b*, B *b*) muſt be likewiſe equal; again, becauſe the *Meridional Differences* of *Latitude* A C, B D are equal, the *Differences of Longitude* A C, B D muſt be likewiſe equal.

Hence, it appears farther, that *Plain Sailing*, or the *Rules* ſo called, are ſtrictly true; for any two given of theſe four; *viz.* the *Courſe*, *Diſtance ſailed*, *Difference of Latitude*, and *Departure*, together with the *Latitude* ſailed from, will determine the true Place of the Ship: yet Plain Sailing, according to the Senſe in which it has hitherto been taken, is falſe; for if the Earth be ſuppoſed to be a Plain, I preſume none will deny its being then falſe; and again, becauſe it has been demonſtrated, that none of the *Writers of Navigation* underſtood the Nature of *Departure*, it follows, that it is every way falſe according to their Precepts.

From

From this Corollary, and the aforegoing one, it appears that the *Departure* is a Quantity of no determinate Length, (and therefore cannot be any way expreſſed upon the Globe ;) but only an Ordinate of a Circle, as ſometimes a *Sine*, and ſometimes a *Tangent*, according to which Side of the Triangle is made *Radius*; and conſequently there is as wide a Difference betwixt the *Departure* and *Meridional Diſtance*, as there is betwixt an *Angle* and a *Side*, or betwixt a *Circle* and a *Right Line*; and the Writers of Navigation might as well take the *Latitude* and *Longitude*, or the *Courſe* and *Diſtance*, &c. for the ſame thing, as the *Departure* and *Meridional Diſtance*.

That if a Ship ſails upon any Rhumb from the Equator towards the Pole, and then returns upon the oppoſite Rhumb, ſhe'll arrive to the Place ſhe departed from, is a Truth ſo evident, that I ſhould not have taken notice of it, had not I met with ſome People of a contrary Notion; to ſupport which, a reputed Author upon this Subject, in a Book entituled, *A Syſtem of the Mathematicks*, Vol. I. page 145, argues thus ;

" If a Ship in ſailing from A, upon a given Rhumb and a given Diſtance, *Plate 3.*
" arrive at *b*, in returning back by the oppoſite Rhumb, ſhe will not arrive *Fig. 2.*
" at A, but fall wide of it nearer to *l*; for becauſe A *l* is greater than
" *a b*, the Angle A *b l* is greater than *a* A *b*, and conſequently the Ship
" in returning, muſt ſail upon a different Rhumb, making a greater Angle
" with the *Meridian*, to arrive at A."

What led People into this Miſtake, muſt be (from their way of arguing) their wrong Notion, that the *Departure* (ſo called, ariſing upon an Oblique Courſe) is the Diſtance between the *Meridian* departed from, and that arrived into ; the Error of which Notion I have already ſhewed. However, I ſhall endeavour to convince them of their Miſtake, from their own way of Reaſoning; for though the *Arch a b* is leſs than the *Arch* A *l*, yet it contains the ſame number of Degrees, becauſe both the *Arches* are intercepted between the ſame *Meridians*; and conſequently if theſe Arches were the Meaſures of the Angles aforementioned, the ſaid Angles would have been equal. But the *Rhumb* A *b* is not any wiſe affected by theſe Arches, for it is the eſſential Property of a *Rhumb-Line* to make the ſame or equal Angles with all the *Meridians* through which it paſſeth ; and let the *Compaſs* be placed in what Point ſoever of the *Rhumb* A *b*, it is very abſurd to ſuppoſe that it will not there point the ſame way, whether the Direction of the Ship be from A towards *b*, or from *b* towards A.

The ſame Author continues; " *And this is one notorious Error that*
" *attends the* Plain Sailing, *and is the Reaſon why thoſe who keep their*
" *Reckoning by the* Plain Chart *are forced to make more Eaſting or Weſting,*
" *as they call it, to arrive at the ſame Place when they ſail towards the* Equa-
" tor, *than when they ſail from it, though they know not from whence the Cauſe*
" *proceeds.*"

That the *Plain Chart* is falſe, is a Truth ſelf-evident, and that *Plain Sailing* is falſe according to the Senſe in which it has hitherto been taken, (though true in itſelf,) has been already ſhewed above; and alſo the ſaid Inference, viz. *That People muſt make more Eaſting or Weſting* (by which is meant the DEPARTURE) *to arrive at the ſame Place, when they ſail towards the* EQUATOR, *than when they ſail from it*, has been likewiſe ſhewed to be falſe; yet as this is a prevailing Notion, I believe univerſally received, at leaſt diſputed by none, I ſhall farther ſhew its Falſity under the ſeventh *Corollary*.

Corol. 5. *If a Ship plies upon any number of Courſes, and then returns homewards upon the oppoſite Rhumbs, and ſails the ſame Diſtance upon each Rhumb reſpectively, as ſhe did in her Voyage outwards, the Departure and Difference of Longitude made good, in both theſe Voyages, will be the ſame.*

For by the preceding *Corollary* to each Courſe and Diſtance *outwards*, and the correſpondent Courſe and Diſtance *homewards*, there will be the ſame *Departure* and *Difference of Longitude*; therefore it follows, that the whole Departure and Difference of Longitude made in both theſe Voyages, will be the ſame.

Corol. 6. *If a Ship from any Place, ſuppoſe from* A *in the Parallel of* 20 *Deg. plies upon any number of Courſes* (A B, B C, C D, *&c.*) *before ſhe arrives to* G *in any other Parallel; the Difference of* {*Latitude* / *Longitude*} *ſhe then makes good, will be equal to the Sum of the Differences of* {*Latitude* / *Longitude*} *correſponding to each ſingle Courſe, when the Differences of* {*Latitude* / *Longitude*} *are all the ſame way: But when the Differences of* {*Latitude* / *Longitude*} *are of different Denominations, the Difference of the Sums of the Differences of* {*Latitude* / *Longitude*} *made* {*Northerly and Southerly* / *Eaſterly and Weſterly*} *will be the Difference of* {*Latitude* / *Longitude*} *made good; which muſt needs be of the ſame Denomination with the greateſt: Alſo the Difference of Latitude and Longitude made good, will be equal to the Difference of Latitude and Longitude the Ship would have made, if ſhe had ſailed from* A *to* G *upon a direct Courſe.*

This

This *Cor.* is pretty evident of itſelf, but as it is an important Article in Navigation, we ſhall not paſs it over without a Demonſtration. 1. When the Ship is at B, A *a* is the *Difference of Latitude* ſhe made upon the Rhumb A B, and *a* B (=H *g*) the *Difference of Longitude.* 2. B *b*=*a c* is the Difference of Latitude made upon B C, and conſequently A *c* is *the Difference of Latitude* made upon the two Courſes A B, B C; again, *b* C (=*g h*) is the Difference of Longitude correſponding to the Rhumb B C, and therefore H *h* (=H *g*—*h g*) is the *Difference of Longitude* made good when the Ship is at C. 3. Becauſe C D is (here ſuppoſed to be) an Arch of a Parallel, the Points D and C are both in the ſame Latitude, and C D=*h i* is the Diff. of Longitude made upon C D, which added to H *h* gives H *i* the *Diff. of Long.* of the Point D (from the Meridian of A.) 4. D E is here ſuppoſed to be an Arch of the Meridian, therefore the Ship has not altered her Longitude in ſailing from D to E; but the Difference of Latitude is the Diſtance ſailed D E=*c d*, and conſequently A *d* (=A *c*+*c d*) is the *Difference of Latitude* made upon the ſeveral Courſes from A to E. 5. E *f*=*d e* is the Diff. of Latitude made upon the Rhumb E F, which becauſe it is towards D muſt be ſubſtracted from A *d*, and the Remainder A *e* is the *Difference of Latitude* made good, when the Ship is at F; again, *f* F =*i k* is the Diff. of Longitude made upon the Rhumb E F, and this added to H *i* gives H *k* the *Difference of Longitude* made good, upon the ſeveral Courſes from A to F. Laſtly, F *k* (=*e* H) is the Difference of Latitude, and *k* G the Difference of Longitude made upon the Rhumb F G; and theſe added to A *e* and H *k*, give A H and H G the whole *Difference of Latitude* and *Longitude* made in ſailing upon the ſeveral Courſes from A to G. And it is evident, that A H (=30°) would be the *Difference of Latitude*, and H G the *Difference of Longitude*, the Ship would have made if ſhe had ſailed directly from A to G.

Although it has been demonſtrated here, that the *Difference of Latitude* and *Longitude* made good, in ſailing any how betwixt any two Places, will be always the ſame; yet the *Sum* of the *Departures* will not always be the ſame, as ſhall be demonſtrated in the following

Corol. 7. *If a Ship plies upon ſeveral Courſes in ſailing between any two Places, the computed Departure may be either greater or leſs than the Departure ariſing upon a ſingle Courſe betwixt the ſaid Places; and that alſo more or leſs, according to the Quality of the Courſes: Alſo the computed Departure, made in ſailing from any Place towards the Equator, may be either greater or leſs than the computed Departure in returning home.*

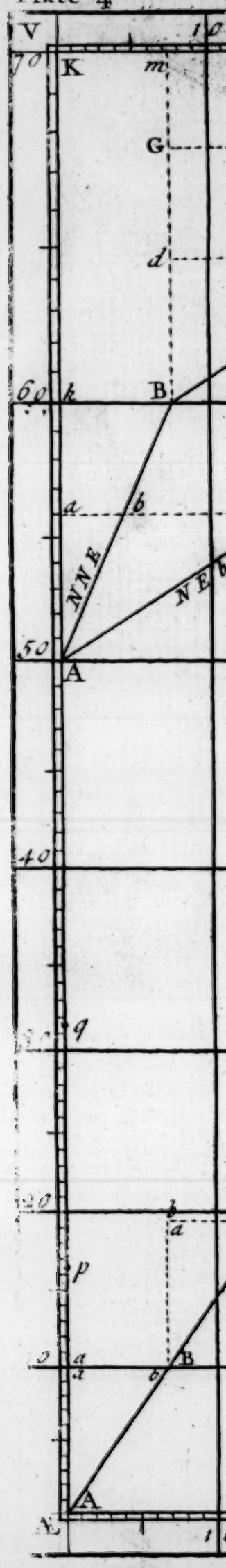
For *Examp.* Suppoſe two Ships ſet ſail from A in the Parallel of 50 Deg. and that one ſails N N E until ſhe comes to B in the Latitude of 60°, and then N E *b* E to C in the Lat. of 70 Degrees; and the other ſails

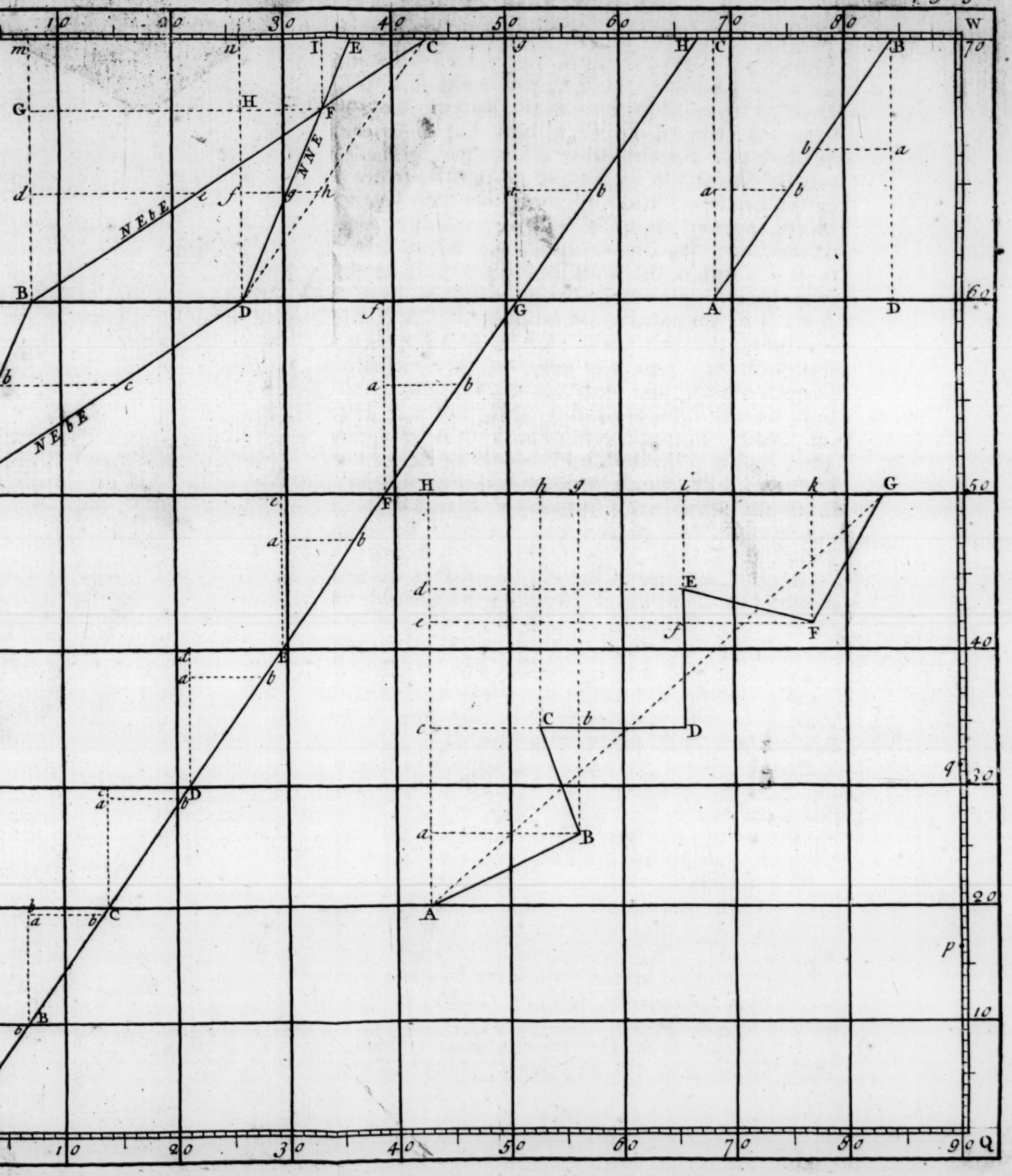
NE *b* E from A to D in the Parallel of 60°, and then NNE till ſhe comes to E in the Parallel of 70 Degrees: Through the Points B and D draw the *Meridians* B *m*, D *n*; then make A *a*, B *d*, D *f* each equal to the *proper Difference of Latitude* of 10 Deg. and through the Points *a*, *d*, and *f*, draw the Lines *a c*, *d e*, *f g* at right Angles to the Meridian. Now it is evident, that the Departure *a b* is equal to the Departure *f g*, and alſo that the Departure *a c* is equal to the Departure *d e*; therefore the *Sum* of the *Departures* made by theſe two Ships are equal, that is, the *Sum* of the *Departures* made in ſailing from A to B, and from B to C, is equal to the *Sum* of the *Departures* made in ſailing from A to D, and from D to E. Through the Point F (where the Rhumbs B C, D E interſect) draw F G at right Angles to the Meridian; now becauſe A D, B F, and A B, D F, are parallel, the alternate ∠ˢ A D B, F B D, and A B D, F D B are equal, therefore the △ˢ A B D, B D F are equiangular, and becauſe they have one Side B D common, they are alſo equilateral, and conſequently D F, and A B are equal; and becauſe A *k* is parallel to D H, and K D to G F; A *k*, D H, and *k* B, H F, and alſo K D, G F are equal. Through the Point F draw F I parallel to the Meridian: Then becauſe D H = A *k*; F I (= H *n*) is the *Exceſs* of the Meridional Difference of Latitude between the Parallels of 60 and 70 Degrees, above the Meridional Difference of Latitude contained between the Parallels of 60 and 50 Degrees. Now, if we make the ſaid Exceſs F I *Radius*, the Exceſs of the Difference of Longitude K C above the Difference of Longitude K E, will be equal to the Difference of the *Tangents* of the ∠ˢ I F C and I F E. And thus it appears in what proportion (in this Example) the Difference of Longitude made by one Ship is greater than the Difference of Longitude made by the other. Again, if the Line D C be drawn, it will be very evident why the *Departure f h* is greater than the *Departure f g*, and conſequently why a Ship in ſailing along the Rhumbs A D, D C, will make more Departure than if ſhe had ſailed along the Rhumbs A B, B C, to the ſame Point C.

Hence appears the Errors of thoſe who ſuppoſe, that a Ship ſailing towards the Equator will always make more Departure than when ſhe ſails betwixt the ſame Places from it. For, If a Ship from A in the Parallel of 50 Degrees ſails towards the *Pole* upon the Rhumbs A D, D C, and then returns back towards the *Equator*, upon the Rhumbs C B, B A; the *Departure* in this Caſe made towards the Pole, will be greater than the *Departure* homewards towards the Equator: But if the Ship had ſailed outwards upon the Rhumbs A B, B C, and homewards upon the Rhumbs C D, D A, the Caſe would have been the contrary: And it has been demonſtrated before, that if the Ship had ſailed outwards and homewards upon the oppoſite Rhumbs, the Departure in both Voyages would have been the ſame.

If the Points B and D were at a greater Diſtance from each other, the Difference betwixt the Departures made upon the Rhumbs A B, B C

Plate 4




1 0
2 0
3 0
4 0
5 0
6 0
7 0
8 0
W
7 0
6 0
5 0
4 0
3 0
2 0
1 0
Q
P
q
m
n
I
E
C
g
H
C
B
G
H
F
d
e
f
g
h
b
a
NEbE
NNE
B
D
G
A
D
b
c
NEbE
f
a
b
H
h
g
i
k
G
c
F
a
b
d
E
F
a
b
c
C
b
D
c
a
b
D
a
B
b
a
b
C
A
b
B
b

and A D, D C betwixt the Points A and C, would have been ſtill greater; and the ſame might happen if the Courſes betwixt A and C were more compounded. And if two Ships ſail upon different Courſes betwixt any two Places in the Chart, there will always be ſome Difference in their *computed Departure*; (except by chance one Error ſhould correct another) and that more or leſs according to the Qualities of the Courſes, and the Latitudes of the Places.

The Error of keeping a Ship's Reckoning by Meridional Diſtance, (as it is called.)

Hence appears the Errors of thoſe who keep their Reckonings by *Meridional Diſtance*; by which they mean the Sum of the ſeveral Departures correſponding to each ſingle Courſe: and if the Ship ſhould ſail under a Parallel, they add that Meridional Diſtance likewiſe to the Sum of the Departures, and call the whole the *Meridional Diſtance* as before.

What led People into this Error, I ſuppoſe may be their miſtaken Notion of *Departure* and *Meridional Diſtance*; for as they took theſe two things to be the ſame, it ſeems they imagined, that the *Sum* of the *ſeveral Departures* would give the *whole Meridional Diſtance* from the Place departed from: but had they conſidered a little farther, they would have found it to be erroneous even in that Senſe; for theſe Meridional Diſtances ought to have been counted ſeverally from the ſeveral Meridians paſſing through the Terms of each reſpective Courſe. But as I have demonſtrated, that the Departure and Meridional Diſtance are different things; and alſo that the Sum of the ſeveral Departures made in two Voyages betwixt the ſame Places can never be the ſame, (except the Ships ſhould ſail exactly in the ſame Tract, or that one Error ſhould correct another, neither of which Chances perhaps ever happened) it follows, that this Method of keeping a Reckoning is groſly falſe; and that it is merely impoſſible by it to make the ſame Reckoning twice betwixt the ſame Places: and ſeeing this is the Method which is (almoſt) univerſally practiſed, the Uncertainty of *Mariners* Reckonings is not to be wondered at.

Since the Nature of the *Departure* was never underſtood, it is no wonder if that Miſtake led People into ſeveral Errors: but having now, I preſume, clearly demonſtrated the true *Theory* of *Navigation*, and from thence the ſeveral Errors of the Writers upon that Subject; I hope for the future, that *Mariners* will be pleaſed to conform themſelves to Truth, and wholly lay aſide their erroneous Practices; and then there will be no room to doubt, but that they will find by Experience, that the Uncertainty of the *Data* in the Practice of Navigation, is not near ſo great as what has hitherto been imagined.

CHAP. II.

NAUTICAL PROBLEMS: *or, The Solutions of all the Caſes in Navigation, that ariſe in ſailing upon a Rhumb.*

FIRST, I ſhall premiſe the following general Rules relating to the *Difference of Latitude.*

Note, A Degree of a Great Circle upon the Superficies of the Earth is divided into 60 *Geographical* or *Nautical Miles*; wherefore the Difference of Latitude in {Degrees / Miles} is reduced into {Miles / Degrees} by {Multiplying / Dividing} by 60.

PROB. I. *Having both Latitudes given, to find the Difference of Latitude.*

Rule. If the Latitudes are of {the ſame / different} Denomination, their {Difference / Sum} is the Difference of Latitude required.

Examp. 1. Let it be required to find the Difference of Latitude betwixt the *Lizard* in Lat. 49° 55′ North, and *Gibraltar* lying in 36° 12′ North.

From the Latitude of the Lizard	49° 55′ N
Take the Latitude of Gibraltar	36 12 N
Remains their Difference of Lat.	13 43 *Deg. & Min.*
	60
	823 *Miles.*

Examp. 2. The Difference of Latitude betwixt *Cape-Finiſter* lying in the Lat. of 48° 06′ North, and the *Cape of Good-Hope* in the Lat. of 34° 25′ South, is required.

To

To the Lat. of Cape-Finiſter	48° 06′	N.
Add the Lat. of the Cape of Good-Hope	34 25	S.
The Sum, is the Diff. of Latitude	82 31	*Deg. & Min.*
	60	
	4951	*Miles.*

PROB. II. *Having one Latitude and the Difference of Latitude given, to find the other Latitude.*

N. B. If a Ship ſails {from / towards} the Equator, ſhe {increaſes / decreaſes} her Latitude juſt ſo much as is the Diſtance ſailed. Wherefore,

1. If the Latitude departed from, and the Difference of Latitude made, are both the ſame way, that is both Northerly or both Southerly; to the Latitude given, add the Difference of Latitude, and the Sum will be the Latitude required.

Examp. If a Ship from the Lat. of 36° 50′ N. ſails Northerly until her Difference of Latitude be 850 Miles; I would know the Latitude ſhe is arrived into.

To the Latitude ſailed from	36° 50′	N.
60)		
Add the Diff. of Latitude made 850 *m.* =	14 10	N^{ly}.
The Sum is the Latitude the Ship is in	51 00	N.

2. When the Latitude departed from, and the Difference of Latitude are of a contrary Denomination, that is, one *Northerly* and the other *Southerly*; then, from the Latitude ſailed from, take the Difference of Latitude made, and the Remainder will be the Latitude the Ship is in.

Examp. Admit a Ship from the Latitude of 46° 30′ N. ſails Southerly until her Difference of Latitude be 960 Miles, and it be required to find the Latitude ſhe is arrived into.

From the Latitude ſailed from	46° 30′	N.
60)		
Take the Diff. of Latitude 960 *m.* =	16 00	S^{ly}.
Remains the Lat. the Ship is in	30 30	N.

In this *Caſe*, when the Difference of Latitude is greater than the Latitude departed from, the Ship hath croſſed the *Equinoctial*; wherefore, from the Difference of Latitude ſubſtract the Latitude ſailed from, and the Remainder will be the Latitude the Ship is arrived into, of a contrary Denomination to the Latitude departed from.

Examp.

Examp. Admit a Ship from the Latitude of 12° 30′ North, ſails Southerly until her Difference of Latitude be 18° 40′, and it be required to find the Latitude ſhe is arrived into.

From the Difference of Latitude made	18° 40′	S^ly.
Take the Latitude departed from	12 30	N.
Remainder is the Lat. arrived into	6 10	S.

SECT. I.

Containing the Solutions of the ſeveral Caſes relating to a ſingle Courſe.

[*Note*, All the *Figures* referred to in this *Section*, are in *Plate* 5.]

Fig. 4. LET BA be the *Proper Difference of Latitude*, BL the *Meridional Difference of Latitude*, and the ∠LBD the *Courſe*; then will AC be the *Departure*, LD the *Difference of Longitude*, BC the *Diſtance* in equal Parts or Miles, and BD the inlarged Diſtance, or the Diſtance upon the *Nautical Chart*.

Now becauſe the △BLD is right-angled at L, and AC is parallel to LD; it follows,

1. BA : AC :: R : T.B; that is,

As the Proper Difference of Latitude, Is to the Departure;
So is the Radius, To the Tangent of the Courſe.

2. BC : AC :: R : S.B; that is,

As the Diſtance, Is to the Departure;
So is the Radius, To the Sine of the Courſe.

3. BC : BA :: R : CS.B; that is,

As the Diſtance ſailed, To the Proper Difference of Latitude;
So is the Radius, To the Co-Sine of the Courſe.

4. R : T.B :: BL : LD; that is,

As the Radius, Is to the Tangent of the Courſe;
So is the Meridional Difference of Latitude, To the Difference of Longitude.

5. BA : BL :: AC : LD; that is,

As the Proper Difference of Latitude, To the Meridional Diff. of Latitude;
So is the Departure, To the Difference of Longitude.

Hence

Hence it follows,

I. Of these four, *viz.* the *Proper Difference of Latitude*, the *Distance*, the *Course*, and the *Departure*, having any two given, we may find the rest.

II. Of these five, *viz. both Latitudes*, the *Distance*, the *Course*, the *Departure*, and the *Difference of Longitude*; any one besides both the Latitudes (which must be either given or found) is sufficient to find the rest.

Again, because it has been demonstrated in *Chap.* I. that the *Difference* of *Longitude*, the *Meridional Distance*, the *Radius*, and the *Co-Sine* of the *Latitude*, are directly proportional: It follows,

III. Of these three, *viz.* the *Latitude*, the *Difference* of *Longitude*, and the *Meridional Distance*, having any two given, we may find the other.

Hence, This Section may be fitly divided into three *Problems*, according to the different Nature of the *Data*; and withal into 13 *Cases*, being properly all the Varieties of the things required.

PROB. I.

Of these three, *viz.* The *Course*, the *Distance sailed*, and the *Difference of Latitude*, any two being given; to find the other, and the *Departure*.

CASE I.

One Latitude, Course, and Distance sailed being given; to find the other Latitude, and the Departure.

Example. A Ship from the *Lizard*, in the Latitude of 49° 55′ N. sails S W *b* S 135 Miles; the Latitude she is arrived into, and the Departure, is required.

Construction. 1. Let B A represent the Meridian, B the Place the Ship departed from, and (for Method's sake) let the Fore-part towards A always represent that part of the Meridian towards which the Ship sailed; *viz.* If she sails towards the Northward, let A represent the *North*, and B the *South* part of the Meridian, then shall the *East* be to the Right-hand, and the *West* to the Left: And on the contrary, when the Ship's Course is Southerly, let A represent the *South*, and B the *North*, then shall the *West* be the Right-hand, and the *East* to the Left. *Fig.* 1.

N. B. This Distinction is not made out of any necessity, for it is not material which way the Triangle lies; neither in Practice is there any necessity of constructing the Cases Geometrically; but yet it is convenient that young Beginners should use themselves to delineate the Figures, and that in their proper Situation.

2. Draw B C forming an Angle of 33° 45′ (= 3 Points) with the Meridian, (in this Case towards the Right-hand, because the Course is S W^{ly}) upon which set off 135 Miles from B to C; then will B C be the Distance sailed.

3. From

3. From C let fall the Perpendicular C A; then will B A be the *Difference* of *Latitude*, (in this Examp. S^{ly}) and A C the *Departure* (W^{ly},) each of which may be measured by the same Scale of equal Parts, that B C was laid off: And by *Calculation*, each of them may be found three different ways by *Case* 3. of *Plain Trigon.* but here it will be sufficient to use only one way, *viz.* that which is judged most apt for Practice; those that desire a greater Variety may have recourse to *Sect.* 2. of *Trigonom.*

And first to find the *Difference* of *Latitude*, it will be,

As R : C.S.B :: BC : BA; that is,

As the Radius	10
To the Co-Sine of the Course 33° 45′	9.919846
So is the Distance sailed 135 *Miles*	2.130334
To the Diff. of Latitude 112.2 *Min.*	2.050180

And so much in this Case is the Ship got to the Southward of the *Lizard*; wherefore, because she is got nearer the Equator,

From the Latitude sailed from	49° 55′ N.
Take the Diff. of Lat. made 112 *M.* =	1 52 S^{ly}.
There remains the Lat. the Ship is in	48 03

2. To find the *Departure*, it will be, R : S. B :: B C : A C; that is,

As the Radius	10
Is to the Sine of the Course 33° 45′	9.744739
So is the Distance sailed 135 *Miles*	2.130334
To the Departure 75 *m.* (in this Case W^{ly})	1.875073

Case 2.

Both Latitudes, and the Course being given; to find the Distance sailed, and the Departure.

Examp. Admit a Ship from the Latitude of 30° 15′N. sailed on a NW*b*W Course until she came into the Latitude of 34° 00′ N; and it be required, to find her *Distance sailed*, and the *Departure*.

Lat. { 34° 00′ N
{ 30 15

3 45
60

Diff. Lat. 225

Fig. 2.

Construction. 1. Make B A = 225 the Difference of Latitude, and erect the Perpendicular A C.

2. Draw the Rhumb-Line B C, making an ∠ = 56° 15′ with the Meridian, and continue it until it meets with the Perpendicular A C in C; then is B C the *Distance* run, and A C the *Departare*;

parture; to find each of which by *Calculation*, it will be by *Case* 1. of Trigonometry.

1. C.S.B : R : : B A : B C; that is,

As the Co-Sine of the Course 56° 15′	9.744739
To the Radius;	10.
So is the Difference of Latitude 225	2.352182
To the Distance sailed 405	2.607443

2. To find the *Departure*, it will be, R : T.B :: B A : A C; that is,

As the Radius,	10.
To the Tangent of the Course 56° 15′	10.175107
So is the Difference of Latitude 225	2.352182
To the Departure 336.7	2.527289

Case 3.

Both Latitudes, and the Distance sailed being given; to find the Course steered, and the Departure.

Example. If a Ship from the Latitude of 2° 16′ N. sails between the South and the East 150 Leagues, and then is found to be in the Latitude of 3° 20′ S. I would know the Course she steered, and the Departure.

Construction. 1. Set the Proper Difference of Latitude =336 M. from B to A, and erect the ⊥ A C.

2. With the Distance sailed =450 *m.* betwixt the Compasses, setting one foot in B, with the other cross the Perpendicular A C in C, and join B C; then is A B C the true Course, and A C the Departure.

Lat. { 2° 16′ N / 3 20 S

5 36

60

P. Diff. Lat. 336

150 *Leagues.*

3

Dist. 450 *Miles.*

Fig. 3.

1. To find the ∠ B by Calculation, it will be,

B C : B A : : R : C.S. B; that is,

As the Distance sailed 450	2.653212
To the Difference of Latitude 336	2.526339
So is the Radius,	10.
To the Co-Sine of the Course 41° 42′	9.873127

Which (becauſe the Ship ſailed between the South and the Eaſt) is S 41° 42′ E^{ly}, or S E *b* S $\frac{1}{2}$ E nearly.

2. To find the *Departure* A C, it will be,

R : S B :: B C : A C; that is,

As the Radius	10.
To the Sine of the Courſe 41° 42′	9.822972
So is the Diſtance ſailed 450.	2.653212
To the Departure 299.3 E^{ly}	2.476184

N. B. When a part of the *Data* is in Leagues, (as in the Examp. above) it muſt be always reduced into Miles; for the Difference of Latitude ought to be always counted in Miles, and not in Leagues.

If the *Departure* had been made a part of the *Data*, this Problem would have admitted of three more Caſes; but the Departure being a Number that can never be given, until it has been firſt found by ſome of the aforegoing Caſes, and is of no other uſe but for finding the *Difference of Longitude*; therefore theſe Caſes do more properly belong to the next Problem.

PROB. II.

Of theſe five, *viz.* 1. *Both the Latitudes*, 2. The *Courſe*, 3. The *Diſtance*, 4. The *Departure*, 5. The *Difference of Longitude*; any one beſides both the Latitudes, or any two whereby both Latitudes may be found, being given; to find the reſt.

Before any of the Caſes in this Problem can either be conſtructed or determined by Calculation, the Meridional Parts anſwering to the Difference of Latitude muſt be found; for the obtaining of which, obſerve the following Rules:

1. When one Place is under the Equinoctial, and the other in North or South Latitude; *or*, which is all one, To find the Meridional Parts anſwering to any given Latitude.

Look in the *Table* of *Meridional Parts* for the Degrees on the top, and the Minutes in the left-hand Column of the Table, and in the common Area you'll find the Meridional Parts required.

Examp. The Meridional Parts anſwering to the *Lat.* of $\left\{\begin{matrix}25° & 50'\\ 62 & 40\end{matrix}\right\}$ is $\left\{\begin{matrix}1605.3\\ 4861.1.\end{matrix}\right.$

2. When

2. When the Places are on the same Side of the Equator, that is, both in North, or both in South Latitude; then the Difference of the Meridional Parts answering to each Latitude, is the Meridional Difference of Latitude required.

Examp. Let one Place be in the Latitude of 60° 00′ N. and the other in the Latitude of 54° 20′ N. their Meridional Difference of Latitude is required.

From the Meridional Parts answering to the Lat. of 60° 00′ = 4527.3
Take the Meridional Parts answering to the Lat. of 54 20 = 3898.8

Remains the Meridional Difference of Latitude 628.5

3. When the Places proposed are on contrary Sides of the Equator, that is, one in North, and the other in South Latitude; then the Sum of the Meridional Parts answering to each Latitude, is the Meridional Difference of Latitude required.

Examp. Let one Place be in the Latitude 30° 20′ N. and the other in the Lat. of 25° 50′ S. required their Meridional Difference of Latitude.

To the Meridional Parts answering to 30° 20′ = 1911.5
Add the Meridional Parts answering to 25 50 = 1605.3

The Sum, is the Meridional Diff. of Latitude = 3516.8

Case 4.

Both Latitudes and the Departure, being given; to find the Course, Distance sailed, and the Difference of Longitude.

Examp. Admit a Ship from the Lat. of 50° 00′ N. sailed between the South and the West until she came into the Lat. of 46° 20′ N. and that her Departure was computed to be 160 *m.* her Course steered, Distance sailed, and Difference of Longitude is required.

Lat. { 50 00 ——— 3474 } *Merid. Parts.*
{ 46 20 ——— 3144 }

3 40 *Merid. Diff. Lat.* 330
60

Prop. Diff. Lat. 220

Construction. 1. Make B A = 220 the proper Difference of Latitude, and upon A raise the ⊥ A C = 160 the Departure, and join B C.

2. From B upon B A produced, lay the Meridional Diff. of Latitude = 330 to L, and draw L D parallel to A C, until it meets B C produced;

then the ∠ B is the *Course steered*, B C the *Distance sailed*, (in equal Parts or Miles) and L D the *Difference of Longitude*. To find each of which by Calculation, it will be,

1. B A : A C :: R : T. B; that is,

As the Proper Difference of Latitude = 220	2.342423
Is to the Departure = 160	2.204120
So is the Radius	10.
To the Tangent of the Course 36° 01′	9.861697

Which (because the Ship sailed between the South and the West,) is S 36° 01′ W^{ly}, or S W *b* S $\frac{1}{4}$ W^{ly} *ferè*.

2. To find the *Distance*, it will be, T. B : *S.* B :: A C : B C; that is,

As the Tangent of the Course = 36° 01′	$\bar{0}$.138303 *Arith. Co.*
To the Departure = 160	2.204120
So the Secant of the Course 36° 01′	10.092134
To the Distance sailed = 272 *Miles.*	2.434557

3. To find the *Difference* of *Longitude*, it will be,

R : T. B :: B L : L D; that is,

As the Radius	10.
To the Tangent of the Course 36° 01′	9.861697
So is the Meridional Difference of Lat. = 330	2.518514
To the Difference of Longitude 240′ (=6° 00′ W^{ly})	2.380211

Or, Because L D is parallel to A C, the Difference of Longitude may be found without knowing the Course, as followeth;

viz. B A : A C :: B L : L D; that is,

As the Proper Difference of Latitude = 220	$\bar{7}$.657577 *Ar. Co.*
To the Meridional Diff. of Latitude = 330	2.518514
So is the Departure = 160	2.204120
To the Difference of Longitude (as before) = 240	2.380211

Case 5.

One Latitude, the Distance sailed, and Departure, being given; to find the other Latitude, Course, and Difference of Longitude.

Example. If a Ship from the Latitude of 45° 40′ N. sailed betwixt the South and the East 600 Miles, and then her Departure was computed to

to be 308 *m.* The Lat. she arrived into, the Course, and Difference of Longitude she made, is required.

Construction. 1. Draw C E parallel to the Meridian B A, at the Distance of the Departure = 308 *m.*

2. With the Distance sailed = 600 *m.* betwixt the Compasses, setting one foot in B, (the Place the Ship departed from) with the other cross the Parallel C E in C; and from C let fall the Perpendicular C A; then is B A the *Proper Difference of Latitude*, and the ∠ B the *Course*. *Fig.* 5.

3. Having found the Latitude the Ship is arrived into, you may find the Meridional Difference of Latitude, which lay from B to L.

4. From L draw L D parallel to A C, and continue B C until it intersects the said Parallel in D; then is L D the *Difference of Longitude.*

1. To find the *Course*, it will be, B C : A C :: R : S. B; that is,

As the Distance sailed 600	2.778151
Is to the Departure 308	2.488551
So is the Radius,	10.
To the Sine of the Course 30° 53′	9.710400

Which is S 30° 53′ E^{ly}, or S S E $\frac{3}{4}$ E nearly.

2. To find the Difference of *Latitude*, it will be,

R : C. S. B :: B C : B A; that is,

As the Radius	10.
To the Co-Sine of the Course 30° 53′	9.933596
So is the Distance 600	2.778151
To the Proper Diff. of Lat. 514.9	2.711747

Which, because the Ship sailed towards the Equator,

From the Latitude sailed from	45° 40′	3086.8	} *Merid. Parts.*
Take the Difference of Lat. 515 (60) =	8° 35′		
The Lat. arrived into	37 05	2398.9	
Merid. Diff. of Lat.		687.9	

3. To find the Difference of *Longitude*, it will be,

R : T. B :: B L : L D; that is,

As the Radius	10.
To the Tangent of the Course 30° 53′	9.776768
So is the Merid. Diff. of Latitude 687.9	2.837525
To the Diff. of Longitude 411.5 (=6° 51′ W^{ly})	2.614293

Or,

Or, Becauſe AC is parallel to LD, the Longitude may be otherwiſe found. For, BA : BL :: AC : LD; that is,

As the Proper Difference of Latitude 514.9	7.288253 *Ar. Co.*
To the Meridional Diff. of Lat. 687.9	2.837525
So is the Departure 308	2.488551
To the Difference of Longitude 411.5	2.614329

CASE 6.

One Latitude, the Courſe, and Departure, being given; to find the other Latitude, Diſtance ſailed, and Difference of Longitude.

Examp. If a Ship from the Latitude of 20° 20′ N. ſteers away NE*b*E until her Departure be 260 *m.* I deſire to know the Latitude ſhe is arrived into, her Diſtance ſailed, and how much ſhe has altered her Longitude.

Fig. 6.

Conſtruction. 1. Having drawn the Meridian BL; make the ∠LBD = 56° 15′.

2. At the Diſtance of 260 the Departure, draw CE parallel to the Meridian BL; and from the Point C, where it interſects the Rhumb-Line, let fall the Perpendicular CA: then is BA the *Proper* Difference of Latitude, by which you may find the Latitude come to; and conſequently the *Meridional* Diff. of Latitude, which lay from B to L, and draw LD parallel to BC; then is LD the Difference of Longitude.

1. To find the *Diſtance* by Calculation, it will be,

S.B : R :: AC : BC; that is,

As the Sine of the Courſe 56° 15′	9.919846
Is to the Radius;	10.
So is the Departure 260	2.414973
To the Diſtance 312.7 *Miles*	2.495127

2. For the Difference of *Latitude*, it will be,

R : C.T.B :: CA : BA; that is,

As the Radius	10.
Is to the Co-Tangent of the Courſe 56° 15′	9.824893
So is the Departure 260	2.414973
To the Proper Diff. of Latitude 173.7	2.239866

Which becauſe the Ship ſailed from the Equator,

To the Latitude ſailed from	20° 20′	1246.4	} *M. Parts.*
Add the Difference of Latitude 173.7 (60) =	2° 54′		
Sum the Latitude the Ship is in	23 14	1433.8	
Merid. Diff. of Latitude		187.4	

3. To find the Difference of *Longitude*, it will be,

R : T.B :: BL : LD; that is,

As the Radius	10.
To the Tangent of the Course 56° 15′	10.175107
So is the Merid. Diff. of Latitude 187.4	2.272770
To the Diff. of Longitude 280.4 (=4° 40′ E^ly) (60)	2.447877

Or, Becauſe the △ˢ BAC, BLD are ſimilar, it will be for the Difference of Longitude. BA : BL :: AC : LD; that is,

As the Proper Difference of Latitude	173.7	$\bar{2}$.760134 *Arith. Co.*
Is to the Meridional Diff. of Latitude	187.4	2.272770
So is the Departure	260	2.414973
To the Difference of Longitude	280.4	2.447877

Case 7.

Both Latitudes, and Difference of Longitude being given; to find the direct Course, and Diſtance.

Examp. Let it be required to find the direct Courſe, and Diſtance between the *Lizard* in the Latitude of 50° 00′ N. and *Bermudas* in the Lat. of 32° 30′ N. their Difference of Longitude being 58° 30′; *Bermudas* lying ſo much to the Weſtward of the *Lizard*.

Firſt, find the Proper and Meridional Difference of Latitude.

Latitudes	50° 00′ N.	3474.4	} *M. Parts.*
	32 30 N.	2063.8	
	17 30	1410.6	*Meridional Diff. of Latitude.*
	60		
Prop. Diff. Lat. in min.	1050		
	58 30		
	60		
Difference of Longitude in min.	3510		

Con-

Fig. 7. *Conſtruction.* 1. Lay the Meridional Difference of Latitude =1410 from B to L; and erect the Perpendicular L D=3510 the Difference of Longitude, and draw B D.

2. Make B A=1050 the Proper Difference of Latitude, and draw A C parallel to L D; then is B C the *Diſtance*, and the Angle B the *Courſe*.

1. To find the *Courſe*, it will be,

B L : L D :: R : T. B; that is,

As the Meridional Difference of Latitude 1410	3.149219
To the Difference of Longitude 3510	3.545307
So is the Radius	10.
To the Tangent of the Courſe 68° 07′	10.396088

Which, (becauſe *Bermudas* is to the Southward of the *Lizard*, and Difference of Longitude Weſterly;) is S 68° 07′ W^{ly}, or W S W a little Weſterly.

2. To find the *Diſtance*, it will be (by *Caſe* 2^{d}.)

C. S. B : R :: B A : B C; that is,

As the Co-Sine of the Courſe 68° 07′	9.571380
Is to the Radius;	10.
So is the Proper Diff. of Latitude 1050 *m.*	3.021189
To the Diſtance 2817 *Miles.*	3.449809

Case 8.

One Latitude, the Courſe, and Diſtance ſailed, being given; to find the other Latitude, and Difference of Longitude.

Example. If a Ship from the Latitude of 50° 30′ N. ſteers upon a S S W Courſe 150 Leagues; I deſire to know the Latitude ſhe is arrived into, and her Difference of Longitude.

Fig. 8. *Conſtruction.* 1. By the Courſe and Diſtance form the Triangle B A C, according to the Directions in *Caſe* 1. then having found the *Proper* Difference of Latitude, and conſequently the *Latitude* arrived into; find the *Meridional* Difference of Latitude, which lay from B to L, and draw L D parallel to A C until it meets B C produced; then L D meaſured on the Scale of equal Parts will be the *Difference of Longitude*.

1. To

1. To find the Proper Difference of Latitude by *Calculation*, it will be by *Case* 1.

		10.
As the Radius		
Is to the Co-Sine of the Course	22° 30′	9.965615
So is the Distance	450 *Miles*	2.653212
To the Diff. of Latitude	415.8	2.618827

From the Latitude sailed from	50° 30′	3521.4	} *Merid. Parts.*
60)			
Take the Diff. of Lat. 415.8 =	6 56		} *Merid. Parts.*
Lat. the Ship is in	43 34	2909.8	} *Merid. Parts.*
	Merid. Diff. Lat.	611.6	

2. To find the Difference of Longitude, it will be,

R : T.B : : BL : LD; that is,

		10.
As the Radius		
To the Tangent of the Course	22° 30′	9.617224
So is the Merid. Diff. of Latitude	611.6	2.786467
To the Diff. of Long.	253.3 *m.* (=4° 13′ W^ly)	2.403691

Case 9.

Both Latitudes, and the Course, being given; to find the Distance, and Diff. of Longitude.

Example. If a Ship from the Latitude of 20° 40′ N. sails N E *b* E until she arrives into the Latitude of 27° 16′ N. and it be required to find the Distance sailed, and Difference of Longitude:

Having constructed the Triangle B A C according to the Directions in *Case* 2. find the Meridional Difference of Lat. and lay it from B to L, and parallel to A C, draw L D the Difference of Longitude. *Fig.* 9.

Lat.	20° 40′	1267.8	} *Meridional Parts.*
	27 16	1701.5	} *Meridional Parts.*
	6 36	433.7	*Meridional Diff. of Latitude.*
	60		
P. Diff. Lat.	396		

Now it will be, R : BL :: T.B : LD; that is,

As the Radius	10.
To the Merid. Diff. of Lat. 433.7	2.637189
So is the Tang. of the Course 56° 15′	10.175107
To the Diff. of Long. 649 (=10° 49′ E^{ly})	2.812296

2. The *Distance* may be found by *Case* 2. Thus,

As the Co-Sine of the Course 56° 15′	9.744739
Is to the Radius;	10.
So is the Proper. Diff. of Latitude 396	2.597695
To the Distance 712.8 *Miles*	2.852956

CASE 10.

Both Latitudes, and Distance sailed, being given; to find the direct Course, and the Difference of Longitude.

Example. Suppose a Ship from the Latitude of 32° 30′ N. sails between the North and the East 960 Miles, and then by Observation is found to be in the Latitude of 44° 00′ N. and it be required to find the direct Course, and Difference of Longitude.

Lat.	44° 00′	2945.8	*Meridional Parts.*
	32 30	2063.8	
	11 30	882	*Meridional Diff. of Latitude.*
	60		
P. Diff. Lat.	690		

Fig. 10. *Construction.* With the Proper Difference of Latitude and Distance, let the △ B A C be constructed by *Case* 3.

2. Make B L = 882 the Merid. Difference of Latitude, and draw L D parallel to A C until it meet B C produced in D; then is L D the Diff. of Longitude, and the Angle B the Course, to find which by *Calculation*, it will be by *Case* 3.

As the Distance 960	2.982271
To the Diff. of Lat. 690	2.838849
So is the Radius	10.
To the Co-Sine of the Course 44° 3′ (or N E nearly)	9.856578

2. To

2. To find the Difference of *Longitude*, it will be,
R : T. B :: B L : L D; that is,

As the Radius	10.
To the Tangent of the Courſe 44° 03′	9.985596
So is the Merid. Diff. of Latitude 882	2.945468
60)	
To the Diff. of Long. 853.2 (=14° 13′ E^ly)	2.931064

Case II.

One Latitude, Courſe, and Difference of Longitude, being given; to find the other Latitude, and Diſtance ſailed.

Example. If a Ship from the Latitude of 45° 30′ S. bears away N NW until her Difference of Longitude be 7° 40′; I deſire to know the Latitude ſhe is arrived into, and her Diſtance ſailed.

Conſtruction. Having formed the ∠ A B C=22° 30′, draw D E parallel to the Meridian B A at the Diſtance of 460 the Diff. of Longitude, and from the Point of Interſection D, let fall the Perpendicular D L; then will B L be the *Meridional* Difference of Latitude.

740
60
460 *m. Diff. Long.*

Fig. 11.

To find which by *Calculation*, it will be,
T. B : R :: D L : B L; that is,

As the Tangent of the Courſe 22° 30′	9.617224
Is to the Radius	10.
So is the Diff. of Longitude 460	2.662758
To the Merid. Diff. of Latitude 1110	3.045534

Becauſe the Ship ſailed from a South Latitude Northerly,

From the Meridional Parts of the Latitude ſailed from	45° 30′	3072.5
Take the Meridional Difference of Latitude		1110
Remains the Merid. Parts of the Latitude came into	31° 04′	1962.5

Whence, to find the Proper Difference of Latitude,

From the Latitude ſailed from	45° 30′
Take the Latitude ſailed into	31 04
Remains the Diff. of Latitude	14 26
	60
Prop. Diff. of Latitude in min.	866

Conſtruction. 2. Set off 866 from B to A, and draw A C parallel to L D; then will B C be the direct *Diſtance* ſailed; to find which by *Calculation*, it will be,

As the Co-Sine of the Courſe 22° 30′	9.965615
Is to the Radius;	10.
So is the Proper Diff. of Latitude 866	2.937518
To the Diſtance 938.4 *Miles*	2.971903

PROB. III.

Of theſe three, *viz.* the *Latitude*, the *Difference* of *Longitude*, and the *Meridional Diſtance*, any two being given; to find the other.

CASE 12.

The Difference of Longitude between two Places, both lying under the ſame Parallel of Latitude, being given; to find their Meridional Diſtance, (or the Length of the Parallel intercepted between them.)

Example. Let it be required to find the Meridional Diſtance betwixt the *Lizard* and the *Iſland* of *Pengwin* on the Coaſt of *Newfoundland*, both lying in the Latitude of 50° 00′ N. and their Difference of Longitude 48° 30′; the *Iſland* of *Pengwin* lying ſo much to the Weſtward of the *Lizard*.

Fig. 12.

48° 30′
60

2910 *m. Diff. of Long.*

Conſtruction. 1. Make Æ Q = 2910 the Difference of Longitude.

2. With the Sine of 90° 00′ (= Rad.) upon Æ Q deſcribe the Iſoſceles △ Æ P Q; then will P repreſent the *Pole* of the World, Æ Q an Arch of the *Equator*, P Q the Meridian of the *Lizard*, and P Æ the Meridian of *Pengwin Iſland*.

3. With the Co-Sine of the Lat. 50° 00′, ſetting one foot of the Compaſſes in P, with the other deſcribe the Arch C L, and join the right Line C L; then will L repreſent the Place of the *Lizard*, and C that of *Pengwin Iſland*; and the right Line C L the *Diſtance* between them; to find which by Calculation, it will be,

P Æ : P C :: Æ Q : C L; that is,

As the Radius	10.
To the Co-Sine of the Latitude 50° 00′	9.808067
So is the Difference of Longitude = 2910	3.463893
To the Merid. Diſtance required = 1870.5 *m.*	3,271960

The

The Number of Miles anſwering to a Degree of Longitude in any Parallel, may be found by this *Caſe*, and thence a Table may be formed for reducing the Minutes of Longitude in any Parallel of Lat. into *Geographical Miles*; *& contrà*. For it will be,

As the Radius,
To the Co-Sine of the Latitude of any Parallel;
So is 60 Miles, (the Length of a Deg. of Longitude upon the Equator)
To the Number of Miles anſwering to a Deg. of Long. in the ſaid Parallel.

After this Manner the following Table was calculated.

A TABLE *ſhewing how many Miles anſwer to a Degree of Longitude, at every Degree of Latitude.*

Lat. D.	Miles.	Lat. D.	Miles.	Lat. D.	Miles.	Lat. D.	Miles.	Lat. D.	Miles.	Lat. D.	Miles.
1	59.99	16	57.67	31	51.43	46	41.68	61	29.09	76	14.51
2	59.96	17	57.37	32	50.88	47	40.92	62	28.17	77	13.50
3	59.92	18	57.06	33	50.32	48	40.15	63	27.24	78	12.48
4	59.86	19	56.73	34	49.74	49	39.36	64	26.30	79	11.45
5	59.77	20	56.38	35	49.15	50	38.57	65	25.36	80	10.42
6	59.67	21	56.01	36	48.54	51	37.76	66	24.41	81	9.38
7	59.56	22	55.63	37	47.92	52	36.94	67	23.45	82	8.35
8	59.42	23	55.23	38	47.28	53	36.11	68	22.48	83	7.32
9	59.28	24	54.81	39	46.62	54	35.26	69	21.50	84	6.28
10	59.08	25	54.38	40	45.95	55	34.41	70	20.52	85	5.23
11	58.89	26	53.93	41	45.28	56	33.55	71	19.54	86	4.18
12	58.68	27	53.46	42	44.59	57	32.68	72	18.55	87	3.14
13	58.46	28	52.97	43	43.88	58	31.79	73	17.54	88	2.09
14	58.22	29	52.47	44	43.16	59	30.90	74	16.53	89	1.05
15	57.95	30	51.96	45	42.43	60	30.00	75	15.52	90	0.00

By this Table the Number of Miles anſwering to a Degree of Longitude, in any Parallel of an even Degree of Latitude, may be found by Inſpection; thus, in the Latitude of 50 Degrees, 38.57 *Geographical* Miles make a Degree of Longitude: whence the Number of Miles anſwering to any Difference of Longitude may be eaſily found; for it will be,

As 60 Miles, To the Miles anſwering to 1 Deg. of Long. in the Lat. propoſed;
So is the Difference of Longitude, To the Meridional Diſtance.

Thus,

Thus, in the Example above, it will be,

60 : 38.57 :: 2910 : 1870 the Diſtance required.

If it be required to find the Number of Miles anſwering to a Degree of Longitude, in any Parallel of Latitude not inſerted in the Table; it will be, *As 60 Miles, To the Difference betwixt the next greater and leſſer Tabular Latitude; So is the Exceſs of the Latitude propoſed, above the next leſs Tabular Latitude, to a proportional Part:* which ſubſtracted from the Miles anſwering to the next leſs Tabular Latitude, will give the Miles anſwering to a Degree of Longitude, in the Latitude propoſed. Thus, if it were required to find the Miles anſwering to a Degree of Longitude, in the Lat. 20° 24′, it will be, *As 60 Miles, To .37 the Tabular Difference between the Miles anſwering to 20° and 21°; So is 24′ the Exceſs of the Lat. given above 20°, to .14:* which therefore ſubſtracted from 56.38 (the Miles anſwering to 1 Deg. in the Lat. of 20°) leaves 56.24 the Number of Miles anſwering to 1 Deg. in the Lat. of 20° 24′.

Note, There is generally placed upon the *Plain Scale*, a Line marked M L; the Uſe whereof is the ſame with the foregoing Table, *viz.* to find the Length of a Degree of Longitude in any Parallel of Latitude, which is thus obtained: Extend the Compaſſes upon the *Line* of *Chords* (adjoining to the ſaid Line M L) from the beginning to the Latitude propoſed; the ſame Extent will reach from 60 upon the *Line* M L, to the Length of a Degree of Longitude in that Parallel. Thus, in the Latitude of 50°, a Degree of Longitude will be found to conſiſt of 38.57 *Geographical* Miles; and in the Latitude of 60 Degrees, 30 *Miles* is equal to a Degree of Longitude, *&c.* as in the aforegoing Table: then if it were required to find the *Meridional Diſtance* anſwering to a given Arch in any Parallel of Latitude; as for inſtance, in the aforegoing Example, where the Latitude was 50 Deg. and the Difference of Longitude 2910′; to find the *Meridional Diſtance*, it will be, as before in the Uſe of the Table;

60 : 38.57 :: 2910 : 1870; the Meridional Diſtance required.

Case 13.

The Diſtance ſailed in any Parallel, being given; to find the Difference of Longitude.

Examp. If a Ship in the Lat. of 50 *Deg.* ſails directly Eaſt or Weſt in that Parallel 1870.5 Miles, and it be required to find how much ſhe has altered her Longitude.

Fig. 12. *Conſtruction.* Upon C L=1870.5 Miles, with the Sine of 40 Degrees (equal to the Co-Sine of the given Latitude) make the Iſoſceles △ C P L, and produce the Sides P C, P L to Æ and Q, until each be equal to the *Radius*, or *Sine* of 90 Deg. and draw the right Line Æ Q, which will be

be equal to the *Difference of Longitude*; to find which by *Calculation*, it will be,

PC : PÆ :: CL : ÆQ; that is,

As the Co-Sine of the Latitude 50° 00′	9.808067
Is to the Radius;	10.
So is the Meridional Distance 1870.5	3.271960
60)	
To the Diff. of Longitude 2910 (=48° 30′)	3.463893

By the Table or the Plain Scale: 1. Find the Length of a Degree of Longitude in the Latitude of 50 Deg. (by the last *Case*) then it will be,

38.57 : 60 :: 1870 : 2910; the Diff. of Longitude required.

The aforegoing are all the *Cases* that properly relate to a *single Course*; but for the sake of the Learner, who would willingly be able to solve all the Varieties of this kind, the two following Cases are added.

I. *The Distance sailed in any Parallel of Latitude, and the Difference of Longitude corresponding thereto, being given; to find the Latitude of the said Parallel.*

Examp. If a Ship in sailing 1870.5 Miles due West, alters her Longitude 2910′ (or 48° 30′) and it be required to find the Latitude she is in.

Construction. 1. Upon ÆQ=2910 (the Diff. of Longitude) with the *Sine* of 90 Deg. describe the Isosceles △ ÆPQ, and let fall the ⊥ PA. *Fig.* 13.

2. Lay half the Distance sailed from A to B, and draw BL parallel to AP; the Point L where it intersects the Meridian, is the Place of the Ship, and QL the *Latitude* of the Parallel required; to find which by Calculation, it will be,

ÆQ : CL :: PQ : PL; that is,

As the Difference of Longitude 2910	3.463893
To the Meridional Distance 1870.5	3.271960
So is the Radius,	10.
To the Co-Sine of the Latitude 50° 00′	9.808067

By the Table or the Plain Scale, first say, 2910′ : 1870.5′ :: 60′ : 38.57; then look in the *Table* for 60.57, and right against it you'll find 50 Degrees the Latitude required. Or, by the *Plain Scale*, extend the Compasses upon the Line ML from 60 to 38.57, the same

Extent will reach upon the *Line* of *Chords*, from the beginning to 50 Deg. the Latitude required.

II. *Having the Diſtance between two Meridians in any given Parallel of Latitude; to find their Diſtance in any other Parallel.*

Examp. 1. Suppoſe two Ships from the Latitude of 60° 00′ N. diſtant aſunder 800 Miles, ſail due South 1200 Miles, or till they arrived into the Latitude of 40° 00′ N. and it be required to find how far they are aſunder.

Fig. 14. *Conſtruction.* 1. Make C L=800 Miles, (the *Meridional* Diſtance given,) and with the Sine of 30° 00′ (the Co-Sine of the Lat. ſailed from) make the Iſoſceles △ C P L.

2. Produce P C and P L to A and B, until each becomes equal to the *Co-Sine* of 40° 00′, (the *Latitude* arrived into) then A and B will be the Places of the two Ships, and the right Line A B their Diſtance; to find which by *Calculation*, it will be,

P C : P A :: C L : A B; that is,

As the Co-Sine of the Lat. ſailed from 60° 00′	0.301030 *Arith. Co.*
To the Co-Sine of the Lat. arrived into 40° 00	9.884254
So is the Meridional Diſtance given 800 *m.*	2.903090
To the Merid. Diſtance required 1225 *m.*	3.088374

By the Table or the Plain Scale. 1. By the Directions given in *Caſe* 12. find the Number of Miles anſwering to a Degree of Longitude in both Latitudes, *viz.* 30 and 45.95; then it will be,

30 : 45.95 :: 800 : 1225; the *Diſtance* required.

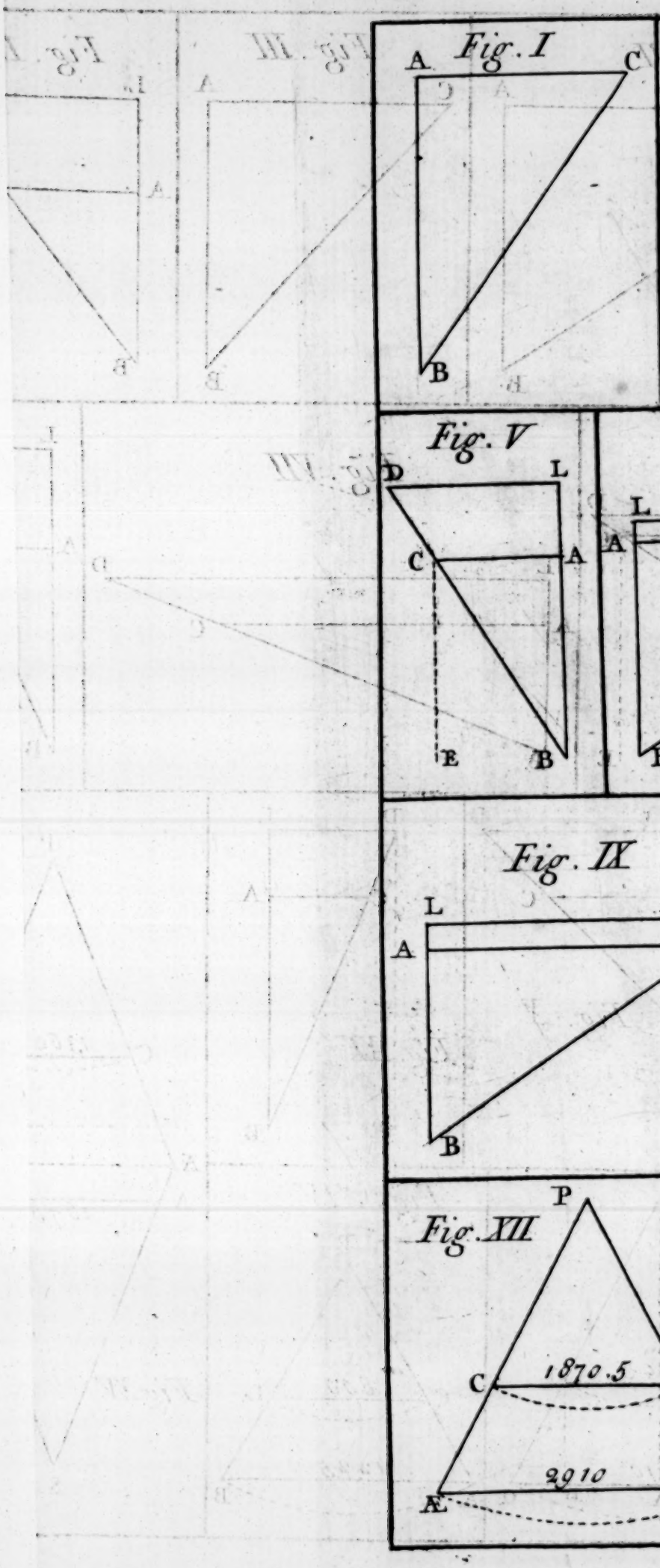
Examp. 2. Suppoſe two Ships from the Latitude of 26° 40′ South, diſtant aſunder 1878 Miles, ſail due North 4560 Miles, or till they arrived into the Latitude of 49° 20′ N. and it be required to find their Diſtance from each other.

Fig. 15. *Conſtruction.* 1. Upon A B=1878 (the Meridional Diſtance given) with the Co-Sine of 26° 40′ make the Iſoſceles △ A B S, and produce S B, S A to Æ and Q until each be equal to the *Radius* or *Sine* of 90° 00′.

2. Upon Æ Q, with the Sine of 90° 00′, make the Iſoſceles △ Æ P Q; and with the Co-Sine of 49° 20′, (the *Lat.* arrived into) from P deſcribe the Arch C L; then Æ Q repreſents the *Equator*; S the *South* Pole; and P the *North* Pole; A B the South Parallel wherein the Ships were at firſt; and C and L their Places in the North Parallel which they arrived into.

N. B.

Plate 5



C

Fig. II

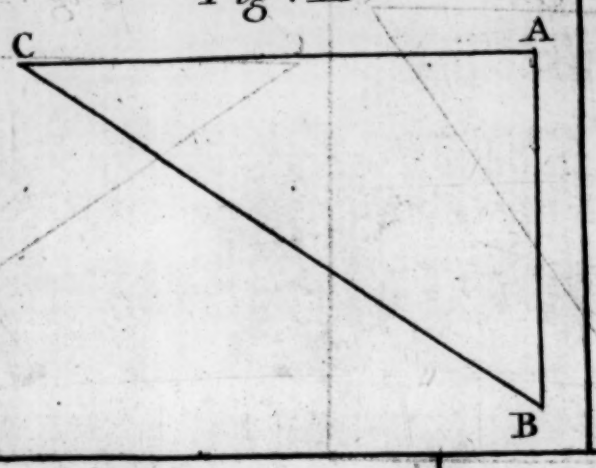


Fig. III

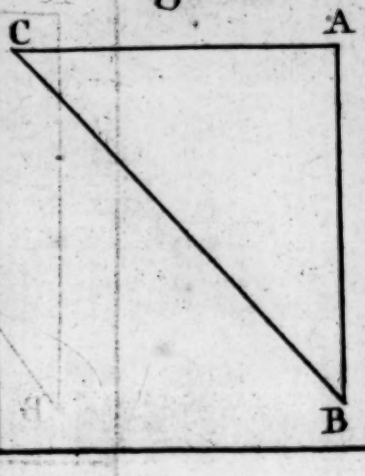
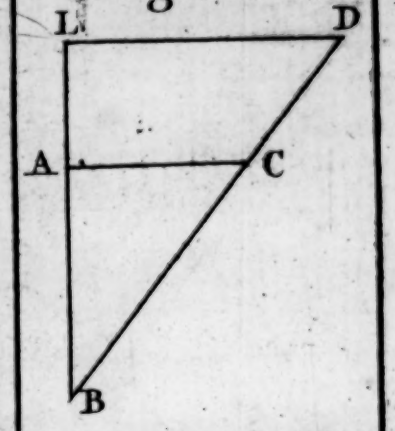


Fig. IV



L

A

Fig. VI

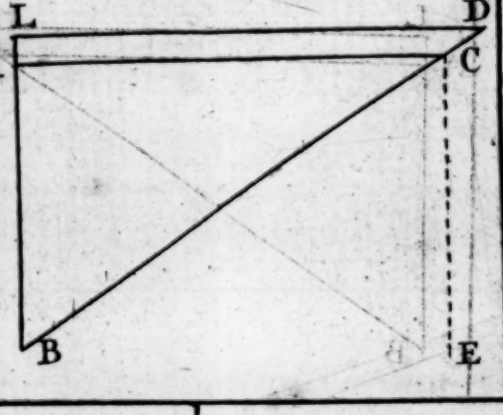


Fig. VII

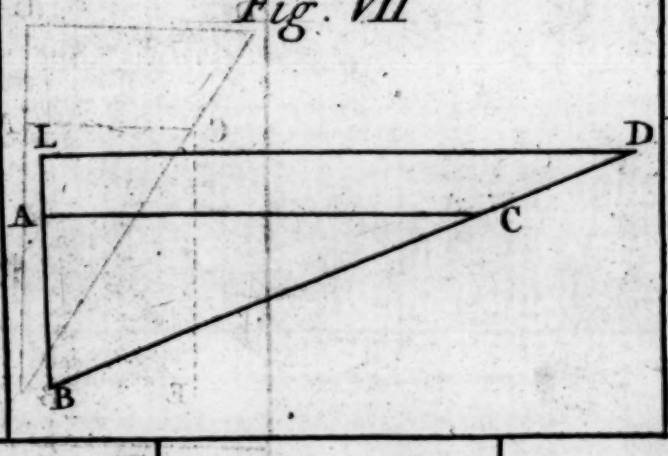
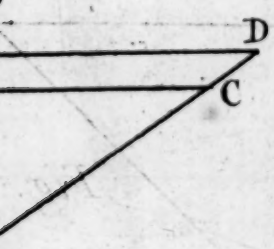




Fig. VIII

g. IX



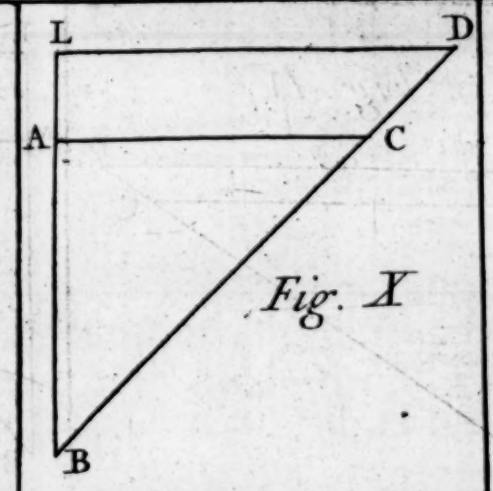


Fig. X

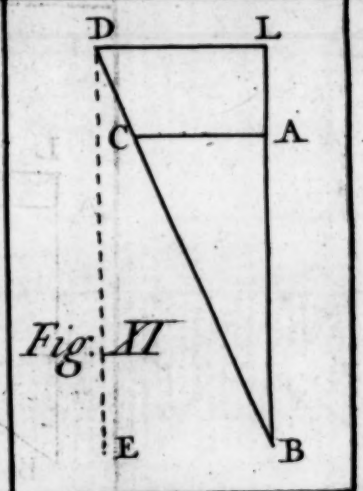


Fig. XI

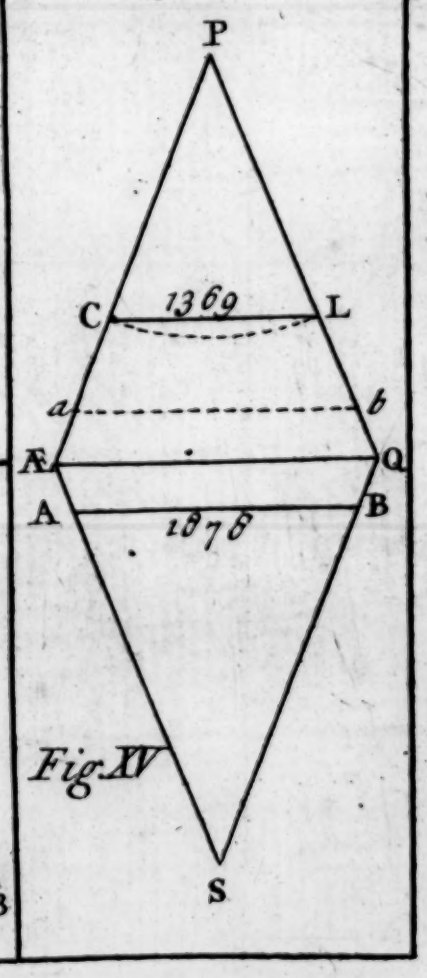


Fig. XV

P

870.5

L

2910

Q

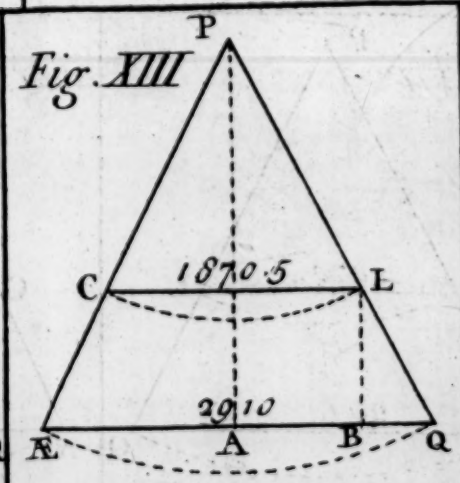


Fig. XIII

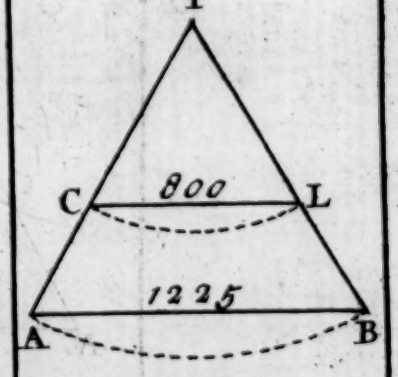


Fig. XIV

N. B. The Merid. Distance C L might have been found by making the Isosceles $\triangle$ *a* P *b* = A S B, and describing the Arch C L, as above; but then the Figure would not represent the true Places of the Ships in both Latitudes.

To find C L by Calculation, it will be,

S A (= P *a*) : P C :: A B (= *a b*) : C L; that is,

As the Co-Sine of the Lat. sailed from 26° 40′	$\bar{0}$.048841 *Arith. Co.*
Is to the Co-Sine of the Lat. sailed into 49 20	9.814019
So is the Distance first given 1878	3.273696
To the Distance required 1369 *Miles*	3.136556

If the Distance between two Meridians in two different Parallels, and the Latitude of one of those Parallels be given, it will be easy to find the Latitude of the other Parallel; for it will be,

As the Distance in the Given Latitude, To the Co-Sine of that Latitude;
So is the other Given Distance, To the Co-Sine of the Latitude required.

The Proportions aforegoing may be all wrought by *Gunter's Scale*, and the *Sliding Rule*, by the Directions given in *Chap.* I. *Sect.* 3. of the *Introduction*; which Directions are so plain and easy, that I thought it needless to repeat them here.

Note, There is usually placed upon *Gunter's Scale* a Meridian Line, whereby the Meridional Difference of Latitude may be found to a sufficient Exactness for working a Proportion upon that Instrument: Thus in *Case* 4. where the Latitudes were $\left\{\begin{smallmatrix}50°\ 00'\ \text{N.}\\ 46\ \ 20\ \ \text{N.}\end{smallmatrix}\right\}$ To find the Meridional Difference of Latitude, extend the Compasses upon the *Meridian Line* from 46° 20′ to 50° 00′, the same Extent applied to the Line of *Equal Parts* (adjoining to the said Merid. Line) will reach 5° 30′; which multiplied by 60 gives 330 the *Meridional Difference of Latitude* required.

There is also commonly placed upon *Gunter's Scale* two Lines marked S R and T R, which are *Artificial Sines*, and *Tangents* to every $\frac{1}{4}$ Point of the Compass; the Numbers 1, 2, 3, *&c.* signifying the Number of Points from the Meridian. These Lines are used after the same manner with the Artificial Sines and Tangents to Degrees and Parts, and therefore need no Explication here.

I shall next shew how the several Cases under *Prob.* II. may be solved by the *Table* of *Artificial Tangents*, without the help of the Meridional Parts.

It is evident, that the *Nautical Meridian Line* is a Scale of the Longitudes answering to each Minute of Latitude on the Rhumb-Line, making an $\angle$ of 45 Degrees with the Meridian: And it has been demonstrated

in the *Phil Tranſ.* N°. — by the celebrated Dr. *Halley*, that the *Meridian Line* is a *Scale* of the *Logarithmick Tangents* of the half Complements of the Latitudes. Wherefore,

1. Becauſe that in every Point of any Rhumb-Line, *the Difference of Latitude is to the Departure*, as *the Radius to the Tangent of the ∠ that Rhumb makes with the Meridian*; and the *Difference of Latitude is every where to thoſe equal Departures*, as *the Meridional Difference of Latitude to the Difference of Longitude*; it follows, that the Differences of Longitude are upon any Rhumb, Logarithms of the ſame Tangents, but of a differing Species; being proportioned to each other, as are the Tangents of the ∠' made with the Meridian.

2. Hence, any Scale of Logarithm-Tangents, (of what Form ſoever) is a Table of the Differences of Longitude, to the ſeveral Latitudes upon ſome determinate Rhumb or other: And the Rhumb which makes an ∠ of 51° 38′ 09″ with the Meridian, has been determined to be that upon which the Logarithm-Tangents of Mr. *Briggs*'s Form, (which are thoſe now commonly uſed) are the true Differences of Longitudes. Hence,

3. *The Difference of the Logarithm-Tangents, or Co-Tangents of half the Complements of the Latitudes of any two Places, Is to the Difference of Longitude a Ship makes in ſailing on any Rhumb from one of thoſe Latitudes to the other; As the Logarithm-Tangent of* 51° 38′ 09″ (viz. 10.101510,) *To the Tangent of the ∠ of the aforeſaid Rhumb with the Meridian.* Wherefore in,

Case 4.

Where the Latitude ſailed from was 50° 00′ *N. the Departure* 160 *m. and the Latitude arrived into* 46° 20′ *N. to find the Difference of Longitude.*

1. By the Difference of Latitude and Departure find the Courſe, by the Directions before given in this Caſe: Then,

Lat. $\left\{\begin{matrix}50^\circ\ 00'\ N.\\ 46\ \ 20\ N.\end{matrix}\right\}$ *Complement* $\left\{\begin{matrix}40^\circ\ 00'\\ 43\ \ 40\end{matrix}\right\}$ $\frac{1}{2}$ *Comp.* $\left\{\begin{matrix}20^\circ\ 00'\\ 21\ \ 50\end{matrix}\right.$

From the Log. Tang. of $\frac{1}{2}$ *Comp. of the leſſer Lat.*	= 21° 50′	9.602761
Take the Log. Tang. of $\frac{1}{2}$ *Comp. of the greater Lat.*	= 20 00	9.561066
	Remainder	416.95

Or,

From the Co-Tangent of 20° 00′		10.438934
Take the Co-Tangent of 21 50		10.397239
	Remainder	416.95

And

And it will be,

As the Tangent of 51° 38′ 09″	9.898490 *Arith. Co.*
To the Tangent of the Course 36° 01′	9.861697
So is the Remainder 416.9	2.620032
To the Difference of Longitude 240 *m.*	2.380219

Note, That Figure of the Remainder which ſtands under the 4th Place in the Logarithm from the left-hand, is to be eſteemed an *Unit*, and the reſt towards the right-hand *Decimals*.

Case 5.

Where the Latitude ſailed from was 45° 40′ *N. the Diſtance ran* 600 *Miles between the South and the Eaſt, and the Departure* 308 *m. to find the Difference of Longitude.*

1. Having found the Courſe 30° 53′, and the Lat. arrived into 37° 05′ N. by the Directions before given in this Caſe: Then,

$$\text{Lat.} \left\{\begin{matrix}45^\circ\ 40'\ N.\\ 37\ \ 05\ \ N.\end{matrix}\right\} \text{Comp.} \left\{\begin{matrix}44^\circ\ 20'\\ 52\ \ 55\end{matrix}\right\} \tfrac{1}{2} \text{ Comp.} \left\{\begin{matrix}22^\circ\ 10'\\ 26\ \ 27\tfrac{1}{2}\end{matrix}\right.$$

From the Log. Tangent of 26° 27′ ½	9.696944
Take the Log. Tangent of 22 10	9.610036
Remainder	869.08

Or,

From the Log. Co-Tangent of 22° 10′	10.389964
Take the Log. Co-Tangent of 26 27 ½	10.303056
Rem.	869.08

And it will be,

As the Tangent of 51° 38′ 09″	9.898490 *Arith. Co.*
To the Tangent of the Course 30° 53′	9.776768
So is the Remainder 869	2.939020
To the Difference of Longitude 411.4 *m.*	2.614278

Case 6.

Where the Latitude ſailed from was 20° 20′ *N. the Courſe* NE *b* E, *and the Departure* 260 *m. to find the Difference of Longitude.*

1. By the Courſe, and Departure, find the Latitude arrived into, *viz.* 23° 14′ N. (by the Directions before given in this Caſe;) then the *Diſtance* (if required) may be eaſily found.

Lat. $\left\{\begin{matrix}20°\ 20'\\23\ \ 14\end{matrix}\right\}$ *Comp.* $\left\{\begin{matrix}69°\ 40\\66\ \ 46\end{matrix}\right\}$ $\frac{1}{2}$ *Comp.* $\left\{\begin{matrix}34°\ 50'\\33\ \ 23\end{matrix}\right.$

From the Log. Tang. of $\frac{1}{2}$ *Comp. of the leſſer Latitude*	34° 50′	9.842535
Take the Log. Tang. of $\frac{1}{2}$ *Comp. of the greater Lat.*	33 23	9.818860
		236.75

Or,

From the Log. Co-Tang. of $\frac{1}{2}$ *Comp. of the greater Lat.*	33° 23′	10.181140
Take the Log. Co-Tang. of $\frac{1}{2}$ *Comp. of the leſſer Lat.*	34 50	10.157465
		236.75

And it will be,

As the Tangent of 51° 38′ 9″	9.898490 *Arith. Co.*
Is to the Tangent of the Courſe 56° 15′	10.175107
So is the Remainder 236.75	2.374289
To the Difference of Longitude 280.4 *m.*	2.447886

Case 7.

Where the Latitude ſailed from was 50° 00′ *N. the Lat. bound to* 32° 30′ *N. and the Difference of Longitude* 58° 30′ = 3510 *min. to find the Courſe, and Diſtance.*

Lat. $\left\{\begin{matrix}50°\ 00'\ N.\\32\ \ 30\ \ N.\end{matrix}\right\}$ *Comp.* $\left\{\begin{matrix}40\ 00\\57\ 30\end{matrix}\right\}$ $\frac{1}{2}$ *Comp.* $\left\{\begin{matrix}20°\ 00'\\28\ \ 45\end{matrix}\right.$

From the Log. Tang. of $\frac{1}{2}$ *Comp. of the leſſer Lat.*	28° 45′	9.739271
Take the Log. Tang. of $\frac{1}{2}$ *Comp. of the greater Lat.*	20 00	9.561066
		1782.05

Or,

Or,

From the Co-Tang. of 20° 00′	10.438934
Take the Co-Tang. of 28 45	10.260729
	1782.05

Then,

As the Remainder 1782	3.250908
Is to the Tangent of 51° 38′ 9″	10.101510
So is the Difference of Longitude 3510	3.545307
To the Tangent of the Course 68° 07′	10.395909

Or, S 68° 07′ W^{ly}, as before: whence the Distance may be easily found (if required.)

Case 8.

Where the Latitude departed from was 50° 30′ *N. the Course* S S W, *and the Distance* 450 *Miles; to find the other Latitude and the Difference of Longitude.*

1. Having (by the former Directions) found the Latitude the Ship is in, *viz.* 43° 34′ N.

Lat. {50° 30′ N. / 43 34 N.} *Comp.* {39° 30′ / 46 26} $\frac{1}{2}$ *Comp.* {19° 45′ / 23 13}

From the Log. Tangent of 23° 13′	9.632401
Take the Log. Tangent of 19 45	9.555138
Rem.	772.63

Or,

From the Log. Co-Tangent of 19 45	10.444861
Take the Log. Co-Tangent of 23 13	10.367598
Rem.	772.63.

And it will be, for the *Difference* of *Longitude*,

As the Tangent of 51° 38′ 09″	$\bar{9}$.898490	*Arith. Co.*
To the Tangent of the Course 22° 30′	9.617224	
So is the Remainder 772.6	2.887955	
To the Diff. of Long. 253.3 *m.* (=4° 13 W^{ly})	2.403669	

Cases

CASE 9.

Where the Latitude sailed from was 20° 40′ *N. the Latitude sailed into* 27° 16′ *N. and the Course* NE*b*E; *to find the Difference of Longitude.*

Lat. {20° 40′ / 27 16} *Comp.* {69° 20′ / 62 44} ½ *Comp.* {34° 40′ / 31 22}

From the Tangent of 34 40	9.839838
Take the Tangent of 31 22	9.785048
	547.90

Or,

From the Co-Tangent of 31 22	10.214952
Take the Co-Tangent of 34 40	10.160162
	547.90

For the Difference of Longitude, it will be,

As the Tangent of 51° 38′ 09″	9.898490 *Arith. Co.*
Is to the Tangent of the Course 56° 15′	10.175107
So is the Remainder 547.9	2.738701
To the Difference of Longitude 649 *m.*	2.812298

CASE 10.

Where the Latitude sailed from was 32° 30′ *N. the Latitude arrived into* 44° 00′ *N. and the Distance sailed* 690 *Miles, between the North and the East; to find the Diff. of Longitude.*

1. Having found the Course 44° 03′ by the Directions already given; then,

Lat. {32° 30′ *N.* / 44 00 *N.*} *Comp.* {57 30 / 46 00} ½ *Comp.* {28° 45′ / 23 00}

From the Log. Tangent of 28° 45′	9.739271
Take the Log. Tangent of 23° 00′	9.627852
	1114.19

Or,

From the Co-Tangent of 23° 45′	10.372148
Take the Co-Tangent of 28° 45′	10.260729
	1114.19

As the Tangent of 51° 38′ 09″	9.898490
To the Tangent of the Course 44° 03′	9.985596
So is the Remainder 1114.2	3.046963
To the Diff. of Long. 853.2 (=14° 13′ E^{ly})	2.931049

Case II.

Where the Diff. of Longitude was given 7° 40′=460 *Min. the Latitude sailed from* 45° 30′ *South, and the Course* N N W; *to find the* Latitude *the Ship is in.*

It will be,

As the Tangent of the Course 22° 30′	9.382776
Is to the Tangent of 51° 38′ 09″	10.101510
So is the Difference of Longitude 460	2.662758
To the Number 1403	3.147044

Which Number	1403
Added to the Log. Tang. of 22° 15′ (½ Comp. of the given Lat.)	9.611841
Gives a Tangent corresponding to 29° 28′	9.752141

Or,

From the Co-Tangent of 22° 15	10.388159
Take the Number before found	.1403
Remains the Co-Tangent of the same Number 29° 28′	10.447859

Which being doubled, and subſtracted from 90° 00′, leaves the Latitude the Ship is in 31° 04′; which being obtained, the Diſtance may be eaſily had.

N. B. If the Ship had ſailed from the Equator, the Number before found muſt have been {ſubſtracted from / added to} the {Tangent / Co-Tang.} of ½ Comp. of the given Lat. and the {Diff / Sum} would have given the {Tangent / Co-Tang.} of half the Complement of the Latitude required.

N. B. Both the Complements of the Latitudes are to be eſtimated from the ſame *Pole* of the World; it is not material from whether: therefore if one Latitude be *North*, and the other *South*, you muſt add 90 Degrees to the one, and ſubſtract the other from 90, and then take their halves, and proceed as before.

Note,

Note, The ſmall Difference betwixt the Logarithms of the Numbers found by the laſt Method, and the Logarithms of the ſame Numbers found before, is owing either to the *Courſe* or the *Latitude*, &c. before found, not being expreſſed to ſmaller Parts than *Minutes*; (which in Navigation would be ſuperfluous) for both theſe Methods are equally true.

N. B. If a Line of the ſame Length with the Line of *Artificial Tangents* upon the Scale, be divided into 1000 equal Parts; the Remainder in the aforegoing Solutions might be found by extending the Compaſſes to ½ Complement of the Latitudes, and applying that Extent to the ſaid Scale of equal Parts: and thence the Proportions worked as before directed, either by the *Sliding Rule* or *Gunter's Scale*.

N. B. If the Remainder found in the foregoing Examples be divided by this Number 1.2633, the Quotient will be the *Meridional Difference of Latitude*. Alſo the *Log.* Co-Tangent (neglecting the Index) of ½ Complement of any given Latitude, divided by the ſaid Number 1.2633, gives the Meridional Parts anſwering to the given Latitude.

In like manner, the Difference betwixt the Sum of the Tangents, or Co-Tangents of ½ the Complements of two Latitudes on different Sides of the *Equator*, and twice the Radius, divided by the aforeſaid Number 1.2633, will give the *Meridional Parts* between the ſaid Latitudes.

I ſhould have here proceeded to give Examples under each of the aforegoing Caſes, but as the *Tables* of Meridional Parts now uſed are ſufficiently exact for all Purpoſes in Navigation, I judged it to be a matter of no great Conſequence to the Mariner.

A TABLE *of Proportions for the Solution of the several Cases relating to a Single Course.*

Fig. A.

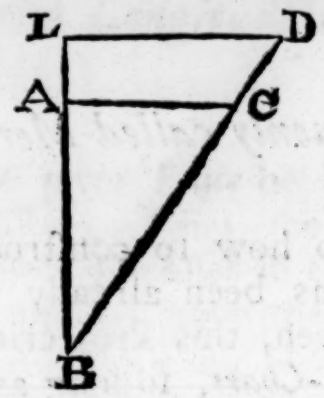
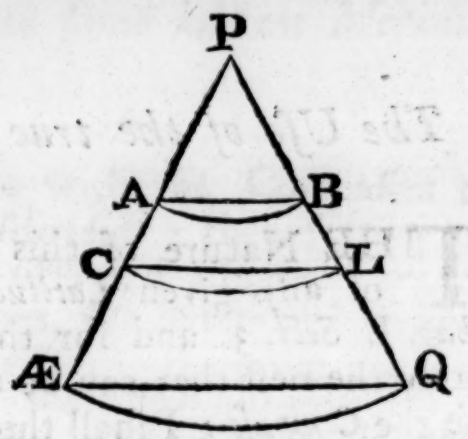


Fig. B.



Giv.	Reqd.	Proportions.	Case.	Prob.
BC B	BA AC	R : C.S.B :: BC : BA. R : S.B :: BC : AC.	1	—
BA B	BC AC	C.S.B : R :: BA : BC. R : T.B :: BA : AC.	2	I
BA BC	B AC	BC : BA :: R : C.S.B. R : S.B :: BC : AC.	3	—
BA AC	B BC LD	BA : AC :: R : T.B. C.S.B : R :: BA : BC. R : T.B :: BL : LD; or BA : BL :: AC : LD; or T.G : T.B :: XT : LD.	4	—
BC AC	B BA LD	BC : AC :: R : S.B. R : C.S.B :: BC : BA. R : T.B :: BL : LD; or BA : BL :: AC : LD; or T.G : T.B :: XT : LD.	5	— II
B AC	BC BA LD	S.B : R :: AC : BC. R : C.T.B :: AC : BA. R : T.B :: BL : LD; or BA : BL :: AC : LD; or T.G : T.B :: XT : LD.	6	—
BA B	B BC	BL : LD :: R : T.B; o XT : LD :: T.G : T.B. C.S.B : R :: BA : BC.	7	—
BC B	BA LD	R : C.S.B :: BC : BA. R : T.B :: BL : LD; o T.G : T.B :: XT : LD.	8	—
BA B	BC LD	C.S.B : R :: BA : BC. R : T.B :: BL : LD; o T.G : T.B :: XT : LD.	9	— II
BA BC	B LD	BC : BA :: R : C.S.B. R : T.B :: BL : LD; or T.G : T.B :: XT : LD.	10	—
B LD	BA BC	T.B : R :: LD : LB; or T.B : T.G :: LD : XT. C.S.B : R :: BA : BC.	11	—
PC ÆQ	CL	PÆ : ÆQ :: PC : CL.	12	— III
PC CL	ÆQ	PC : PÆ :: CL : ÆQ.	13	—
CL ÆQ	PC	ÆQ : CL :: PÆ : PC.		
PC PA CL	AB	PC : PA :: CL : AB.		

Note, The eleven first Cases are referr'd to Fig. A, and the rest to Fig. B.

Note, In Prob. II. one Latitude is understood to be at least given; and when BA or BL is known, the other is known.

Note, { TG *signifies the Tangent of* 51° 38′ 09″
{ XT *the Difference of the Tangents or Co-Tang. of* ½ *Complements of the Latitudes.*

SECT. II.

The Uſe of the true Sea-Chart, commonly called Mercator's.

THE Nature of this *Projection*, and alſo how to conſtruct a *Chart* of any given *Latitude* and *Longitude*, has been already ſhewed in *Chap.* I. *Sect.* 3. and for the Reaſons there given, this Projection is abſolutely the beſt that can be thought of for a *Sea-Chart*, ſo long as Mariners uſe the *Compaſs*: I ſhall therefore be very particular touching its Uſe, and alſo ſhew how all the Caſes in Navigation may be eaſily and ſpeedily wrought thereon; hoping thereby to remove the Prejudice which ſome have conceived againſt it, for want of having its Uſe ſufficiently explained.

PROB. I.

To find the *Latitude* and *Longitude* of any Place, and alſo the *Bearing* and *Diſtance* of any two Places, within the Limits of the Chart.

CASE I.

To find the Latitude.

Plate 7. Let the Place whoſe Latitude is required, be the Point F: Take with the Compaſſes the neareſt Diſtance between the given Place F, and ſome Parallel of Latitude, as (here) the Parallel of 35°; then ſetting one foot of the Compaſſes at the end of the ſaid Parallel, as at *a*, apply that Extent to the graduated Meridian, (the ſame way that the given Place lies from the ſaid Parallel) and the other Foot will fall upon 36°, the *Latitude* required.

Or, Lay a Thread or the Edge of a Ruler over the given Place, ſo that it may cut the ſame Number of Degrees in both the graduated *Meridians*, (on each Side of the Chart) and the ſaid Number of Degrees, *&c.* will be the *Latitude* required.

Or, If you have a *Parallel Ruler* of a convenient Length, lay the Edge thereof along the next Parallel to the given Place; then open the Ruler until the Edge of it touches the ſaid Place, and where the ſaid Edge cuts the graduated *Meridian*, is the *Latitude* required.

Case 2.

To find the Longitude of any Place from ſome known Meridian in the Chart.

Let the given *Place* be F, as before: Take with the Compaſſes the neareſt Diſtance from the Point F to ſome *Meridian*; then ſetting one Foot of the Compaſſes in the beginning of one of the graduated Parallels, the other will fall upon the Number of Degrees, *&c.* contained between the aforeſaid Meridian and the given Place; which if the given Place lies to the {Right / Left} hand from the ſaid Meridian, the Difference of Longitude is {*Eaſterly.* / *Weſterly.*} Or, if you apply a Thread ſo over the given Place to the graduated Parallels, as above directed to the graduated Meridians; you'll have the *Difference* of *Longitude* between the given Point, and the firſt Meridian in the Chart; which in this Examp. is 24° *Weſterly.*

By the Converſe of the two foregoing *Caſes*, having the *Latitude*, and alſo the *Longitude* from ſome known Meridian, of any Place given; the Point where it lies in the *Chart* may be readily found.

Case 3.

To find the mutual Bearings of any two Places in the Chart.

That the Cuſtom of drawing Rhumb-Lines upon Charts is abſurd.

For this Purpoſe, it is cuſtomary to draw *Rhumb-Lines* for every *Point* of the *Compaſs*, from ſeveral Places in the *Chart*. But this Cuſtom is certainly very abſurd, for theſe Rhumbs do very much deface and obſcure the Chart, ſo that the Places laid thereon are not ſo readily diſcerned as otherwiſe they would; beſides, theſe Rhumbs are but of very little (I might ſay of no manner of) uſe, for it is evident, except the Places whoſe Bearings are required lie exactly under ſome one or other of them, (which is a great chance when it ſo happens) their exact Bearings cannot be found: but the uſual Method is, to take with a Pair of Compaſſes the Diſtance betwixt one of the Places, and the Rhumb which lies neareſt parallel to the two Places; then you are to carry one foot along that Rhumb, and as near as you can gueſs, the other perpendicular to it, until it is brought near the other Place, and ſo gueſs how much the Bearing of the two Places is wide of the Rhumb whereon you carried the Compaſſes. This being but gueſs-work at the beſt, (though in ſome Caſes it may indifferently ſerve) I ſhall ſhew another Method whereby the Bearings of any

two Places, may be readily and exactly found, without the Perplexity and Confuſion of Rhumb-Lines.

Let there be made a *Protractor* of about 4 or 5 Inches diameter, graduated and numbered like the Specimen, *Plate 6. Fig.* 1. and right over the Center A let there be faſtened a ſmall cylindrical *Pin*; then having a fine Thread, make a ſmall Nooſe at the end of it, which place about the ſaid Pin.

The Inſtrument being thus prepared, when you would know the Bearings of ſome two Places in the Chart, which for example, let be F and G; apply the *Center* of the *Inſtrument* to one of them, as F, ſo that the *North* and *South* Points thereof may lie exactly in the *Meridian*, (which you may eaſily do by turning it about until the neareſt Meridian to the given Point [F,] cuts the ſame Number of Degrees on the ſame Side of the North and South Points of the Inſtrument,) then extend the Thread over the Point G, and it will ſhew in the Limb the Bearings of the two Places to be N E *b* N and S W *b* S.

In like manner you may readily find the Bearings of any two Places to the greateſt Exactneſs, remembring, (what I need not have mentioned) that the North Point of the *Inſtrument* be to the North Part of the *Meridian*, &c.

For want of ſuch an Inſtrument made in *Braſs*, with a *Pin* fixed over the Center, as above deſcribed; there ought to be one at leaſt printed upon *Vellom* or *Paſt-board*, &c. to be had along with the *Charts*; by which the *Bearings* of Places might be much readier and exacter found, than by the *Rhumb-Lines*: Thus, hold a ſmall *Pin* or a Needle having a Thread about it, perpendicularly over one of the given Places, then having applied the Center cloſe to this Pin, turn the *Protractor* about until it lies in its due Poſition, (as above directed) the Thread extended over the other Place, will ſhew in the Limb the *Courſe* required.

If it be required to find the Courſe between any two Places, according to the *Compaſs*, the Variation being known; having fixed the Protractor as above directed, make a prick at the North Point thereof, then if the *Variation* be { Eaſterly / Weſterly } turn the Inſtrument about towards the { Right / Left } hand, until you bring the quantity of the Variation to the ſaid Point; then the Thread being extended to the other Place, gives the *Courſe* required. For *Examp.* Let the two Places be F and G, (as before) and let the mean *Variation* in that Sea betwixt them be 1 *Point* or 11° 15′ *Weſterly*: having placed the Inſtrument at F, and made a prick at the North Point thereof, (according to the Directions above) turn it about *Weſtward* (or to the left-hand) until you bring N *b* E or 11°¼ Eaſterly to the ſaid prick; then the Thread being extended to G, will ſhew in the Limb that the *Courſe* from F to G, according to the *Compaſs*, is N E.

By this *Inſtrument* any Angle may be laid down and meaſured much readier than by the *Line* of *Chords*. Thus,

1. *To*

1. *To lay down or to make an Angle:* Place the Center to the angular Point, and turn the Inſtrument about until the North or South Point thereof cuts the given Line; then from the ſaid Line count ſo many Degrees in the Limb equal to the propoſed Angle, and there make a prick; a Line drawn from the Center through this prick, will form the *Angle* required.

Note, If the Angle ſhould be *Obtuſe,* you muſt count beyond 90°, calling 80,100 *Deg.* 70,110 *Deg.* &c.

2. *To meaſure an Angle:* Place the Center of the Inſtrument to the angular Point, and turn it about until the North or South Point cuts one of the given *Legs*; then the number of Degrees intercepted betwixt that Leg and where the other cuts the Inſtrument, is the quantity of the *Angle* required.

Note, The double Scale of Degrees upon the Limb, is intended for pricking the Points of the *Compaſs* the more exact; thus, if it were required to draw a N N E Rhumb; right againſt N N E in the innermoſt *Scale* of Degrees I find 22° 30′, wherefore a prick made at 22° 30′ in the outermoſt *Scale,* (the Inſtrument being rectified to the Meridian) will be the Point through which the N N E Rhumb muſt paſs.

4. *To find the Diſtance between any two Places in the Chart, wherein are three Caſes.*

Case 4.

To find the Diſtance between two Places both lying under the ſame Meridian.

In this Caſe, the Difference of Latitude converted into Miles or Leagues, gives the Diſtance required. As if one of the Places be in the Latitude of 40 *Deg.* and the other in the Lat. of 45 *Deg.* the Diſtance between them is 300 *Miles* or 100 *Leagues.*

Case 5.

To find the Diſtance betwixt two Places differing both in Latitude and Longitude.

Let the Places propoſed be F and G. Having found their Latitudes by *Caſe* 1. *viz.* the Lat. of F 36°, and of G 47°; take their Difference of Latitude = 11 *Deg.* from one of the graduated *Parallels,* a *Ruler* being laid over the two Places, apply one foot of the Compaſſes to the Edge thereof, ſo that the other being turned about, may juſt touch ſome Eaſt and Weſt Line croſſed by the ſaid Ruler; then take the Diſtance from the Point *b,* the

the Place where the Compaſſes reſted, to the Point *e*, the Place where the Ruler croſſes the ſaid Eaſt and Weſt Line; that Extent meaſured upon the *graduated Parallel* gives 13° 14′, which multiplied by 60 gives 794 Miles the *Diſtance* required.

The Reaſon of this Operation is evident; for *b d* the Diff. of Lat. taken off the graduated Parallel, is the *Proper Difference of Latitude*, (becauſe the Degrees of Longitude in this *Projection* are every where equal to the Degrees of Longitude upon the *Equator*) and becauſe you are directed to carry one foot of the Compaſſes along the Line G *e*, until the other turned about may juſt touch ſome Eaſt and Weſt Line, (croſſed by the Ruler, as 35 35) the Line *b d* between the Compaſſes muſt be perpendicular to the ſaid Eaſt and Weſt Line, and conſequently parallel to the *Meridian*; wherefore the ∠ *e b d* (= *f* F G) is the *Courſe*, and *e b* (= F *c*) the *Diſtance* in equal Parts or Miles.

In the Year 1724. I conſtructed a *Scale* for meaſuring Diſtances in this Projection, by one Extent of the Compaſſes; which Scale was then engraved by Mr. *Senex*, and ſoon afterwards ſhewed by him to ſeveral People, though never directly publiſhed. But in about four Years afterwards, the very ſame Scale was publiſhed by one Mr. *Haſelden* as his own Invention; with this only difference, that the Rhumbs drawn upon his Scale are Oblique to each other, whereas in mine I made them all parallel Lines for the ſake of taking leſs room, which I take to be an advantage; (but it was ſo much the more trouble, becauſe I was obliged to draw them firſt after the ſame manner as he has done.) And indeed Mr *Haſelden*'s Scale is made for ſmaller Arches than mine, which yet makes no difference in the Nature of the two Scales. Mr. *Haſelden* ſays, he begun his in the Year 1721; but as to that I have no more to ſay, but that I never heard of ſuch a one till about four Years after mine was done, and ſhewed to ſeveral People, and particularly to one Mr. *Scatliff*, an intimate Acquaintance of Mr. *Haſelden*'s. However, I do by no means diſpute the Invention with Mr. *Haſelden*, nothing more probable than that two Perſons might both hit upon ſo eaſy a thing; neither ſhould I have taken this notice of it, but that under the next *Caſe* I uſe a part of the ſaid *Scale* for meaſuring Diſtances in a *Parallel*; nor do I think it any Degradation to me, if now I do not approve of the reſt, (eſpecially ſince I never publiſhed it) for I think any Diſtance upon an *Oblique* Rhumb may be found by the Directions above, as ready, and perhaps more exact, (and I am ſure with leſs Perplexity) than by the fore-mentioned Scale. And, I hope, the Gentlemen who recommended Mr. *Haſelden*'s Scale, have no reaſon to take it amiſs that I differ from them in Opinion; eſpecially, ſince I only diſapprove of what I had done myſelf before.

Mr. *Haſelden* in his Preface to the foreſaid Scale, makes mention of ſome Errors in the Practice of Navigation; but as he gives us no manner of reaſon for what he aſſerts, he leaves the World in ſuſpenſe as to the Truth

Truth of it; and therefore whatever I have advanced upon theſe Heads, I ſtill claim as my own.

Case 6.

To meaſure the Diſtance betwixt two Places, both lying in the ſame Parallel of Latitude.

If the Places are near the *Equator*, and not very far aſunder; you may take the Diſtance between them, and apply the Compaſſes to the graduated *Meridian*, ſo that one foot may ſtand as many *Minutes* above the given *Latitude*, as the other is below it; and the Degrees intercepted may be taken for the *Diſtance*: Or, thus, take the Length of a Degree upon the Meridian at the given Latitude, and turn that Extent between the two Places in a right Line, and ſo often as you find that Extent between the given Places, you may reckon ſo many Score *Leagues* or 60 *Miles*, for the *Diſtance* between them.

The Proportion uſed in the aforegoing Methods, for finding the Length of an Arch of a Parallel, is; *As the Meridional Parts contained between two Latitudes, both equidiſtant from the given Latitude, To the Proper Difference of Latitude; So the Difference of Longitude, To the Meridional Diſtance.* But the true Proportion is; *As the Radius, To the Co-Sine of the Lat. of the given Parallel; So is the Difference of Longitude to the Diſtance required.* Wherefore it would be convenient, that there ſhould be a *Scale* annexed to all *Charts* for meaſuring of Diſtances in all Parallels within their Limits. This *Scale* may be graduated as followeth:

Let it be required to form a *Scale* for this *Chart* that ſhall contain 100 *Leagues*.

N. B. It is convenient the Breadth of the Scale ſhould not contain above 10 *Deg.* and ſo many 10 *Deg.* of Latitude there is in the Chart, ſo many times (at leaſt) the Scale ought to be repeated. Wherefore, 1. Let us make a *Scale* for meaſuring Diſtances in the Parallels from 30 to 40 Degrees.

Having drawn a right Line *a b*, let it be divided into 10 equal Parts, and upon one end thereof erect a Perpendicular *b c*; then through each of the Diviſions in the Line *a b*, draw right Lines parallel to *b c*: Next find how many *Minutes* of Longitude anſwer to 110 *Leagues* or 330 *Miles*, in the extreme Parallels, (*viz.* of 30° and 40°) and alſo in as many of the intermediate Parallels as you judge convenient. To find the Length of 330 Miles in the Parallel of 30 *Deg.* the Proportion will be,

As the Co-Sine of the Latitude 30° 00′	9.937531
To the Radius;	10.
So is 330 *Miles*	2.518514
To the correſpondent Diff. of Long. = 381 *m.*	2.580983

In

In like manner you may find the Length of 110 *Leagues*, in the ſeveral Parallels of 32, 34, 36, 38 and 40 Degrees; which being found, lay theſe ſeveral Diſtances from the Line *a b* in their reſpective Parallels, and thro' the Points of Extenſion (by the help of a *Bow*) draw the Curve Line 100 *c*; then ſubdivide each of theſe Lines into 10 equal Parts, and through the correſpondent Diviſions draw Curve Lines, and let the firſt be numbered 0, the next 10, *&c.* Now the ſeveral Parallels are divided to every 10 Leagues, from 0 to 100. Next, ſubdivide the Breadth of the firſt 10 *Leagues* from the Line *a b*, into 5 or 10 equal Parts, according as it will admit of, and through the correſpondent Diviſions draw Curve Lines, and the *Scale* is finiſhed. *Note*, It is convenient that the middlemoſt Parallel be *dotted*, for the more ready diſtinguiſhing of the reſt.

In like manner may the Parallels from 40 to 50 Deg. be divided, and ſo of any other: And if the Breadth of a Degree in the *Chart* be pretty large, this *Scale* may be very well divided to a ſingle Mile.

The Scale being thus conſtructed, let it be required to find the Diſtance between *f* and G, both Places lying in the Parallel of 47°; becauſe *f* G is greater than the Length of the Parallel of 47° in the Scale, I take off 100 Leagues in that Parallel, and lay it from *f* to *g*; then I take the Remainder *g* G between the Compaſſes, and ſo carry it along the Parallel of 47° until one foot being placed in one of the large Diviſions, the other may fall among the ſmall Diviſions: Thus, when one foot ſtands in 40, the other reaches to 1, which ſhews *g* G to be 41 *Leagues*, and conſequently *f* G is 141 *Leagues* or 423 Miles, the *Diſtance* required.

This *Scale* may be divided *Geometrically*, thus; If you make 100 Leagues or 5 Degrees in the *Equator*, the *Radius* of a Circle; the *Secants* of the ſeveral Degrees in that Circle will be the reſpective Lengths of 100 Leagues in the correſpondent Parallels of Latitude in the *Scale*: whence the intermediate Diviſions may be found, and Curve Lines drawn thro' them, as before. Or, If you make *a b* equal to the Difference of the *Secants* of 30 and 40 Deg. alſo the Breadth of *a* 31°, *a* 32°, *&c.* equal to the Difference of the Secants of 30° and 31°, 32°, *&c.* and through theſe Diviſions draw right Lines perpendicular to *a b*; the extreme Parallels being limited and divided as before, right Lines drawn through theſe Diviſions, will divide the intermediate Parallels into equal Parts: but then the Breadth of the Scale will not be ſo convenient, neither can you eſtimate the Parts of a Degree ſo well, as when the Diſtance between the extreme Parallels is divided into equal Parts.

PROB.

PROB. II.

From any ſufficient *Data*, to find the Ship's Place upon the Chart.

I ſhall begin with *Caſe* 4. of the foregoing *Section*, (becauſe the three firſt are only ſubſervient to thoſe which follow) and ſo run through the remaining Caſes in the ſame Order as they have been there treated of.

I. *Having both Latitudes and the Departure; to find the Ship's Place.*

Examp. Admit a Ship from B in the *Lat.* of 42° 00′ N. ſailed between the North and the Eaſt until ſhe came into the *Lat.* of 48° 00′ N. and that then her *Departure* was computed to be 149 *m.* and it be required to find the *Ship*'s Place upon the *Chart.* Having laid a Ruler over the Point B parallel to the Meridian, take the Difference of Latitude = 6° 00′ from the graduated Parallel, and lay it cloſe to the Edge of the Ruler from B to A; then take 149 Miles (= 2° 29′) from the graduated Parallel, and lay that Diſtance from A, towards the right-hand, as near as you can perpendicularly to the Meridian; then apply the Edge of the Ruler cloſe to the Point C, (where the Foot of the Compaſſes falls) and to the Point B. The Ruler lying in this Poſition, ſee where the Parallel of the Ship's Latitude interſects it, as in D, and the Point D is the *Ship*'s Place. Whence by *Prob.* I. the Courſe will be found to be N N E, the Diſtance ſailed 390 Miles nearly, and the Difference of Longitude 3° 31′.

II. *Having one Latitude, Diſtance ſailed, and the Departure; to find the Ship's Place.*

Let the Place departed from be B (as before) the *Diſtance* ſailed 390 Miles in the N E *Quarter*, and the *Departure* 149 *m.* to find the Ship's Place in the *Chart.* Take 149 *m.* from the graduated Parallel, and lay it directly Eaſt from B to E; then lay the Ruler cloſe to the Point E and parallel to the Meridian, and take 390 (= 6° 30′) from the graduated Parallel, and ſetting one Foot of the Compaſſes in B, turn the other about until it touches the Edge of the Ruler, as in C; then take B E betwixt the Compaſſes, and ſetting one Foot in C, turn the other about until it touches the Meridian B L, as in A: The Line A B meaſured upon the graduated Parallel, gives the Difference of Latitude, by which find the Latitude the Ship is in; a Ruler being laid from B to C, where it interſects the Parallel of Latitude arrived into, as in D, is the *Ship*'s Place required.

III. *Having one Latitude, Courſe, and the Departure; to find the Ship's Place.*

Let B be the Place departed from, the Courſe NNE, and the Departure 149 *m.*

Lay a Ruler from B along the Courſe, then taking 149 Miles from the graduated Parallel, carry one Foot of the Compaſſes along the Meridian B L, ſo that the other Foot may be perpendicular to it, until the ſaid Foot touches the Ruler; then the Length from A (where the Compaſſes reſted) to B being meaſured in the graduated Parallel, will give the *Difference* of Latitude, by which find the *Latitude* arrived into; the Ruler lying ſtill in its former Poſition, ſee where the Parallel of the Ship's Latitude interſects it, as in D; the *Point* D is the *Ship's* Place required.

IV. *Both Latitudes and the Difference of Longitude, being given; to find the Ship's Place.*

Let B be the Place departed from, the Latitude arrived into 48° 00′, and the Difference of Longitude 3° 31′ E^ly. Having laid a Ruler from B parallel to the Meridian, take 3° 31′ from the graduated Parallel, and apply that Extent due Eaſt from the Edge of the Ruler, in the Parallel of 48° 00′; the *Point* D where the ſaid Extent reaches to, is the *Ship's* Place required.

V. *Having one Latitude, the Courſe, and Diſtance; to find the Ship's Place.*

Admit a Ship ſailed from B NNE 390 Miles, and it be required to find her Place upon the Chart.

Having laid a Ruler from B along the Courſe, take 390 Miles (= 6° 31′) from the graduated Parallel, and apply that Extent cloſe to the Edge of the Ruler, from B to C; then take the neareſt Diſtance from C to the Parallel of (B) 42°, and apply that Extent from B to A; then A B meaſured in the graduated Parallel will give the *Difference* of Latitude, by which find the *Latitude* the Ship is in; the *Point* D where the ſaid Parallel interſects the Ruler (it lying ſtill in its former Poſition) is the *Ship's* Place required.

VI. *Both Latitudes, and the Courſe, being given; to find the Ship's Place.*

Admit a Ship ſailed from B NNE, until ſhe came into the Lat. of 48° 00′, and it be required to find her Place in the *Chart.*

Lay a Ruler from B along the Courſe, and where the Parallel of 48° 00′ interſects it, as in D, is the *Ship*'s Place required.

VII. *Having both Latitudes and the Diſtance ſailed; to find the Ship's Place.*

Admit a Ship from B ſails in the N E *Quarter* 390 *Miles*, and then is found to be in the *Lat.* of 48° 00′; and it be required to find her Place in the *Chart*.

Having laid a Ruler from B parallel to the *Meridian*, take 6 *Deg.* (the Diff. of Lat.) from the *graduated Parallel*, and lay that Extent cloſe to the Edge of the Ruler from B to A; then turn the Ruler cloſe to the Point A until it lies directly *Eaſt* and *Weſt*, and take 390 Miles from the *graduated* Parallel, and ſetting one Foot of the Compaſſes in B, turn the other about E^ly until it touches the Edge of the Ruler; the Compaſſes reſting in this Poſition, apply the Ruler cloſe to both the Legs, then where the fiducial Edge of the Ruler interſects the Parallel of 48 *Deg.* as in D, is the Ship's *Place* required.

VIII. *Having one Latitude, the Courſe, and Difference of Longitude; to find the Ship's Place.*

Admit a Ship from B, ſails NNE until her *Difference* of Longitude be 3° 31′; and it be required to find her Place in the *Chart*.

Having laid the Edge of a Ruler from B along the Courſe, take 3° 31′ from the *graduated Parallel*, and carry one Foot of the Compaſſes along the *Meridian* BL, keeping the other due *Eaſt* from it, until it touches the Ruler, as in D; and the Point D is the Ship's *Place* required.

N. B. In the aforegoing Examples, the Rhumb-Line may be repreſented by the Thread extended from the Center of the *Protractor* adjuſted to the Place departed from; and the Ruler may be very well laid either directly *North* and *South*, or *Eaſt* and *Weſt*, by the help of the *Meridians* and *Parallels* already drawn upon the *Chart*.

IX. *Admit a Ship from* L *ſails due Eaſt, until ſhe has altered her Longitude* 3° 31′; *and it be required to find the Place ſhe is in.*

Take 3° 31′ from the *graduated Parallel*, and lay it due Eaſt from L to D; the Point D is the *Place* of the Ship.

X. *Admit a Ship from* L *in the* Lat. *of* 48° 00′ *ſails due* Eaſt 47 *Leagues; and it be required to find her* Place *in the* Chart.

From the *Scale* (for meaſuring Diſtances in a Parallel) take 47 Leagues in the Parallel of 48°; then apply that Extent directly Eaſt from L to B; and the Point D is the Ship's *Place* required.

If the Diſtance ſailed in any Parallel, ſuppoſe 47 Leagues, and the Difference of Longitude correſponding thereto, which ſuppoſe to be 3° 31′, be given; and it be required to find the Latitude of the Parallel.

Take 3° 31′ from the graduated Parallel, and ſetting one Foot of the Compaſſes in 40 Leagues upon the annexed *Scale*, carry it along that Line (keeping both Feet parallel to the Lines of Latitude) until the other Foot falls upon 7 Leagues; the Line wherein the Feet of the Compaſſes ſtand, will be the Latitude required; which in this Example is 48 Degrees.

Thus all the *Caſes* in Navigation may (if required) be eaſily ſolved by this *Projection* and the annexed *Scale:* But for Practice, it is beſt to keep a daily Account of the Ship's Difference of *Latitude* and *Longitude* in Numbers; by which her Place in the *Chart* may be readily found, and conſequently the *Bearing* and *Diſtance* of the Place ſhe is intended for.

The Uſe of this *Projection* being made ſo very plain and eaſy, even to the meaneſt Capacity; it might be thought ſtrange that People ſhould ſtill uſe that abſurd and erroneous Projection called the *Plain Chart:* but for this I do not think that Mariners are ſo much to be blamed, for they muſt make uſe of what *Charts* are already done to their hands, let the Projection be what it will. And ſeeing the Situation of Places are ſo erroneouſly repreſented in moſt of our *Sea-Charts*, it is not at all ſurprizing if People cannot ſo much experience the Excellency of one Projection above the other. But in a ſhort time there will be done a compleat Sett of *Sea-Charts* by Mr. *Senex*, upon that only true Projection invented ſo long ſince by Mr. *Wright*, (tho' commonly called *Mercator's*) and which I do not doubt will exceed any thing of that kind which has hitherto been publiſhed.

SECT. III.

Of a Traverſe, or a Compound Courſe.

A Traverse, is when ſeveral Courſes are conſidered as making one *direct Courſe:* or, when ſeveral Courſes are given, to find the direct *Courſe* and *Diſtance*, &c. made good in all the ſaid Courſes.

When

When the Diſtances ran are but ſmall, it is ſufficient for Practice to find only the *Difference* of *Latitude*, and *Departure* correſponding to each ſingle Courſe (by *Caſe* 1. of *Sect.* 1. of this *Chap.*) and from thence the Difference of Latitude and Departure made good; by which the true Place of the Ship may be found (by *Caſe* 4. of the ſaid *Sect.*) and conſequently the *Bearing* and *Diſtance* of the Place departed from.

Examp. Admit a Ship takes her Departure from the *Lizard* in the *Lat.* of 50° 00′ N. it bearing N N W Diſtance (by Eſtimation) 5 Leagues; and ſails S E 34 Miles, then W *b* S 16 Miles, W N W 39 Miles, and laſtly S *b* E 40 Miles; and it be required to find the Latitude ſhe is arrived into, and her Bearing and Diſtance from the *Lizard.*

[*Note, Taking the Departure from the Land*, is a Phraſe in Navigation; But the Word *Departure* here has no manner of relation to what we call the *Departure* in working a Day's Account. This I mentioned as a farther Caution not to miſtake the Term Departure concerned in a Day's Work, for the Diſtance from the Meridian of the Place departed from.]

1. Find the ſeveral Differences of Latitudes and Departures anſwering to each Courſe and Diſtance, (by *Caſe* 1. *Sect.* 1. of this *Chap.*) as followeth.

Note, Becauſe the *Lizard* bore from the Ship when ſhe took her Departure N N W 5 Leagues, the firſt Courſe from the *Lizard* is S S E 15 Miles.

For the Diff. of Latitude.		*For the Departure.*	
1. Courſe S S E 15 Miles.			
As the Radius	10.	*As the Radius*	10.
To C.S. of Courſe 22° 30′	9.96561	*To the S. of Courſe* 22° 30′	9.58284
So is the Diſtance 15	1.17609	*So is the Diſtance* 15	1.17609
To the Diff. Lat. 13.86	1.14170	*To the Departure* 5.74	0.75893
2. Courſe S E 34 Miles.			
As the Radius	10.	*As the Radius*	10.
To C.S. of Courſe 45° 00′	9.84948	*To the S. of Courſe* 45° 00′	9.84948
So is the Diſtance 34	1.53147	*So is the Diſtance* 34	1.53147
To the Diff. of Lat. 24.04	1.38095	*To the Departure* 24.04	1.38095
3. Courſe W *b* S 16 Miles.			
As the Radius	10.	*As the Radius*	10.
To C.S. of Courſe 78° 45′	9.29023	*To the S. of Courſe* 78° 45′	9.99157
So is the Diſtance 16	1.20412	*So is the Diſtance* 16	1.20412
To the Diff. of Lat. 3.12	0.49435	*To the Departure* 15.69	1.19569

For

For the Diff. of Latitude. *For the Departure.*

4. Course W N W 39 Miles.

As the Radius	10.	*As the Radius*	10.
To C.S. of Course 67° 30′	9.58284	*To the S. of Course* 67° 30′	9.96561
So is the Distance 39	1.59106	*So is the Distance* 39	1.59106
To the Diff. of Lat. 14.92	1.17390	*To the Departure* 36.03	1.55667

5. Course S *b* E 40 Miles.

As the Radius	10.	*As the Radius*	10.
To C.S. of Course 11° 15′	9 99157	*To the S. of Course* 11° 15′	9.29023
So is the Distance 40	1.60206	*So is the Distance*	1.60206
To the Diff. of Lat. 39.23	1.59363	*To the Departure* 7.83	0.89229

Now the ſeveral Differences of Latitudes and the Departures, being thus found, they muſt be diſpoſed of, as in the following Table, called the *Traverſe Table*; which conſiſts of 6 Columns: In the two firſt of which muſt be placed the ſeveral Courſes and Diſtances, and in the other four, the ſeveral Differences of Latitudes and Departures deduced from each Courſe and Diſtance, each in its reſpective Column; that is, if the Courſe be N {Eaſterly / Weſterly} the *Difference of Latitude* muſt be placed under Column N, (or Column for the *North*) and the *Departure* under Column {*Eaſt*; / *Weſt*;} but if the Courſe be S {Eaſterly / Weſterly} then the Departure muſt be placed as before, and the Difference of Latitude under the *South*; and being thus diſpoſed, it is manifeſt, that the Totals of the ſeveral Columns ſhew the Difference of Latitude *North* and *South*, and the Departure *Eaſt* and *Weſt* that the Ship has made; which ſome call the *Northings*, *Southings*, *Eaſtings*, and *Weſtings*.

Wherefore, Having added all the Northings, as likewiſe the Southings, and ſo the Eaſtings, and alſo the Weſtings; the Difference of the Totals of the Northings and Southings gives the *Difference of Latitude*, which if the {Northing / Southing} is greateſt, the Ship is got to the {*Northward* / *Southward*} of her firſt Place; alſo the Difference of the Totals of the Eaſtings and Weſtings gives the *Departure*, which if the {Eaſting / Weſting} be the greateſt, the *Departure* is accordingly {Eaſterly / Weſterly.

The

The Traverse Table.

Courses.	*Dist.*	*Diff. of Lat.*		*Departure.*	
		N.	S.	E.	W.
SSE.	15	- -	13.86	5.74	- -
SE.	34	- -	24.04	24.04	- -
W *b* S.	16	- -	3.12	- -	15.69
WNW.	39	14.92	- -	- -	36.03
S *b* E.	40	- -	39.23	7.83	- -
From Sum	- -	14.92	80.25	37.61	51.72
Take	- -	- -	14.92	- -	37.61
Rests.	- -	- -	65.33	- -	14.11

Now, because the Sum of the *Southings* exceeds that of the *Northings* by 65.33 *Miles*, or 1° 07′.33, it follows, that the Ship has got so far to the Southward of the *Lizard*. Wherefore,

From the Latitude sailed from	50° 00′ N.
Take the Difference of Latitude made	1 07 S^{ly}.
Remains the Latitude the Ship is in	48.53

Again, Because the Sum of the *Westings* exceeds the Sum of the *Eastings* by 14.11 *m.* it follows, that the *Departure* is 14.11 *m.* Westerly; whence to find the direct Course, or the Bearing of the *Lizard* from the Ship; it will be by *Case* 4. of *Sect.* 1.

As the Difference of Latitude 65.33	1.815113
To the Departure 14.11	1.149527
So is the Radius	10.
To the Tangent of the Course 12° 11′	9.334414

Which, because the Difference of Latitude is S^{ly}, and the Departure W^{ly}, is S 12° 11′ W; whence the *Lizard* bears from the Ship N 12° 11′ E^{ly}, or N *b* E 56′ E^{ly}.

Again, for to find the direct Distance, it will be by the same *Case*,

As the Sine of the Course 12° 11′	9.324366
Is to the Radius	10.
So is the Departure 14.11	1.149527
To the direct Distance 66.86 *Miles*	1.825161

By

By the ſame Caſe may the Difference of *Longitude*, if required, be found.

A *Traverſe* may be Protracted three different ways; *viz.*

1. By drawing new *Meridians* through the Extremity of every Courſe, (parallel to the firſt Meridian) then laying off the Courſe and Diſtance, as in *Caſe* 1. of *Sect.* 1.

2. By laying down the Quantity of the Angle (after the firſt Courſe is protracted) between the Point next to be laid down, and the Point oppoſite to the Courſe laſt protracted; and ſo proceeding till the whole is finiſhed.

3. Having a Circle deſcribed, and divided like the *Mariners Compaſs*, you may draw the ſeveral Courſes parallel to the *Rhumbs* in the *Compaſs*, and lay off the Diſtances as uſual: Which being done, a Line drawn from the Place the Ship departed from, to the Place ſhe arrived at, is the *Diſtance*, and the Angle which this Line makes with the Meridian, is the *direct Courſe*; the ſame is to be underſtood in the two former Methods.

For the ſake of the young Learner, I ſhall protract the Example above after theſe three different Methods; tho' in Practice the things required ought to be found *Arithmetically*, and then there will be no neceſſity for a *Geometrical* Conſtruction.

Plate 6. Fig. 2. 1. Having drawn the Meridian L M, take ſome convenient Point therein L, for the *Lizard*; then becauſe the *Lizard* bore N N W 15 *Miles* from the Ship, draw the S S E Line L A, and lay thereon the Diſtance 15 Miles from L to A; then will A be the Place of the Ship when ſhe took her Departure.

Through A draw the Meridian A *a* parallel to L M, and make the ∠ *a* A B=4 Points, and draw the S E Line A B, and lay thereon 34 *Miles* from A to B; then is B the Place of the Ship at the end of her firſt Courſe.

3. Through B draw the Meridian B *b*, and make the ∠ *b* B C= 7 Points, and upon the W *b* S Line B C lay 16 *Miles* from B to C.

4. Through C draw the Meridian C *c*, and make the ∠ *c* C D=6 Points, and upon the W N W Line C D lay 39 *Miles* from C to D.

5. Through D draw the Meridian D *d*, and make the ∠ *d* D E=1 Point, and upon the S *b* E Line D E lay 40 *Miles* from D to E; then is E the Place of the Ship at the end of her laſt Courſe.

Laſtly, Draw the Right Line E L, and let fall the Perpendicular E M; then is L E the *Diſtance* of the Ship from the *Lizard*, L M her *Difference of Latitude*, E M her *Departure*, and the ∠ E L M the *Courſe* ſhe made good.

Note, If from the ſeveral Points A, B, C, *&c.* be drawn Perpendiculars upon the Meridians, you will have the ſeveral *Differences of Latitudes* and *Departures* to each ſingle Courſe; but that would be ſuperfluous, ſince the Difference of Latitude and Departure, *&c.* made good, is the only thing wanted.

Note,

Note, In laying off the Angles, regard muſt be had to the *Cardinal* Points, by conſidering that the Top is the *North*, the Bottom the *South*, the Right-hand the *Eaſt*, and the Left the *Weſt*.

II. Having drawn the *Meridian* L M, and fixed upon ſome Point within it L for the *Lizard*; draw the SSE Line L A, and lay 15 Miles from L to A. *Fig.* 3.

2. Make the ∠ L A B = 14 Points, (the Quantity between the NNW Line A L, and the next Courſe S E,) and make A B = 34 Miles.

3. Make the ∠ A B C = 5 Points, (the Quantity between the N W Line B A, and the next Courſe W *b* S) and lay 16 Miles from B to C.

4. Make the ∠ B C D = 13 Points, (the Quantity between the E *b* N Line C B, and the next Courſe W N W) and lay 39 Miles from C to D.

5. Make the ∠ C D E = 5 Points, (the Quantity between the E S E Line D C, and the next Courſe S *b* E) and lay 40 Miles from D to E; then is E the Place of the Ship; from whence if you draw the right Line E L, and let fall the Perpendicular E M, you'll have the *Courſe*, *Diſtance*, *Difference of Latitude*, and *Departure*, made good, as before.

Note, When the Angle between both Rhumbs is above 8 Points, you may eaſily find its Quantity, by taking both Courſes as they are given (inſtead of taking the oppoſite to the laſt Rhumb) and finding the Angle between them; the Complement whereof to 16 Points or 180 Deg. will be the Angle ſought. Thus, the Arch between the SSE Line L A, and the S E Line A B is evidently two Points, the Supplement whereof = 14 Points, is the ∠ L A B.

III. Having drawn the *Meridian* L M, and fixed on ſome Point within it L for the *Lizard*; either from thence, or from ſome other convenient Place, deſcribe a Circle, which divide after the ſame manner with the *Mariners Compaſs*; then draw the S S E Line L A = 15 Miles. *Fig.* 4.

2. Parallel to the S E draw A B = 34 Miles; again, from B parallel to the W *b* S, draw B C = 36 Miles; next through C parallel to the WNW, draw C D = 39 Miles; Again, from D draw D E = 40 Miles, parallel to the S *b* E: then is E the Place of the Ship at the end of her Traverſe, E L the *Diſtance* from the *Lizard*, L M her *Difference of Latitude*, E M her *Departure*, and the ∠ E L M the *Courſe* ſhe made good.

N. B. You need not draw any more *Rhumbs* in the *Compaſs* than you have Courſes to lay down.

By this Method *Surveyors* who take their Angles by the *Needle*, (or from the *Meridian*) may eaſily protract their Field; by having a large Circle divided into Degrees, *&c.* printed upon the Paper which they intend for their rough Draft; and then by the help of a *Parallel Ruler* may all the Angles be moſt readily laid down.

The moſt expeditious way of working a *Traverſe* of ſhort Diſtances, is by a *Table*, entituled, *A Table of Difference of Latitude and Departure.*

This Table is calculated by *Case* 1. of *Sect.* 1. of this *Chap.* ſhewing the Difference of Latitude and Departure to every $\frac{1}{4}$ Point for the *Courſe*, and for any *Diſtance* not exceeding 100 Miles; and may be made to ſerve for any other Diſtance, provided it be firſt divided into Parts not exceeding the Limits of the Table.

Look the *Courſe*, if {under / above} four Points from the Meridian, in the {Head / Bottom} of the Table, and the *Miles* ſailed in one of the outermoſt Columns, (which if it be {leſs / more} than 50 Miles, you'll find in the {Left / Right} hand Page) and in the common Angle of meeting {under / above} Lat. you'll find the *Difference of Latitude*; and {under / above} Dep. the *Departure* from the Meridian. If the Diſtance ſhould exceed 100 *m.* it muſt be broken into Parts not exceeding 100, and therewith the Table entered, and the Diff. of Latitudes, and the Departures taken out proper to each Part of the Diſtance; the Totals of theſe will be the *Difference of Latitude* and *Departure* required.

Note, The Courſe muſt be always counted from the *Meridian*, or the North or South Points of the *Compaſs*, according as it happens to be Northerly or Southerly.

Examp. 1. Admit a Ship ſails N N E $\frac{1}{2}$ E^{ly} (that is, 2 $\frac{1}{2}$ Points from the Meridian) 68 Miles; and it be required to find the *Difference of Latitude* and the *Departure*.

In the right-hand Page (becauſe the Diſtance is greater than 50) find the Diſtance 68; and at the Top (becauſe the Courſe is leſs than 4 Points) look for the Courſe 2 $\frac{1}{2}$ Points, and right under it againſt 68 the Diſtance ſailed, you'll find in the Column Lat. 59.96 for the *Difference of Latitude*, and in the Column Dep. 32.05 for the *Departure*.

If the Courſe had been N E *b* E $\frac{1}{2}$ E^{ly}, and the Diſtance as before 68; then the ſaid *Courſe* muſt have been ſought for at the Bottom, and the *Difference of Latitude* would have been found to be 32.05, and the *Departure* 59.96.

Examp. 2. Admit a Ship ſails S W *b* W 150 Miles, and it be required to find the *Difference* of *Latitude* and the *Departure*.

At the Bottom of the right-hand Page find out the Courſe 5 Points, and againſt the Diſtance 100 you'll find 55.56 for the *Diff. of Latitude*, and 83.15 for the *Departure*. Again, find out 50 (the remaining Part of the Diſtance) in the left-hand Page, and right againſt it, and above 5 Points, you'll find 27.77 for the Difference of Latitude, and 41.57 for the Departure.

Diſt.

Fig. I

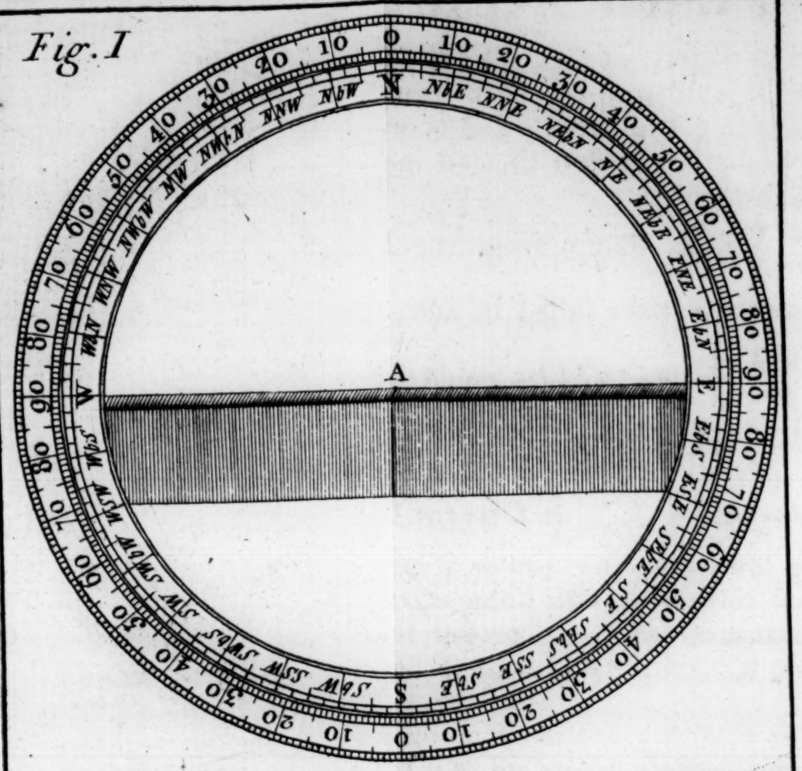


Fig. II

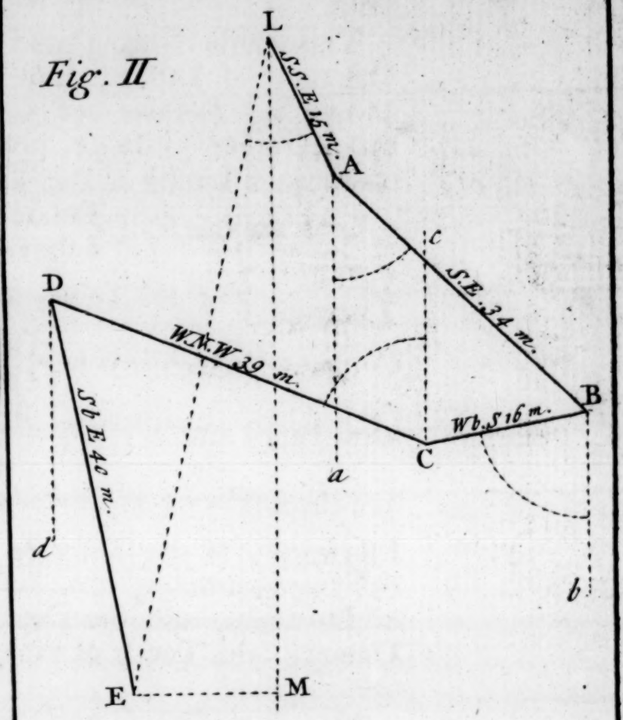


Fig. III

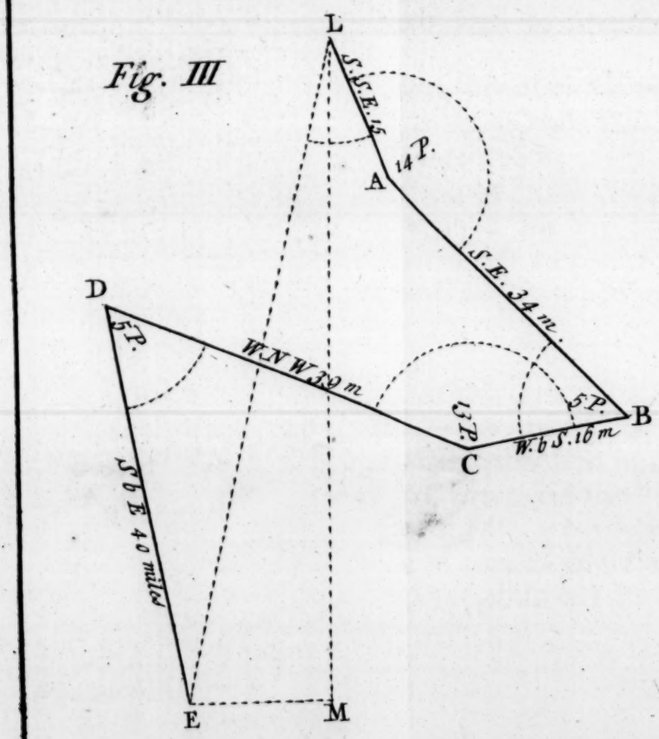
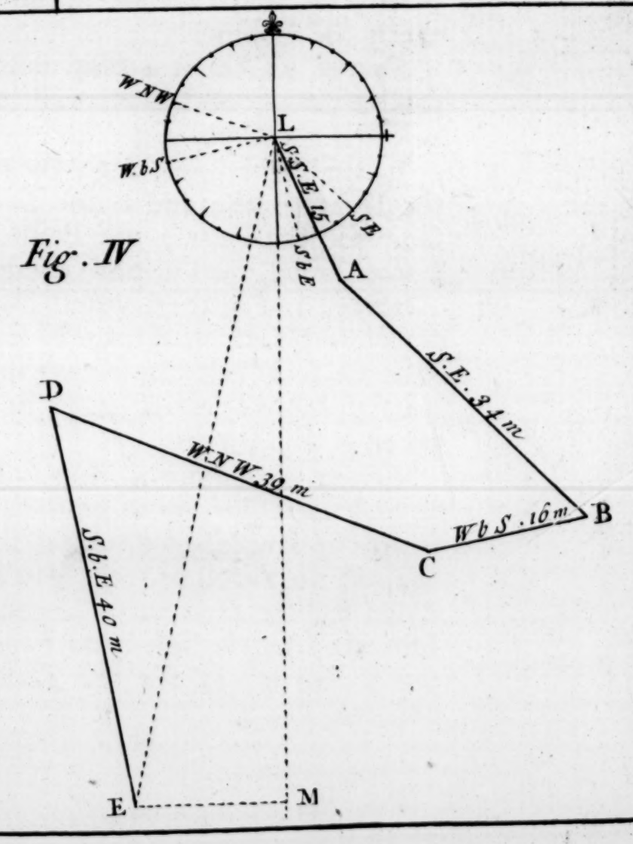


Fig. IV



Diſt.	*Diff. Lat.*	*Dep.*
100	55.56	83.15
50	27.77	41.57
	83.33	124.72

So that the *Difference of Latitude* anſwering to the Courſe S W *b* W, and the whole Diſtance 150 Miles, is 83.33 *min.* and the *Departure* 124.72 *m.*

After the ſame manner the ſeveral Diff. of Latitudes, and the Departures anſwering to each Courſe and Diſtance, in the foregoing *Traverſe*, will be found, as ſet down in the *Traverſe-Table*; and ſo of any other.

When the Diſtances ran are pretty large, inſtead of finding the Departure, you ought to find the *Difference of Longitude* correſponding to each ſingle Courſe: Thus,

1. Find the *Difference of Latitude* anſwering to each Courſe, which ſet down in a Table, as in the Example above; then in another Column ſet down the ſeveral *Latitudes* arrived into, in their proper Places, againſt which, in two ſeparate Columns, ſet the *Meridional Parts* anſwering to each Latitude, and alſo the *Meridional Difference of Latitude:* then have you at one View all the Numbers requiſite for finding the *Difference of Longitude*; which being obtained, either by the 8th or the 13th *Caſe* of *Sect.* 1. ſet in proper Columns, as before you did the *Departures*; then add the *Eaſt* Longitudes together, and alſo the *Weſt* Longitudes, and their Difference will be the *Difference of Longitude* required.

Example. Admit a Ship from the *Lizard* in the Lat. of 50° 00′ N. is bound to the Weſt-end of *Madera*, lying in the Lat. of 32° 20′ N. Longitude from the *Lizard* 12° 15′ Weſt; and ſteers as followeth, *viz.* 1. SSW 250 Miles; then due Weſt 156 Miles, SE *b* S 300 Miles, W *b* N 180 Miles, and laſtly due South 185 Miles; and it be required to find the *Latitude* ſhe is arrived into, and the *Bearing* and *Diſtance* of the Weſt-end of the *Madera*.

The Traverse Table.

Courses.	*Dist.*	*Diff. of Lat.*		*Latitude.*	*M.Parts.*	*Diff. of*	*Diff. of Long.*	
		North.	*South.*	50° 00′ N.	3474.5	*M.Parts.*	*East.*	*West.*
S S W.	250		231	46 09	3128.5	346		143.3
West.	156			46 09				225.2
S E *b* S.	300		249.4	42 00	2781.7	346.8	231.7	
W *b* N.	180	35		42 35	2829	47.3		237.8
South.	185		185	39 30				
							231.7	606.3
								231.7
Diff. of Lat.				*Diff. of Longitude.*				374.6

Thus, having proceeded according to the above Directions, I find that the Ship is arrived into the *Lat.* of 39° 30′ N. and that she has made 374.6 *m.* (=6° 14′.6) of *Difference of Longitude* Westerly; whence the direct Course and Distance from the *Lizard* may be had by *Case* 7. of *Sect.* 1.

To find the Bearing and Distance of the West-end of Madera *from the Ship.*

From the Latitude the Ship is in 39° 30′ *N.* ——— 2583.6
Substract the Latitude of Madera 32 20 *N.* ——— 2052 } *M. Parts.*

7 10 — 531.6 *M. Diff. Lat.*
60

Prop. Diff. Lat. 430

Again,

From the Longitude of Madera *from the* Lizard 12° 15′ W (=M *m*)
Take the Longitude of the Ship *from the* Lizard 6° 15′ W (=E *e*)

Remains the Long. of Madera *from the Ship* { 6 00 W^ly (=MS)
60
360 *m.*

Then say,

As the Merid. Diff. of Lat. (E S) 531.6 — 2.725585
To the Difference of Longitude (S M) 360 — 2.556302
So is the Radius — 10.
To the Tang. of the Course (S E M) 34° 06′ — 9.830717

Hence

Hence, the Weſtermoſt Point of *Madera* bears from the Ship S 34° 06′ W^{ly}.

To find the Diſtance, it will be,

As the Co-Sine of the Courſe 34° 06′	9.918062
Is to the Radius;	10.
So is the Difference of Latitude 430	2.633468
To the direct Diſtance 519.3 *Miles*	2.715406

Having found the ſeveral Latitudes arrived into, this *Traverſe* may be delineated in the *Sea-Chart* by *Caſe* 6. and 10. of *Prob.* II. of the laſt *Section*; and thence the Courſe and Diſtance from the Point E (the Place of the Ship at the end of her Traverſe) to the Point M (repreſenting the Weſtermoſt Part of *Madera*) may be found by *Caſe* 3. and 5. of *Prob.* I. of the ſame *Sect.*

I ſhall next ſhew how a Ship may ſhape her Courſe, ſo as to bring any known Place, upon any required Bearing and Diſtance from her; which many times is very neceſſary to be known, in order to gain the advantage of *Tides* or conſtant *Winds*, &c. when a Ship is near entering into a *Harbour*, &c.

1. Let it be required to find how far, and upon what Courſe the Ship muſt ſteer from the Point E, to bring *Madera* or the Point M to bear S E *b* S 120 Miles.

Conſtruction. Thro' M draw the Meridian M *d*, and alſo the N W *b* N Line M N; then taking 120 Miles from the graduated Parallel, apply that Extent from M to *b*, and let fall the ⊥ *b a*: then *a* M meaſured on the graduated Parallel, will give the Difference of Latitude, by which find the Latitude of the Point *d*, (or the Latitude wherein the Ship would be after ſhe had ſailed N W *b* N 120 Miles from M) through which draw the Parallel N *e*, and where this Parallel interſects the Rhumb-Line M N, as in N, is the Point which the Ship muſt ſail to; from whence draw the Rhumb-Line N E: then is the ∠ N E *e* the required *Courſe*, E *e* the *Diff. of Latitude*, and N *e* the *Difference of Longitude*; whence the *Diſtance* may be found by *Caſe* 5. of *Prob.* I. of the laſt *Section*. But to determine theſe by *Calculation*,

Firſt, the Difference of Latitude and Longitude betwixt *Madera* or the Point M, and the Point N where the Ship is required to ſail into; muſt be found as followeth, *viz.* R : C.S. *d* M N : : M *b* : M *a*; that is,

As the Radius	10.
To the Co Sine of the Courſe 33° 45′	9.919846
So is the Diſtance 120 *m.*	2.079181
To the Difference of Latitude 99.78 *m.*	1.999027

Which,

Which, becauſe the Difference of Latitude is Northerly, and the Latitude of *Madera* is N^ly.

To the Latitude of Madera	32° 20′ N.	2052	*M. Parts.*
Add the Diff. of Lat. 99.78 =	1° 40′ N^ly.		
The Latitude of the Point N.	34 00	2171	
		Merid. Diff. Lat. 119	

To find the Difference of Longitude, it will be,

As the Radius	10.
To the Tangent of the Courſe 33° 45′	9.824892
So is the Merid. Difference of Lat. 119	2.075547
To the Diff. of Longitude (*d* N) = 79.5 *m.*	1.900439

Which, becauſe it is Weſterly, and *Madera* lies Weſtward from the Ship,

To d e *the Diff. of Long. of* Madera *from the Ship*	360′
Add the Difference of Longitude d N	79.5
The Sum N e *is the Diff. of Long. the Ship muſt make*	439.5

Next,

From the Latitude of the Point E	39° 30′	2583	*M. Parts.*
Subſtract the Latitude of the Point N	34 00	2171	
	5 30	412 *M. Diff. Lat.*	
Prop. Diff. of Latitude	60		
	330		

To find the direct *Courſe* from E to N, it will be,

E *e* : *e* N :: R : T. *e* E N; that is,

As the Merid. Diff. of Latitude 412	2.614897
Is to the Diff. of Longitude 439.5	2.642959
So is the Radius	10.
To the Tang. of the Courſe 46° 51′ (S W^ly)	10.028062

Whence to find the *Diſtance*, it will be,

As the Co-Sine of the Courſe 46° 51′	9.834999
Is to the Radius	10.
So is the Proper Diff. of Latitude 330	2.518514
To the Diſtance 482.5 *m.*	2.683515

2. If

2. If it be required to find how far, and upon what Courſe the Ship muſt ſteer from E, to bring the Point M to bear due Weſt diſtant 90 Miles.

Firſt, find the Difference of Longitude correſponding to 90 Miles in the Lat. of the Point M, (or the Weſt-Point of *Madera*) as followeth;

As the Co-Sine of Lat. (of the Point M) 32° 20′	9.926831
Is to the Radius	10.
So is the Merid. Diſtance 90 *Miles*	1.954242
To the Diff. of Longitude 106.5	2.027411

Which 106.5 (=1° 46′½) take from the graduated Parallel, and lay Eaſtward from M to O; then is the ∠OES the *Courſe* the Ship muſt ſteer; and if E*f* be made equal to the *Proper Difference of Latitude* from E to S, *g* E will be the *Diſtance* required.

Or, the Point O might be found only by help of the *Scale* deſcribed in the laſt *Section*, for meaſuring of *Meridional Diſtances*; whence the Courſe and Diſtance might be found by *Caſe* 3 and 5. of *Prob.* I. of the ſaid *Sect.*

Next, becauſe the Points O and E are both to the Eaſtward of *Madera*,

From the Longitude of the Ship, or the Point E *from* Madera	360′
Subſtract the Long. of the Point O *from* Madera	106.5
Remains OS *the Diff. of Long. the Ship muſt make*	253.5

To find the ∠OES, or the *Courſe* from F to O, it will be,

ES : SO :: R : T. SEO; that is,

As the Merid. Diff. of Lat. ES=531.6	2.725585
To the Diff. of Longitude OS=253.5	2.403978
So is the Radius	10.
To the Tang. of the Courſe 25° 30′ (S W^{ly})	9.678393

Whence, to find the *Diſtance*, it will be,

C.S. OES : R :: E*f* : E*g*; that is,

As the Co-Sine of the Courſe 25° 30′	9.955488
Is to the Radius:	10
So is the Proper Diff. of Latitude 430	2.633468
To the Diſtance required 476.4 *m.*	2.677980

After the ſame manner it may be eaſily found, how far and upon what Courſe a Ship muſt ſteer, in order to bring any known Place to any Bearing

Bearing and Diſtance required; *viz.* 1. Finding the Diff. of Latitude and Longitude (from ſome known Place) of the Point the Ship muſt ſail into; then by the Difference of Latitude and Longitude from the Ship to the ſaid Point, may the *Courſe* and *Diſtance* required be found.

SECT. IV.

Of Finding the Bearings and Diſtances of Places.

ALthough this *Section* is of no immediate Uſe in keeping a Ship's Reckoning, yet is it very uſeful for deſcribing of *Sea-Coaſts*, laying down of *Shoals*, &c. in *Sea-Charts*; alſo Surveying of Land, and Mapping of Countrys may be performed by the Rules herein given.

It would be needleſs here to trouble the Reader with a Deſcription of the ſeveral Inſtruments uſed for taking of *Angles*; thoſe uſed by *Surveyors* of *Land* being of various kinds, as the *Theodolite*, *Circumferentor*, &c. But the *Bearings* of Places at Sea may be readily found by the new *Azimuth Compaſs* deſcribed in *Sect.* 2. of *Chap.* IV. Thus, turn the Inſtrument about until you can ſee the Object through both the Sights, and the Point of the *Card* which cuts the Meridian, is the *Bearing* required.

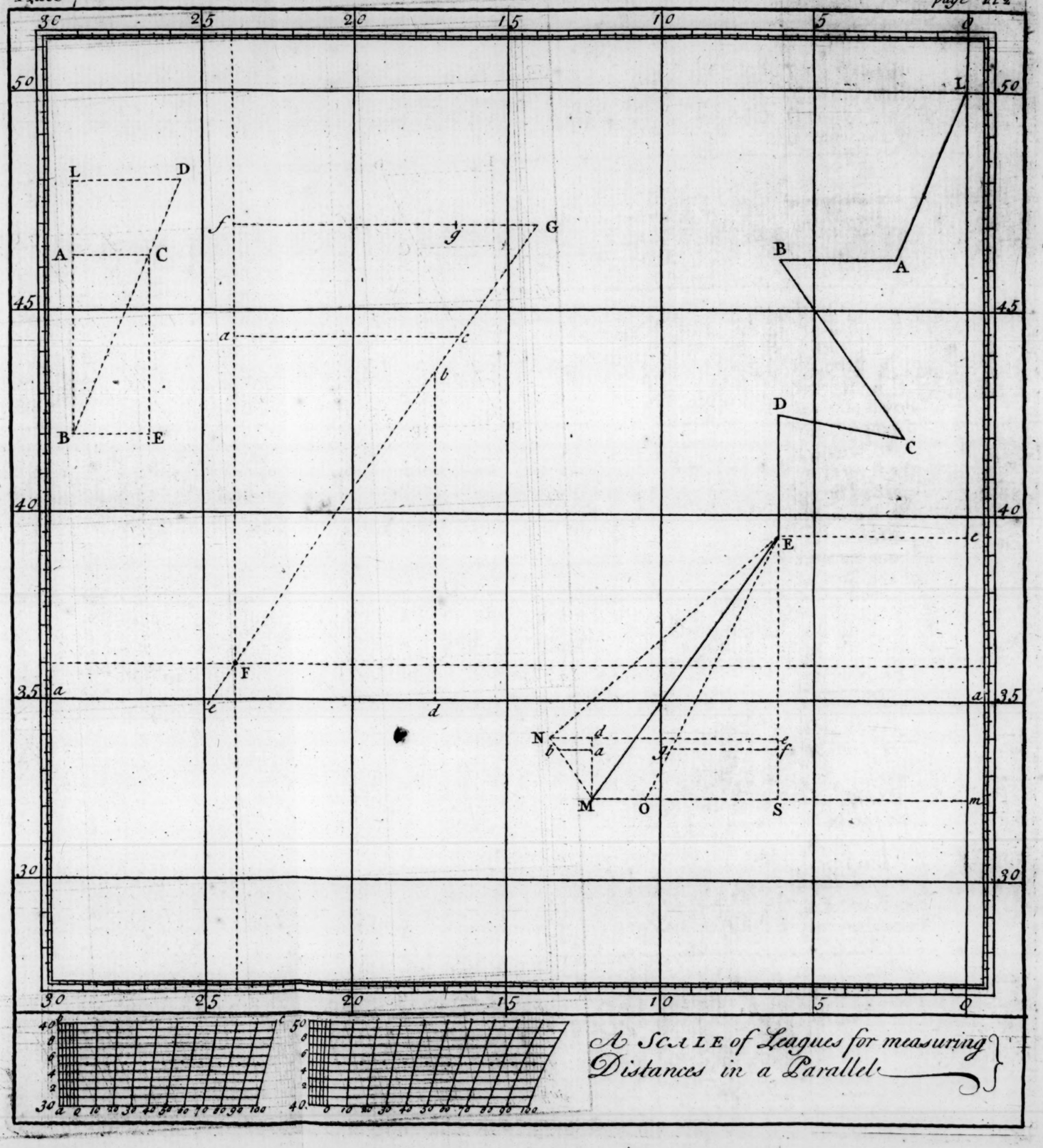
CASE 1.

The Bearing and Diſtance of two Places from each other, as alſo the Bearing of each of them from a third Place, being given; to find the Diſtance from the ſaid third Place to each of the other two Places.

Examp. 1. Coaſting along the Shore I ſaw a *Cape* of Land which bore from me N N E, then I ſtood away W *b* N 15 Miles, and the ſame *Cape* bore from me N E *b* E; I would know the Diſtance of the Ship at both Stations from the *Cape*.

Plate 8. *Fig.* 1. *Conſtruction.* 1. Having drawn the *Compaſs* N, E, S, W; let A repreſent the Place of the Ship in her firſt *Station*, from whence through the N N E Point draw the indefinite right Line A C, alſo draw the W *b* N Line A B =15 Miles; then will B repreſent the Place of the Ship in her ſecond *Station*.

2. From B draw the Line B C parallel to the N E *b* E till it meets A C in C; then will C repreſent the *Cape* of *Land*, A C the Diſtance of the



A SCALE of Leagues for measuring Distances in a Parallel

the Ship from the *Cape* in her firſt Station, and BC the Diſtance in her ſecond Station. To find which by Calculation,

In the △ ABC are given the Side AB = 15 Miles, the ∠ BAC = 101° 15′, (=9 Points, the Diſtance between the NNE Line AC and the W *b* N Line AB) the ∠ B = 45° 00′, (= 4 Points, the Diſtance between the E *b* S Line BA and the NE *b* E Line BC) the ∠ C = 33° 45′, (the Supplement of the other two to 180°, or the Diſtance between the SW*b*W Line CB and the SSW Line CA;) whence, firſt to find the *Diſtance* AC, it will be by *Caſe* 8. of *Trigonometry*.

S.C : AB :: S.B : AC; that is,

As the Sine of the ∠ C = 33° 45′	9.744739
To the Diſtance ſailed AB = 15 *Miles*;	1.176091
So is the Sine of the ∠ B = 45° 00′	9.849485
To the Side AC, *the Diſtance between the Ship and the Cape in her firſt Station.* = 19.09*m.*	11.025576
	1.280837

2. To find BC the Diſtance of the Ship in her ſecond Station from the *Cape*, it will be by the ſame *Caſe* of Trigonometry,

S.C : S.A :: AB : BC; that is,

As the Sine of the ∠ C = 33° 45′	0.255261 *Arith. Co.*
Is to the Sine of the ∠ A = 101° 15′ (its Sup. 78° 45′)	9.991574
So is the Diſtance run AB = 15 *Miles*	1.176091
To the Diſtance required BC = 26.48 *Miles*	1.422926

This *Solution* will ſerve for many other *Queſtions* dreſſed in different Terms; of which the following ones are a *Specimen*.

Examp. 2. Being at Sea I ſaw two *Head-lands*, whoſe Bearing from one another I find by the *Chart* to be W *b* N and E *b* S Diſtance 15 Miles; the Northermoſt bore from me SW *b* W and the Southermoſt SSW; I demand my Diſtance to each of the ſaid *Head-lands*.

Anſ. The Diſtance from the Northermoſt *Head-land* is 26.48 Miles, and from the Southermoſt 19.09 Miles.

This Example and the firſt are of uſe for finding the Diſtance from a *Head-land*, &c. when a Ship is about to take her *Departure* from the *Land*.

Examp. 3. Two Ships ſet ſail from the ſame *Port*, one ſails W *b* N 15 Miles, the other ſails NNE ſo far until ſhe finds the firſt to bear SW*b*W; I demand the Diſtance between the two Ships, and the ſecond Ship's Diſtance ſailed.

Anſ. The Diſtance between the two Ships is 26.48 Miles, and the ſecond Ship's Diſtance ſailed is 19.09 Miles.

Examp. 4. There are three *Iſlands* in ſight of each other, a Ship at A finds C to bear N N E, then ſailing W *b* N 15 Miles to B, C bears N E *b* E; I demand the Diſtance from A to C, and from B to C.

Anſ. The Diſtance from A to C is 19.09 Miles, and from B to C is 26.48 Miles.

Examp. 5. There are two *Ports* that lie N N E and S S W, a Ship from the Southermoſt ſails W *b* N 15 Miles, another from the Northermoſt ſails S W *b* W until ſhe meets the firſt Ship; I would know the Diſtance between theſe two *Ports*, and the ſecond Ship's *Diſtance* ſailed.

Anſ. The Diſtance between the two *Ports* is 19.09 Miles, and the Diſtance ſailed by the ſecond Ship is 26.48 Miles.

Examp. 6. A Ship under ſail diſcovers a dangerous *Shoal*, and finds it to bear from a *Mark* on the Shore N N E; then ſailing W *b* N 15 Miles finds the *Mark* to bear N E *b* E; required the Diſtance from the *Shoal* to the ſaid *Mark*.

Anſ. The Diſtance between the dangerous *Shoal* and the *Mark* on Shore is 19.09 Miles.

This Example is uſeful for laying down of *Rocks* or *Sands*, &c. in *Sea-Charts*.

Examp. 7. Admit there are two *Ports* bearing from each other W *b* N and E *b* S diſtant 150 Leagues, and that a Ship from the Northermoſt ſteers away ſometimes N E *b* E, and ſometimes S S W, and then arrives at the Southermoſt Port; I demand the *Diſtance* ſailed upon each of theſe Courſes.

Anſ. She ſailed upon the N E *b* E Courſes 264.8 *Leagues*, and upon the S S W Courſes 190.9 *Leagues*.

Examp. 8. Being at A by a River-Side, I would know how far it is to C a *Steeple* on the other Side. In order thereto, I obſerve the Angle contained between a Mark B on this Side the River, and the *Steeple* C, to be 101° 15′; then leaving a Mark at A, I meaſure from thence the direct Diſtance to B, which I find to be 150 *Perches*; then at B, I obſerve the ∠ contained between the Point A and the *Steeple* C, to be 45° 00′. Now in the △ A B C, I have given the Side A B, and all the ∠ˢ, to find the Side A C, which by *Calculation* I find to be 190.9 *Perches*, the *Diſtance* required; and the *Diſtance* B C is 264.8 *Perches*.

After the ſame manner may the Diſtance from any Place, to any other Place within ſight, be determined.

Case 2.

The Bearing and Diſtance of two Places from each other, and the Diſtance of one of theſe Places, and the Bearing of the other from a third Place, being given; to find the Bearing of the firſt, and the Diſtance of the ſecond, from the third Place.

Examp. 1. Admit two Ships ſail from the ſame *Road*, one ſails N E $\frac{1}{2}$ E^{ly} 16 Miles, the other ſails 20 Miles, and then finds the firſt to bear N N W;

N N W; I demand the Diſtance between the two Ships, and the Courſe which the ſecond ſteered.

Conſtruction. 1. Having drawn the *Compaſs* E, N, W, S; let A be the Place the Ships departed from, and draw the N E ½ E Line A B = 16 Miles. *Fig.* 2.

2. From B draw the right Line B C parallel to the S S E, then with 20 Miles betwixt the Compaſſes, ſetting one Foot in A, with the other croſs the Line B C as in C, and join A C; then is the ∠ B A C the *Courſe* which the ſecond Ship ſteered, and the Side B C the *Diſtance* between the two Ships. To find which by *Calculation*; in the △ A B C are given the Side A B = 16 Miles, the Side A C = 20 Miles, and the ∠ A B C = 73° 07′½ (= 6 ½ *Points*, the Diſtance between the S W ½ W Line B A and the S S E Line B C;) whence to find the ∠ C, it will be by *Caſe* 9. of *Trigonometry*.

A C : A B :: S. B : S. C; that is,

As the Side A C 20 *m.*	8̄.698970
To the Side A B 16 *m.*	1.204120
So the Sine of the ∠ B 73° 07′ ½	9.980885
To the Sine of the ∠ C 49° 57′	9.883975

Which being counted from the N N W to the left-hand or W^ly, (becauſe C A is to the left-hand of C B) gives the *Bearing* of C A, N 72° 27′ W^ly; whence, the *Courſe* ſteered by the ſecond Ship is S 72° 27′ E^ly or E S E ½ E nearly.

2. To find the Side B C the Diſtance between the two Ships, it will be by *Caſe* 10. of *Trigonom.*

S. B : S. A :: A C : B C; that is,

∠ B = 73 07½	*As the Sine of the* ∠ B 73° 07′½	0.019115
∠ C = 49 57	*Is to the Sine of the* ∠ A 56° 55′½	9.923222
123 04½	*So is the Side* A C 20 *m.*	1.301030
∠ A = 56.55½	*To the Diſt. required* C B 17.51 *m.*	1.243367

This *Solution* will give Anſwers to the ſeveral Queſtions following.

Examp. 2. Suppoſe two *Ports* lie N N W and S S E, and a Ship from the Northermoſt ſails S W ½ W 160 Miles, and another from the Southermoſt ſails 200 Miles, and then meets the firſt Ship; I would know the Courſe ſteered by the laſt, and the Diſtance of the *Ports*.

Anſ. The laſt Ship ſteered N 72° 27′ W^ly, or W N W ½ W nearly; and the Diſtance of the Ports is 175.1 Miles.

Examp. 3. A Ship under ſail deſcries an *Head-land* diſtant from her 20 Miles, and after ſhe had ſailed N E ½ E 16 Miles, the ſame *Head-land* bore

from her S S E. How did the *Head-land* bear from the Ship at firſt, and what is her preſent Diſtance from it?

Anſ. The *Head-land* at firſt bore S 72° 27′ E^{ly}, and now it is diſtant from the Ship 17.51 Miles.

Examp. 4. Being at Sea the Wind at N E *b* N, I ſaw a *Head-land* bearing N E ½ E diſtant 16 Miles; then I ſtood away between the South and the Eaſt 20 Miles, and then the ſame *Head-land* bore from me N N W; I demand the true *Courſe*.

Anſ. The true Courſe is S 72° 27′ E^{ly} or E S E ½ E nearly.

Hence, knowing how near the Wind the Ship can lie, her *Leeway* or *Driſt* may be found; for admitting ſhe ſailed E ½ S by the *Compaſs*, or within 5 ½ Points of the Wind, then in this Caſe ſhe has made one Point *Leeway*.

Examp. 5. Admit there are two *Ports* bearing N E ½ E and S W ½ W diſtant 16 Miles, and a Ship from the Northermoſt ſails S S E, and then ſailing 20 Miles farther, arrives at the Southermoſt Port; I would know the firſt Diſtance ſailed, and Courſe to the ſecond Port.

Anſ. The firſt Diſtance ſailed is 17.51 Miles, and her ſecond Courſe N 72° 27′ W^{ly} or W N W ½ W nearly.

Either of the Examples of the two preceding Caſes might have been ſo propoſed, as to have been ambiguous, and admitted of two Anſwers, as appears by the 9th and 10th *Caſes* of *Trigonometry*; where is ſhewed at large when ſuch Ambiguities may happen, together with the Solution of the double Anſwers.

Case 3.

The Bearings and Diſtances of any two Places from a third Place, being given; to find the Bearing of the ſaid Places, and their Diſtance from each other.

This *Caſe* is the ſame with *Caſe* 11. of *Trigonometry*, where two Sides and the ∠ included, are given; to find the reſt.

Examp. 1. Admit two Ships ſet ſail from the ſame *Port*, one whereof ſails N W 30 Miles, the other ſails N E *b* N 40 Miles; I demand their *Bearing* and *Diſtance* from each other.

Fig. 3. *Conſtruction.* Having drawn the *Compaſs* N, E, S, W; let B be the Place from whence the Ships departed; then draw the N W Line B C = 30 Miles, alſo the N E *b* N Line B D = 40 Miles, and join the Points C and D: then will C D be the mutual Diſtance of the Ships, and the ∠ C the *Bearing* from the S E Line C B.

To find which by *Calculation*; in the △ B C D are given the Side B C =30 Miles, the Side B D=40 Miles, and the ∠ B=78° 45′, (=7 Points, the Quantity between the N W and N E *b* N;) whence to find the ∠ C it will be by *Caſe* 11. of *Trigonometry*.

180°

	180° 00′	*As the Sum of the Sides* B C *and* C D = 70	8̄.154902
∠ B	78 45	*Is to their Difference* = 10	1.
	101 15	*So is the Tang. of* ½ Z ∠ˢ B+D = 50° 37′½	10.085827
½ Z ∠ˢ C+D	50 37½	*To the Tang. of* ½ *their Diff.* = 9° 52′½	9.240729

Which added to 50° 37′½ gives the ∠ C = 60° 30′ = ∠ *b* B *d*; whence the Bearing of the Ship at D from the Ship at C is N. 74° 30′ E^{ly}, or E *b* N 4° 15′ N^{ly}, and consequently the Bearing of C from D is S. 74° 30′ W^{ly}, or W *b* S 4° 15′ S^{ly}. Whence, to find the mutual Distance C D, it will be,

As the Sine of the ∠ C = 60° 30′	0̄.060303
Is to the Sine of the ∠ B = 78° 45′	9.991574
So is the Side B D = 40	1.602060
To the mutual Distance C D = 45.07 *m.*	1.653937

This *Solution* will likewise give Answers to the several Questions following.

Examp. 2. There are two *Islands* D and C, a Ship from the Northermost sails S W *b* S 40 Miles, another from the Southermost sails S E 30 Miles, and then meets the former; I demand the Bearing and Distance between the two *Islands*.

Ans. The *Islands* bear from each other S 74° 30′ W, and N 74° 30′ E; Distance 45.07 Miles.

Examp. 3. Coasting along the Shore I see two *Head-lands*, the first bears from me N W, Distance by Estimation 10 Leagues, or 30 Miles, the other bears N E *b* N Distance 40 Miles; I would know the Bearing and Distance between the two *Head-lands*.

Ans. The Bearing between the two *Head-lands* is N 74° 30′ E, and S 74° 30′ W; Distance 45.07 Miles.

Examp. 4. Coasting along the Shore I saw a *Cape* of Land bearing from me N W, Distance about 10 Leagues or 30 Miles, then I stood away N E *b* N 40 Miles; I desire to know how the *Cape* bears from me then, and how far I am from it.

Ans. The *Cape* bears from me S 74° 30′ W or W *b* S 4° 15′ S^{ly}; Distance 45.07 Miles.

Examp. 5. A Ship from a certain *Port* sails S E 30 Leagues, then N E *b* N 40 Leagues; I demand the direct Course and Distance from the *Port*.

Ans. The direct Course is N 74° 30′ E, or the Port bears S 74° 30′ W; Distance 45.07 *Leagues*.

Case 4.

The mutual Diſtances of three Places from each other, and alſo the Bearing between any two of them, being given; to find the ſeveral Bearings of theſe two from the third Place.

This *Caſe* is the ſame with *Caſe* 12. of *Trigonom.* where the three Sides of a Triangle are given to find the Angles.

Examp. 1. Admit there are two *Ports* lying E *b* N and W *b* S diſtant 400 Miles; a Ship from the Eaſtermoſt ſails N^{ly} 450.7 Miles, another from the Weſtermoſt ſails 300 Miles, and then meets the firſt: I demand the *Courſes* ſteered by theſe two Ships.

Fig. 4. *Conſtruction.* 1. Having drawn the *Compaſs* N, E, S, W; let B repreſent the Weſtermoſt *Port*, and draw the E *b* N Line B D = 400 Miles; then will D be the Eaſtermoſt *Port.*

2. With 300 *m.* betwixt the Compaſſes, ſetting one Foot in B, with the other deſcribe an Arch ſomewhere above the Line B D, as at C; then taking 450.7 *m.* betwixt the Compaſſes, ſetting one Foot in D, with the other interſect the former Arch in C, and join B C and D C: then is the ∠ D the Quantity from the W *b* S which the firſt Ship ſteered, and the ∠ B the Courſe from the E *b* N which the ſecond Ship ſteered. To find which by *Calculation*, it will be by *Caſe* 12. of *Trigonometry.*

D C = 450.7	*As the Baſe* B D = 400	7.397940
B C = 300	*To the Sum of the* 2 *Sides* D C & B C = 750.7	2.875466
D C + B C = 750.7	*So is their Difference* = 150.7	2.178113
D C − B C = 150.7	*To* D *a* = 282.8	2.451519

Which ſubſtracted from B D leaves *a* B = 117.2, the half whereof is A B = 58.6; then, to find the Courſe which the firſt Ship ſteered, it will be,

	As the Hypothenuſe D C = 450.7	2.653888
D *a* = 282.8		
a A = 58.6	*Is to the Side* D A = 341.4	2.533263
	So is the Radius	10.
D A = 341.4	*To the Co-Sine of the* ∠ D = 40° 45′	9.879375

Which being counted from *b* towards the right hand, gives the Bearing of *d c* (‖ D C) N 60° 30′ W^{ly}, the *Courſe* which the firſt Ship ſteered.

To

To find the *Courſe* which the ſecond Ship ſteered, it will be,

BC : BA :: R : C.S. B ; that is,

As the Hypothenuſe B C = 300	2.477121
Is to the Side B A = 58.6	1.767898
So is the Radius	10.
To the Co-Sine of the ∠ B = 78° 45	9.290777

Which being counted from the E *b* N towards the left-hand, it ſhews that the ſecond Ship ſteered due North.

This *Solution* will give Anſwers to the following Queſtions, being of the ſame Nature with the preceding *Example*, only expreſſed in different Terms.

Examp. 2. A Ship from a certain *Port* ſails E *b* N 40 Leagues, then between the North and the Weſt 45.07 Leagues, and from thence ſhe is drove back to the *Port* from whence ſhe departed 30 Leagues ; I demand the ſecond Courſe ſteered, and her laſt Courſe back to the *Port*.

Anſ. The ſecond Courſe ſteered is N 60° 30′ E^{ly}, and her laſt Courſe to the *Port* is directly South.

Examp. 3. There are two *Ports* that lie North and South diſtant 300 Miles, a Ship from the Southermoſt *Port*, cloſe hauled upon the Wind at N *b* E, with her *Larboard-Tacks* on *Board*, making one or more Trips, ſails 400 Miles; then ſailing 450.7 Miles with her *Starboard-Tacks* on *Board*, ſhe arrives at the Northermoſt *Port* ; I demand the Courſe upon each *Tack*, and how near the Wind ſhe made her way good.

Anſ. The Courſe upon the *Larboard-Tack* is E *b* N, and conſequently ſhe has made her Courſe good upon this *Tack* within 6 Points of the Wind ; and her Courſe upon the *Starboard-Tack* is N 60° 30′ W^{ly}, and conſequently upon this *Tack* ſhe has made her way good within 71° 45′ or 6 Points 4°$\frac{1}{4}$ of the Wind.

Hence, knowing how near the Wind a Ship can lie or bear up her Head, or, which is all one, knowing the Courſe given by the *Compaſs*, we may find the Ship's *Drift* or *Leeway* ; for admitting in the Example above, that ſhe bore up her Head within 5 $\frac{1}{2}$ Points of the Wind, it is evident, that upon the firſt *Tack* ſhe made juſt $\frac{1}{2}$ Point *Leeway*, and upon the ſecond *Tack* her *Drift* or *Leeway* was 9° 52′$\frac{1}{2}$, or almoſt 1 Point.

Examp. 4. Admit there are two *Ports* lying exactly under the Meridian, differing in Latitude 5 Degrees, and that there is an *Iſland* to the Eaſtward of theſe Ports, diſtant from the Northermoſt 450.7 Miles, and from the Southermoſt 400 Miles ; I demand the Courſe from both *Ports* to the *Iſland*.

Anſ.

Anſ. The Courſe from the Northermoſt *Port* to the *Iſland* is S 60° 30′ E^{ly}, and from the Southermoſt *Port* the Courſe is E *b* N.

CASE 5.

The Bearings of two or more Places from two different Stations, as alſo the Bearing and Diſtance of the ſaid Stations from each other, being given; to find the Bearing and Diſtance of the ſaid Places from each other.

This *Caſe* is only a Compound of the 1^{ſt} and 3^{d} *Caſe* aforegoing.

Examp. 1. Coaſting along Shore I ſaw two *Head-lands*, the firſt bore from me N E, and the ſecond E N E; and after I had ſailed E *b* S 10 Miles, the firſt bore from me N *b* E, and the ſecond N E *b* N; I demand the mutual *Bearings* and *Diſtance* of the two *Head-lands*.

Fig. 5. *Conſtruction.* 1. Having drawn the *Compaſs* N, E, S, W; let A repreſent the Place of the Ship, from whence draw the N E Line A C, the E N E Line A D, and the E *b* S Line A B=10 Miles: then will B be the Ship's ſecond *Station*.

2. From B draw the Line B C parallel to the N *b* E, where this interſects the N E Line as in C, gives the firſt *Head-land*.

3. Alſo, from B draw the Line B D, parallel to the N E *b* N, where this interſects the E N E Line as in D, gives the ſecond *Head-land*.

4. Join the Points C and D, then will C D be the mutual Diſtance between both *Head-lands*, and the ∠ A C D their Bearing from the N E Line. To find which by *Calculation*;

Firſt, you muſt find the Diſtance of both *Head-lands* from the Ship, either at firſt or laſt ſetting their Bearings; for the obtaining of which,

1. In the △ A B C are given the ∠ C A B=56° 15′, (=5 *Points*, the Diſtance between the N E and the E *b* S;) the ∠ A B C=90° 00′ (=8 *Points*, the Diſtance between the N *b* E and W *b* N) and the Side A B=10 Miles; whence, to find the Diſtance A C, it will be,

As the Sine of the ∠ A C B=33° 45′	9.744739
Is to the Sine of the ∠ B A C=90° 00′	10.
So is the Side A B 10	1.
To the Side A C 18	1.255261

Or the Diſtance between the Ship in her firſt Station, from the firſt *Head-land*.

2. Again, in the △ A B D are given the ∠ B A D=33° 45′ (=3 *Points*, the Diſtance between the E N E and the E *b* S) the ∠ B=112° 30′ (=10 *Points*,

Points, the Distance between the W*b*N and the N E *b* N) the ∠ D=33° 45′ (=3 *Points*, the Supplement of the other two ∠ˢ, or equal to the Distance between the W S W and the S W *b* S) and the Distance A B=10 Miles; whence to find the Distance A D between the Ship in her first *Station*, and the second *Head-land*, it will be,

As the Sine of the ∠ D=33° 45′	9.7 44739
Is to the Side A B=10 *Miles*;	1.
So is the Sine of ∠ B=112°30′ (its Sup. 67°30′)	9.965615
To the Side A D 16.63 *Miles*	1.220876

After the same manner may the Distances from the second *Station* of the Ship, to each of the *Head-lands*, be found, if required: but in this *Examp.* it happens, that the Side B D is = B A, because B A D is an *Isosceles* Triangle.

3. In the △ A C D are given the Side A C = 18 Miles, the Side A D =16.63 Miles, and the contained ∠ A=22° 30′ (= 2 *Points*, the Distance between the N E and the E N E;) whence to find the opposite ∠ˢ, it will be by *Case* 11. of *Trigonometry*.

A C = 18	*As the Sum of the Legs* 34.63	1.539452
A D = 16.63		
Sum 34.63	*Is to their Difference* 1.37	0.136721
Diff. 1.37	*So is the Tang. of* ½ *Sum opp.* ∠ˢ 78° 45′	10.701338
		10.838059
The ∠ A = 22° 30′		
Sum opp. ∠ˢ 157 30	*To the Tang. of* ½ *their Diff.* 11° 15′	9.298607
½ *Sum* 78 45		

Which subtracted from the ½ Sum 78° 45′, gives 67° 30′ the ∠ A C D, whence the second *Head-land* bears from the first S S E, and consequently the first bears from the second N N W: Now to find their mutual *Distance*, it will be,

As the Sine of the ∠ C=67° 30′	9.034385
Is to the Sine of the ∠ A=22° 30′	9.582840
So is the Side A D=16.63	1.220876
To the Side A D *the Dist. required* =6.89 *m.*	0.838101

Or, because the △ A D C happens to be Right-Angled, the Proportion might have been made, *viz.* R : T. A :: A D : D C.

This Solution will give the *Distances* required in the following *Example*, and also the *Bearing* of the first and second Ship; and their *Latitudes* may be found by *Case* 1. of *Sect.* 1. of this *Chapter*.

Examp. 2. Three Ships ſet ſail from a certain *Port* in the *Latitude* of 46° 30′, the firſt ſailed N E, the ſecond E N E, and the third ſailed E *b* S 10 Miles; then the firſt Ship bore from the third N *b* E, and the ſecond Ship bore from the third N E *b* N; I demand the firſt and ſecond Ship's *Diſtance* ſailed, the *Latitude* they are arrived into, and their *Bearing* and *Diſtance* from each other.

Anſ. The firſt Ship ſailed 18 Miles, the ſecond Ship ſailed 16.63 Miles, their Bearing is N N W and S S E diſtant 6.89 Miles; the *Lat.* of the firſt Ship is 40° 43′ N, and of the ſecond Ship is 40° 36′ N.

He that underſtands the Solution of the two preceding Examples, will eaſily ſolve the following.

Examp. 3. Admit there are three *Iſlands* ſituated as followeth; *viz.* A in the *Latitude* of 40° 45′ N; B bears from A E *b* S 40 Miles, C in the N E Quarter diſtant from A 72 *Miles*, and D in the ſame Quarter 66.52 Miles; C bears from B N *b* E, and D from C S S E: I demand the *Bearings* of C and D from A, and likewiſe of B and D from each other, as alſo the ſeveral *Latitudes* of B, C, and D.

Anſ. The *Bearing* of C from A is N E, and of D from A is E N E, of B and D from each other is N E *b* N and S W *b* S, the *Lat.* of B is 40° 37′ N, of C 41° 36′ N, and of D 41° 10′ N.

Fig. 6. If the Point B ſhould happen to be between the Lines A C, A D, you may find the *Bearing* and *Diſtance* of C D with the ſame eaſe; for in the $\triangle^s$ B A C, B A D, there will be given the Side A B in both, and all the $\angle^s$, to find the Sides A C and A D, (or B C and B D;) which being found, you have in the $\triangle$ C A D (or B C D) two Sides and the $\angle$ included, to find the remaining $\angle^s$ and the Side C D, as before. And if B had been the firſt Station, and A the ſecond, the Caſe would have been directly the ſame.

N. B. The ſecond Station muſt not be in either of the Lines A C or A D, for then the Side C D, *&c.* cannot be determined.

Fig. 7. *Examp.* 4. A Ship at A at her Entrance into a *Bay*, C, D, E, F, *&c.* obſerved the Bearings of the ſeveral Points or Corners C, D, E, F, *&c.* to be as followeth; *viz.* C W $\frac{1}{4}$ S, D W N W $\frac{1}{2}$ W, E N W *b* W, F N W *b* N $\frac{1}{2}$ W, G N *b* W $\frac{1}{2}$ W, H N *b* E, I N E *b* N, K N E *b* E $\frac{1}{4}$ E, L E *b* S $\frac{1}{4}$ S; and after ſhe had ſteered away N W 1 $\frac{1}{2}$ Mile unto B, the Point C bore S W *b* W, D W *b* S, E W N W, F N W *b* N, G N *b* E, H N E *b* E, I E *b* N, K E *b* S, and L E S E $\frac{1}{2}$ E; I demand the Bearings and Diſtances of the ſeveral Points C and D, D and E, E and F, *&c.* from each other.

This *Example* is *conſtructed*, and the ſeveral Requiſites *calculated* juſt after the ſame manner with *Example* 1. of this *Caſe*; for, having choſe ſome convenient Point A for the Ship's Place at firſt, draw a *North* and *South* Line A N, and lay therefrom the ſeveral *Bearings* of the Points C, D, E, F, *&c.* (either by the *Protractor* or a Line of *Chords*) and draw the *Viſual* Lines A C, A D, A E, *&c.* then draw the N W Line A B = 1 $\frac{1}{2}$ Mile, and having drawn the *Meridian* B N parallel to the former A N, lay the Center of the *Protractor* ſo to the Point B, that the *North* Point

thereof may cut the Line B N; then by the *Limb* thereof prick the ſeveral *Bearings* of the Points C, D, E, F, *&c.* from B, and draw the Lines B C, B D, B E, *&c.* until they interſect their correſpondent Lines A C, A D, A E, *&c.* and join C D, D E, E F, F G, *&c.* then is the Figure C, D, E, F, G, H, I, K, L the *Bay*; and the Sides C D, D E, E F, *&c.* the ſeveral *Diſtances* required, and the ∠ˢ A C D, A D E, *&c.* the *Bearings* from the Lines A C, A D, *&c.* To find which by *Calculation*;

In the ſeveral △ˢ A B C, A B D, A B E, *&c.* there are given the Side A B, and all the ∠ˢ, to find the Sides A C, A D, A E, A F, *&c.* which being found; then in the ſeveral △ˢ C A D, D A E, E A F, *&c.* will be given two Sides and the ∠ included at A, to find the remaining ∠ˢ, and the other Side: Now the ∠ˢ A C D, A D E, *&c.* being obtained, you may find the Bearings of the ſeveral Lines C D, D E, *&c.* by conſidering how much theſe ∠ˢ lie from the Lines C A, D A, *&c.* whoſe Bearings are already found.

But in Examples of this kind (which are very uſeful for deſcribing of *Bays*, &c.) a *Geometrical* Conſtruction is ſufficient without any *Arithmetical* Calculation.

I ſhall next ſhew how the *Diſtances* of Places may be eſtimated by *Sound* and *Lightning*.

From ſome Experiments that have been made, it has been determined, that *Sound* travels at the rate of 1142 Feet in one *Second* of Time; which being granted, and becauſe the Motion of *Light* is in a manner Inſtantaneous; (it moving at leaſt at the rate of 10000 Miles in a Second of Time) it follows, that if the Difference between the time an *Exploſion* is firſt ſeen, and the time that the *Sound* of it reaches the Ear, be given; the *Diſtance* betwixt the Place where the *Exploſion* was made, and the *Station* of the Obſerver, may be eaſily found, as followeth:

An *Engliſh* Mile contains 5280 Feet, and conſequently *Sound* requires 4.623 Seconds to travel 1 Mile: And a *Geographical* Mile (according to Mr. *Norwood's* Experiment) contains 6120 *Engliſh* Feet; wherefore, 5280 : 6120 :: 4.623 *Seconds* : 5.358 *Seconds*; the time that *Sound* requires to travel over a *Geographical* Mile. And conſequently, *as* 1, *is to* $\left\{\begin{matrix}4.623;\\5.358;\end{matrix}\right\}$ *ſo is any given Diſtance in* $\left\{\begin{matrix}\text{Engliſh}\\\text{Geograp.}\end{matrix}\right\}$ *Miles, To the time that Sound requires to travel over that Diſtance.* On the contrary, *as* $\left\{\begin{matrix}4.623\\5.358\end{matrix}\right\}$ *is to* 1; *So is the number of Seconds between the time a Sound is generated, and the time it arrives at any Place, To the Diſtance betwixt theſe two Places in* $\left\{\begin{matrix}\text{Engliſh}\\\text{Geograp.}\end{matrix}\right\}$ *Miles.*

Examp. 1. Being at Sea, I counted 40 Seconds between the time that I ſaw the *Flaſh* of a Gun, and heard the *Report*; I demand the *Diſtance.*

If 40 Seconds be divided by 5.358, the Quotient 7.46 will be the Diſtance in *Geographical* Miles required; Or, if the ſame number of Seconds be divided by 4.623, the Quotient 8.65 will be the Diſtance in *Engliſh* Miles: ſo that the *Gun* when it was fired was 7.46 *Geographical* Miles, or 8.65 *Engliſh* Miles diſtant from the Obſerver.

Examp. 2. Admit I counted 70 Seconds between the time I ſaw a *Flaſh* of *Lightning*, and heard the next *Clap* of *Thunder* immediately following it; I demand how far off was the *Thunder*.

Anſ. 15.14 *Engliſh* Miles.

The beſt way for meaſuring a ſmall quantity of Time, is by a *Clock* having a ſecond *Hand* or *Index*; or rather by ſuch a *Watch*, made alſo to ſtop at pleaſure. But for want of ſuch *Apparatus*, you may take a String or Wire of 39.2 Inches long, and having faſtened a Plummet at one end of it, hang the other end upon a Nail or Peg; then giving it a ſmall ſwing, the number of *Vibrations* (if the deſcribing Arches are very ſmall) will give the number of *Seconds* of *Time* elapſed during the Experiment: Or, if you take a String of 9.8 Inches long, and prepare it as before, the number of *Vibrations* will be ſo many $\frac{1}{2}$ *Seconds* of Time.

It is generally found, that the number of *Pulſations* of the *Artery* in a Man of good Habit of Body, is about 75 in a *Minute*; this being ſuppoſed, it follows, that *Sound* travels at the rate of 1 $\left\{\begin{matrix} Geographical \\ Engliſh \end{matrix}\right\}$ Mile in $\left\{\begin{matrix} 6.7 \\ 5.78 \end{matrix}\right\}$ *Pulſations.* Wherefore, if the number of Pulſations between the time a Man ſees an *Exploſion* and hears the *Report*, be divided by $\left\{\begin{matrix} 6.7 \\ 5.78 \end{matrix}\right\}$ the Quotient will give the number of $\left\{\begin{matrix} Geograp. \\ Engliſh \end{matrix}\right\}$ Miles of *Diſtance*, nearly.

SECT.

SECT. V.

Of Computing the true Courſe and Diſtance ſailed in Currents; *Of Finding their Direction and Velocity: Of Turning to Windward.*

BY CURRENTS are here meant certain *Streams* in the Sea, which force Ships ſailing therein to alter either their *Direction* or *Velocity*, or both; and ſo form a *Courſe* compounded of the *Ship*'s Motion and that of the *Current*: Whence it appears,

1. If the *Current* ſets with the Courſe of the *Ship*, her *Velocity* will be increaſed juſt ſo much as is the *Drift* of the Current.

2. If a *Current* ſets directly againſt the Ship, the Difference of the *Velocities* of the Ship and the Current, will be the true *Motion* of the Ship, the ſame way that the Velocity is greateſt; that is, if the Velocity of the { *Ship* / *Current* } be the greateſt, the Ship { goes a *Head* / falls a *Stern* } juſt ſo much as is the Difference of the *Current's* and *Ship's* Velocity: And if their *Velocities* ſhould be equal, the *Ship* ſtands ſtill; for ſo much as ſhe is drove forwards by the *Wind*, ſhe is forced back by the *Current*.

3. If the *Current* thwarts the *Courſe* of the Ship, it not only leſſens or augments her Velocity, but gives her a new Direction, compounded of the *Courſe* ſhe ſteers, and the *Setting* of the Current; as by the following

LEMMA.

If a Body be agitated by two Motions at the ſame time, the one with a certain Velocity that will carry it according to the Direction of the Line A B, the Length A B, in a given ſpace of time; the other according to the Direction of the Line A C, with a Velocity that will carry it the Diſtance A C in the ſame time; the Body thus agitated will move in the *Diagonal* A D, and at the end of that time will be found in the Point D. *Plate* 8. *Fig.* 8.

PROB.

PROB. I.

The *Courſe* and Diſtance run by a *Ship* in any given Space of Time, and the *Direction* and *Setting* of the *Current* in the ſame time, being given; to find the *Ship's* true Courſe and Diſtance made good in that time.

CASE 1.

When the Courſe of the Ship, and Setting of the Current, are the ſame way, the Sum of their Velocities is the Diſtance run.

Examp. If a Ship ſails N E 30 Miles in 6 Hours, in a *Current* that ſets N E, at the rate of 2 ½ Miles an hour; the Ship's true *Courſe* in this Caſe is N E, and her *Diſtance* ſailed 45 Miles.

CASE 2.

When the Current ſets directly againſt the Head of the Ship.

Examp. 1. If a Ship ſails N N E 20 Miles in 4 Hours, in a *Current* that ſets S S W, at the rate of 2 Miles an hour; the Ship's true *Courſe* will be N N E, *Diſtance* ſailed 12 Miles.

Examp. 2. If a Ship ſails N N W, at the rate of 4 Miles an hour, in a Current that ſets S S W, at the rate of 4 Miles an hour; It is evident in this Caſe, that the Ship makes no way, but ſtands ſtill; for ſo much as ſhe is forced forward by the *Wind*, ſhe is driven backward by the *Current*.

Examp. 3. Admit a Ship ſails S W 6 Hours, at the rate of 4 Miles an hour, in a *Current* that ſets N E, at the rate of 6 Miles an hour; it is evident in this Caſe the Ship falls a Stern, and her true Courſe is N E, at the rate of 2 Miles an hour, or 12 Miles in 6 Hours.

CASE 3.

When the Current thwarts the Ship's Motion; to find the Courſe and Diſtance made good.

Examp. 1. Admit a Ship ſails N E *b* E 360 Miles in 3 *Days*, and a *Current* is found to ſet S *b* W, at the rate of 2 Miles an *hour*; and it be required to find the Ship's true Courſe and Diſtance ſailed in that time.

Con-

Conſtruction. 1. Having drawn the *Compaſs* N, E, S, W; let A repreſent the Place the Ship ſailed from, and draw the N E *b* E Line A C = 360 Miles, (then will C repreſent the Place the Ship caped at.) *Fig.* 9.

2. From C draw C B parallel to the S *b* W Line = 144 Miles, (the Drift of the Current in 3 Days, or 72 Hours;) then will B be the Place of the Ship, A B her true *Diſtance* ſailed, and the ∠ N A B the true *Courſe*; to find which by *Calculation*,

In the Δ A B C are given A C=360 Miles, C B=144 Miles, and the ∠ A C B = 45° 00′ (= 4 *Points*, the Diſtance between the N *b* E and the N E *b* E;) whence to find the ∠ at A, it will be by *Caſe* 11. of *Trigonometry*.

A C = 360	∠ C = 45° 00′		
B C = 144	135.00		
AC+BC 504	½ Z *opp.* ∠ˢ 67 30		
AC—BC 216			
½ Z = 67° 30′	*As the Sum of the Sides* A C *and* B C = 504		7.297570
½ X = 45 58	*Is to their Difference* = 216.		2.334454
∠ A = 21 32	*So is the Tang. of* ½ *Sum* ∠ˢ A & B = 67° 30′		10.382776
N A C = 56 15	*To the Tang. of* ½ *their Diff.* = 45° 58′		10.014800
N A B = 77 47			

Hence, the Ship's true *Courſe* is N 77° 47′ E^{ly}, or E *b* N nearly. Next, to find the direct *Diſtance* ſailed, it will be by *Caſe* 8. of *Trigonom.*

As the Sine of the ∠ A = 21° 32′	9.435284
Is to the Drift of the Current B C = 144	2.158362
So is the Sine of the ∠ C = 45° 00	9.849485
To the Diſtance run = 277.4 *Miles*	2.443131

The *Example* above might have been thus expreſſed;

A Ship ſails N E *b* E 360 Miles, in a *Current* that ſets S *b* W 144 Miles in the ſame time; what is the true Courſe and Diſtance ſailed? Or thus;

A Ship ſails N E *b* E 360 Miles, in a *Current* that ſets S *b* W, at the rate of 2 Miles in the time the Ship ſails 5 Miles; what is the true Courſe, *&c.*

Examp. 2. If a Ship from the Latitude of 46° 30′ N, ſails N N E 40 Miles in 7 Hours, then N E *b* E 30 Miles in 6 Hours, in a Current that ſets S E, at the rate of 2 ½ per hour: I deſire to know the direct *Courſe* and *Diſtance* made good, as alſo the *Latitude* the Ship is in.

Conſtruction. 1. Having drawn the Compaſs N, E, S, W; let A repreſent the Place the Ship ſailed from, and draw the N N E Line A B = 40 Miles, alſo draw B C parallel to the N E Line = 17 ½ Miles, (the Current's Drift in 7 Hours.) *Fig.* 10.

2. From

2. From C draw C D parallel to the N E *b* E Line = 30 Miles, and D G parallel to the N E Line = 15 Miles, (equal to the Drift of the Current in 6 Hours,) and draw the Line A G; then will G repreſent the Place of the Ship at the end of her Traverſe, A G the direct *Diſtance* ſailed, and the ∠ N A G the true *Courſe* from the *Meridian*.

In this *Example*, the true Courſe and Diſtance anſwering to each Courſe ſteered, may be found as in the laſt *Example*; and thence the direct Courſe and Diſtance made good. But the eaſieſt and readieſt way is to conſider it as a *Traverſe* of four Courſes; and then finding the *Difference* of *Latitude*, and the *Departure* to each Courſe, and thence the *Latitude* the Ship is in, after the manner taught in *Sect.* 3. of this *Chapter*. Or, the two *Settings* of the *Current* may be added together, and then it may be conſidered as a *Traverſe* of three Courſes.

See the Operation both ways, as follows:

Courſes.	*Diſt.*	*Diff. Lat.* N.	*Diff. Lat.* S.	*Depar.* E.	*Depar.* W.
N N E.	40	36.9		15.3	
Cur. S E.	17 ½		12.3	12.3	
N E *b* E.	30	16.7		24.9	
Cur. S E.	15		10.6	10.6	
		53.6	22.9	63.1	
		22.9			
		30.7		63.1	

Courſes.	*Diſt.*	*Diff. Lat.* N.	*Diff. Lat.* S.	*Depar.* E.	*Depar.* W.
N N E.	40	36.9		15.3	
N E *b* E.	30	16.7		24.9	
Cur. S E.	32 ½		22.9	22.9	
		53.6	22.9	63.1	
		22.9			
		30.7		63.1	

Hence, the *Latitude* the Ship is in (becauſe ſhe ſailed from a North Latitude Northerly) is 47° 00′.7 N, and the direct Courſe and Diſtance will be found by *Caſe* 4. of *Sect.* 1. of this *Chapter*, to be, *viz. Courſe* N 64° 04′ E'ly, *Diſtance* 70.2 Miles.

After the ſame manner may the *direct* Courſe and Diſtance, the *Latitude* the Ship is in, (and thence her *Difference* of *Longitude* if required) be found, after ſhe has ran any number of *Courſes* in a *Current*.

Case 4.

How to compute the Diſtance from the Land, when a Ship is about taking her Departure, in a Current or Tide's Way.

Examp. Taking my *Departure* from the Land, I ſaw a certain *Cape* bearing N N W; then I ſtood away E N E 20 Miles in 4 Hours, and then the *Cape* bore from me W *b* N, but there was a *Current* all the while.

while under Foot, ſetting N *b* E, at the rate of 2 Miles an hour; I demand the Ship's true *Courſe* and *Diſtance* ſailed, as alſo her Diſtance from the *Cape*.

Conſtruction. 1. Having draw the Compaſs N, E, S, W; let A be the firſt Station of the Ship, from whence draw the E N E Line A B = 20 Miles; from B draw B D parallel to the N *b* E=8 Miles, (the Current's Drift in 4 Hours) and join A D; then is A D the true *Diſtance* ſailed, and the ∠ N A D the true *Courſe*. *Fig.* 11.

2. Draw the N N W Line A C, and likewiſe D C parallel to the W*b*N, which continue until they interſect each other in C; then is C the *Head-land*.

To find the ſeveral Requiſites by *Calculation*.

1. In the △ A B D are given the Side A B = 20 Miles, the Side B = 8 Miles, and the ∠ A B D = 123° 45′, (= 11 *Points*, the Diſtance between the W S W Line B A, and the N *b* E Line B D;) whence, 1. to find the ∠ A, it will be by *Caſe* 11. of *Trigonom.*

	180° 00′	*As the Sum of the Sides* A B *and* B D=28 *m.*	8̄.552842
∠ B =	123° 45′	*Is to their Difference* = 12 *Miles*;	1.079181
	56 15	*So the T. of* ½ *the Sum of* ∠ˢ A & D=28°07′½	9.727957
½ Z *op.* ∠ˢ	28°07′½	*To the Tang. of half their Diff.*=12° 54′½	9.359980

Hence, the Ship's true *Courſe* will be found to be N 52° 17′ E^{ly}: And to find A D the true *Diſtance* ſailed, it will be,

As the Sine of the ∠ A = 15° 13′	9̄.580921
To the Sine of ∠B=123° 45′ (its Sup.=56°15′)	9.919846
So is the Drift of the Current B D = 8 *Miles*	0.903090
To the Diſtance required A D =25.34 *Miles.*	1.403857

Laſtly, To find D C the Diſtance between the Ship and the *Head-land.*

In the △ A D C are given the ∠ C=56° 15′, (= 5 *Points*, the Diſtance between the S S E Line C A and the E *b* S Line C D) the ∠ A = 74° 47′, (the Diſtance between the N N W and the N 52° 17′ E^{ly}) and the Side A D (the Diſtance ſailed by the Ship) = 25.34 Miles; whence to find D C, it will be,

As the Sine of the △ C= 56° 15′	9̄.080154
Is to the Side A D= 25.34 *m.*	1.403857
So is the Sine of the ∠ A= 74° 47	9.984500
To the Side required C D= 29.41 *Miles*	1.468511

Case 5.

To hit a Port, or double a Cape in a Current *or* Tide's *Way: Or, the Setting and Drift of the Current, the Rate of the Ship's Sailing, and the Bearing and Distance of any Place from the Ship, being given; to find how far and upon what Course the Ship must sail for to hit the said Place.*

Examp. Admit a Ship descries a certain *Cape* bearing N W *b* N, Distance 6 Leagues, and that she is bound to a *Port* bearing from the said *Cape* E *b* N, Distance 50 Miles, and all the way there is a *Current* under foot setting N W, at the rate of 2 Miles an hour; the Ship sailing at the rate of 5 Miles an hour: I demand upon what Course, and how far the Ship must sail to fetch the said *Port*.

Fig. 12. *Construction.* 1. Having drawn the *Compass* N, E, S, W; let A represent the Place of the Ship, from whence draw the N W *b* N Line A B = 18 Miles; then will B represent the *Cape*.

2. From B draw the Line B C parallel to the E *b* N = 50 Miles; then will C represent the *Port* the Ship is bound to.

3. Draw C D parallel to the S E, and make C *d* = the Current's Drift in any given space of time; then taking the Ship's Drift in the same time betwixt the Compasses, set one Foot in *d*, and turn the other about until it intersects the Line A C, as in *a*, and draw the Line A D parallel to *a d*: then will A D be the *Distance* required, and the ∠ N A D the true *Course* from the Meridian. To find which by *Calculation*;

1. In the △ A B C are given A B = 18 Miles, B C = 50 Miles, and the ∠ A B C = 67° 30′ (= *6 Points*, the Distance betwixt the S E *b* S Line B A, and the E *b* N Line B C;) to find the ∠ B A C the *Course*, and the Side A C the *Distance*, from the Ship to the *Port*.

1. To find the ∠ B A C, it will be,

180°00′	*As the Sum of the two Sides* A B & B C = 68	1.832509
67 30	*Is to their Difference* = 32	1.505150
112.30	*So is the Tang. of* ½ *Sum op.* ∠ˢ = 56° 15′	10.175107
56° 15′ ½ Z *op.* ∠ˢ		11.680257
	To the Tang. of ½ *their Diff.* = 35° 09′	9.847748

Hence, the ∠ A is 91° 24′, and consequently the direct *Course* from the Ship to the *Port* is N 57° 39′ E^{ly}. And for the *Distance*, it will be,

As the Sine of the ∠ A = 91° 24′ (its Sup. 88° 36′)	c̄.000130
To the Sine of the ∠ B = 67° 30′	9.965615
So is the Side BC = 50 *m.* (the Diſt. from the *Cape* to the *Port*)	1.698970
To A C *the Diſtance from the* Ship *to the* Port = 46.21 *m.*	1.664715

Again, in the △ *a d* C are given the ∠ C = 102° 39′, (the Diſtance between the S E Line C D, and the S 57° 39′ W^{ly} C A) the Side C *d* = 10 Miles, (the *Current's* Drift in 5 Hours, and the Side C *a* = 25 Miles, the *Diſtance* ran (by the Ship) in the ſame time; whence, to find the ∠ C *a d* (= C A D) it will be,

As the Side d a = 25 *m.*	1.397940
To the Side C d = 10 *m.*	1.
So is the Sine of ∠ a C d = 102° 39′ (its Sup. 77° 21′)	9.989328
To the Sine of the ∠ C a d = 22° 58′	9.591388

Hence, becauſe the ∠ C A D is = C *a d*, the Courſe the Ship muſt ſteer for to gain the Port is N 80° 37′ E^{ly}, (= N A C 57° 39′ + C A D 22° 58′ = N A D 80° 37′.)

To find A D the *Diſtance* the Ship muſt ſail, it will be,

As the Sine of the ∠ D = 54° 23′	9.910054
Is to the Sine of the ∠ A C D 102° 39′ (its Sup. 77° 21′)	9.989328
So is the Diſt. betwixt the Ship and the Port A C = 46.21 *m.*	1.664715
	11.654043
To A D *the Diſt. the Ship muſt ſail to hit the ſaid* Port = 55.46 *m.*	1.743989

Case 6.

To find the Bearings and Diſtances of Head-lands, when the Ship ſails in a known Current: *Or, the Bearings of two or more Head-lands from two different Stations, the Courſe and Diſtance betwixt theſe two Stations, and alſo the Setting and Drift of a Current under foot, being given; to find the Bearing and Diſtance of the two Head-lands from each other.*

Examp. Being at Sea, I ſaw two *Head-lands*, one bearing from me N NW, the other N *b* E; then I ſtood away E N E 20 Miles in 4 Hours, and then the firſt bore from me W *b* N, and the ſecond N W *b* N, there was all the while a *Current* under Foot ſetting N *b* E, at the rate of 2

Miles an hour: I demand the mutual *Bearing* and *Diſtance* of the two *Head-lands*.

This *Caſe* is only a Compound of *Caſe* 3. of this *Prob.* and *Caſe* 5. of the laſt *Section*; wherefore,

Fig. 13. 1. With the Courſe and Diſtance ſailed, and the Setting and Drift of the *Current*, conſtruct the Δ A B C by *Caſe* 3. of this *Prob.* then will A and C be the true Places of the Ship at both Stations, A C the true *Diſtance* ſailed, (which in *Caſe* 4. has been determined to be 25.34 Miles) and the ∠ N A C the true *Courſe* made good, (which in the ſame *Caſe* has been alſo determined to be N 52° 17′ E^{ly}.)

2. With the ſeveral Bearings of the two *Head-lands* from A and C, find the Points D and G by *Caſe* 5. of the laſt *Section*; then will D and G repreſent the two *Head-lands*, whoſe mutual *Bearings* you'll find (by the ſaid *Caſe*) to be N 40° 05′ E^{ly}, and S 40° 05′ W^{ly}, *Diſtance* 21.74 Miles.

If but one of the *Head-lands* could be ſeen from either *Station*, and inſtead of the Bearing of the other *Head-land*, the Diſtance of the former (which *Mariners* take by Eſtimation) be given; the *Bearing* and *Diſtance* of the two *Head-lands* from each other may be eaſily found; that is, if inſtead of the Bearing of G from A, you have the Bearing and Diſtance of D from A, and inſtead of the Bearing of D from C, you have the Bearing and Diſtance of C G; to find the Bearing and Diſtance of D G: Having by the *Courſe* and *Diſtance* ſailed, and the *Setting* and *Drift* of the *Current*, determined the Point C, you may by the ſimple Bearing and Diſtance of the *Head-lands* from A and C, determine the Points D and G by *Conſtruction*. By *Calculation*, having determined the Bearing and Diſtance of A C; in the Δ A D C will be given the two Sides A D, A C, and the ∠ included at A, to find the Bearing and Diſtance of D C; which being found: In the Δ D C G will be given the two Sides C D, C G, and the ∠ included at C; to find the Bearing and Diſtance of D G required. Or,

If the Bearing and Diſtance of D from A, and alſo the ſeveral Bearings of G from D and C, be given; the Diſtance of D and G from each other may be eaſily had.

The uſual way of trying of *Currents* at Sea, is to let down a heavy Weight faſtened to a Rope or Cord, to a great Depth, out of the Head of a *Boat*; then by the *Log* and *Compaſs* is found the *Setting* and *Drift* of the *Current*, (after the ſame manner as is found the Ship's *Courſe* and *Diſtance* ſailed.) But this Method muſt needs be very uncertain, for the ſame *Current* as drives the *Ship* muſt alſo drive a Body having ſo great a hold upon the Water as a *Boat*, (tho' undoubtedly not ſo faſt, by reaſon of the Weight underneath:) Beſides, this Experiment cannot well be repeated ſo often as in many Caſes it ought to be, by reaſon of the too much trouble attending it. I ſhall therefore ſhew in the two enſuing *Problems*, how the Quality of the *Current* may be otherwiſe found ſeveral ways.

PROB.

Plate 8

Fig. I

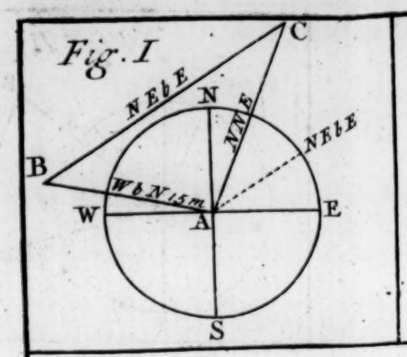


Fig. II

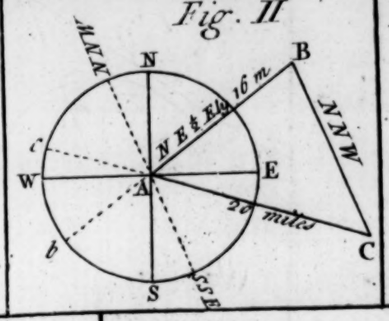


Fig. III

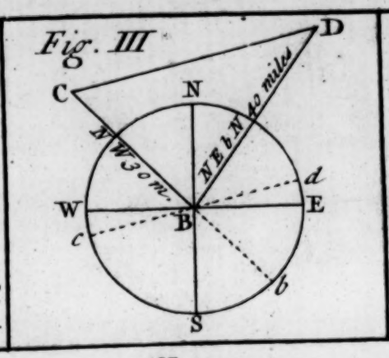


Fig. IV

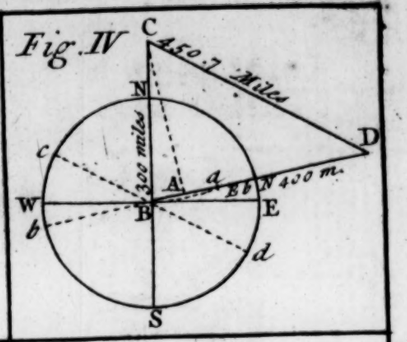


Fig. V

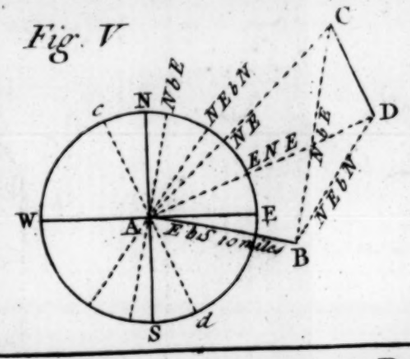


Fig. VI.

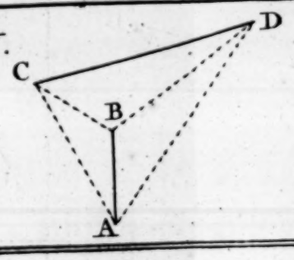


Fig. VII

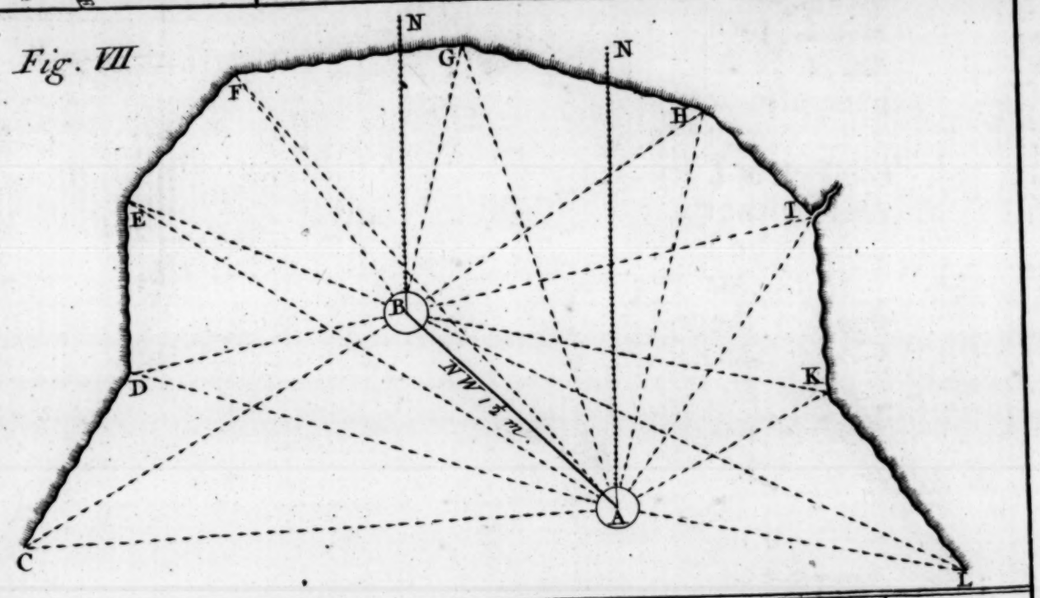


Fig. VIII

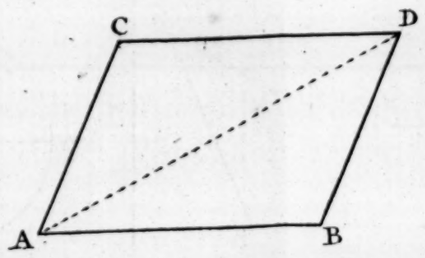


Fig. IX

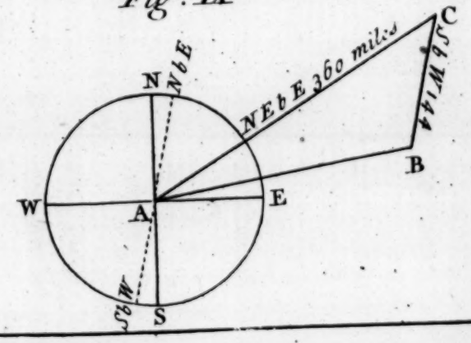


Fig. X

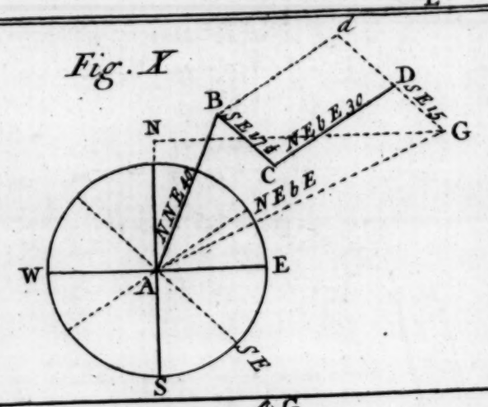


Fig. XI

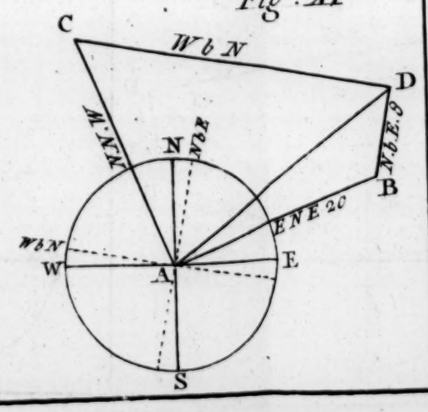


Fig. XII

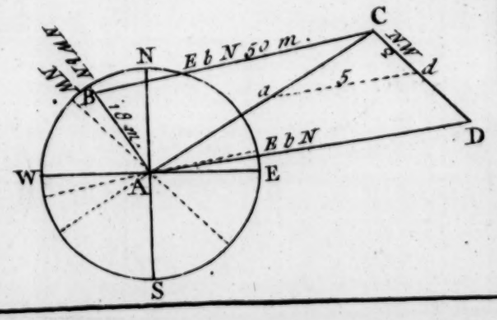
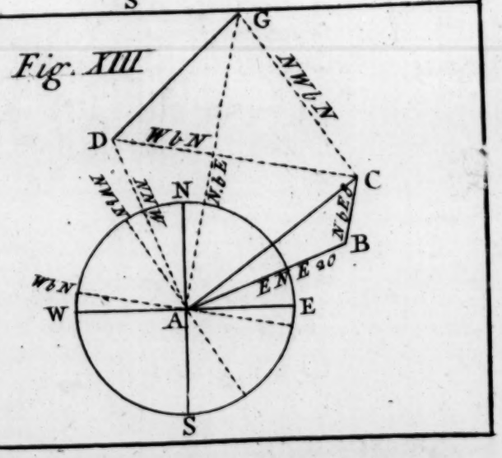


Fig. XIII



PROB. II.

To find the Setting and Drift of a *Current*, when the Ship is within ſight of Land.

CASE I.

The Courſe, Diſtance ſailed, the Difference of Latitude, and the Bearing of the Place departed from, being given; to find the Setting and Drift of the Current.

Examp. A Ship from a certain *Head-land* in the Latitude of 40° 00′ N, ſails E *b* S 20 Miles in 4 Hours, and then the ſame *Head-land* is found to bear W *b* S, and the Ship is found to be in the Latitude of 40° 05′ N; I demand the *Setting* and *Drift* of the *Current*, which is ſuppoſed to be in this Place.

Conſtruction. 1. Having drawn the *Compaſs* N, E, S, W; let A repreſent the Place the Ship departed from, and draw the E *b* S Line A B = 20 Miles, alſo the E *b* N Line A C. *Plate 9. Fig. 1.*

2. At the Diſtance of 5 Miles, (the Difference of Latitude N^ly) draw the Line D C parallel to the *Eaſt* and *Weſt*, where this interſects the E *b* N Line A C in C, gives the Place of the Ship; join the Points C and B; then will B C be the Setting and Drift of the *Current*. To find which by *Calculation*;

1. In the Right ∠^d Δ A D C are given the *Difference of Latitude* A D=5 Miles, the ∠ A= 78° 45′; whence to find A C, it will be,

As the Co-Sine of the ∠ A (the Courſe) = 78° 45′	9.290236
Is to the Radius:	10.
So is the Diff. of Latitude A D = 5	0.698970
To the Diſtance run A C = 25.63 *m.*	1.408734

2. In the Δ A B C are given the Side A B=20, the Side A C=25.63, and the ∠ A = 22° 30′; whence to find the ∠ B, it will be,

22° 30′	*As the Sum of the Sides* AB & AC=45.63	$\bar{8}$.340750
157 30	*Is to their Difference* = 5.63	0.750508
78 45 ½ Z ∠^s B+C	*So is the Tang. of* ½ *Sum of* ∠^s B & C=78° 45′	10.701338
	To the Tangent of ½ *their Diff.* = 31 48	9.792596
	∠ B = 110.33	

To

To the W *b* N Line B A, (the ∠ W A *b* = 11° 15′) add the ∠ B = 110° 33′, the Sum W A *c* is N 31° 48′ E^{ly}, the *Setting* of the *Current*.

To find B C the Current's *Drift* in 4 Hours, it will be,

As the Sine of the ∠ B = 110° 33′ (its Sup. = 69° 27′)	0.028554
Is to the Sine of the ∠ A = 22° 30′	9.582840
So is the true Diſtance run A C = 25.63	1.408734
To the Drift of the Current B C = 10.48 *Miles.*	1.020128

And conſequently the *Current's Drift* per hour is 2.62 Miles.

Case 2.

The Courſe and Diſtance run, and the true Bearing and Diſtance between the Place departed from, and the Place arrived at, being given; to find the Setting and Drift of the Current.

Examp. A Ship ſails from a certain *Port* E *b* S 200 Miles, at the rate of 5 Miles an hour, and then arrives at a *Port* diſtant from the former 256.3 Miles, bearing E *b* N; I demand the *Setting* and *Drift* of the *Current*.

Fig. 1. Having drawn the E *b* N Line A C = 256.3, and the E *b* S Line A B = 200 Miles, join the Points B and C; then will the ∠ A B C be the *Setting* of the Current from the W *b* N Line B A, and the Side B C the Current's *Drift* in 40 Hours: To find which by *Calculation*; in the △ A B C are given the two Sides A B, A C, and the ∠ included at A, to find the oppoſite ∠^{s}, and the Side B C; which being determined, the Setting and Drift of the Current will be found to be as in the laſt *Example*, *viz.* the *Setting* is N 31° 48′ E^{ly}, at the *rate* of 2.62 Miles an hour.

Case 3.

The Bearing and Diſtance of any Place from two other Places, and the Courſe and Diſtance run (according to the Log and Compaſs) *between theſe two Places, being given; to find the Ship's true Courſe and Diſtance ſailed, and alſo the Setting and Drift of the Current.*

Examp. A Ship takes her Departure from a certain *Cape* bearing WSW, Diſtance by Eſtimation 4 Leagues, and having ſailed N W *b* N 18 Hours, at the rate of 6 Knots an hour, arrives at an *Iſland* bearing N W from the

the former *Cape*, Diſtance 130 Miles, there is a Weſterly *Current* ſuppoſed to be under Foot; required the Ship's true *Courſe* and *Diſtance* ſailed, and the *Setting* and *Drift* of the Current.

Conſtruction. 1. Having drawn the *Compaſs* N, E, S, W; let A repreſent the *Cape* from whence the Ship took her Departure, and draw the N W Line A D=130 Miles, alſo the E N E Line A B=12 Mies; then will D repreſent the *Iſland*, and B the Place of the Ship when ſhe took her Departure. *Fig.* 2.

2. Draw the Line B C parallel to the N W *b* N = 108 Miles, (the Diſtance ſailed in 18 Hours) and join D C and D B; then will B D be the true Diſtance ſailed, the ∠ D B C the true Courſe from the N W *b* N, the ∠ B C D the *Setting* of the *Current* from the S E *b* S (Line C B) and C D its Drift in 18 Hours. To find each of which by *Calculation*;

1. In the △ A D B are given the Side A D=130, the Side A B=12, and the ∠ A=112° 30′, to find the ∠ D, which will be found to be 4° 43′, whence the *Ship's* true *Courſe* is N 49° 43′ W^ly; and the Side B D the true *Diſtance* failed, will be found to be 135 Miles.

2. In the △ B C D are given the Side B D = 135, the Side B C=108, and the ∠ B = 15° 58′; whence the ∠ C will be found to be = 120° 24′, and conſequently the *Setting* of the *Current* is S 86° 39′ E^ly, the Side C D the *Current's Drift* in 18 Hours, will be found to be = 43.08 Miles, and conſequently the *Current's Drift* per hour is 2.39 Miles.

In the two following *Caſes* I ſhall give the *Geometrical Conſtructions*, and alſo Directions for finding the ſeveral Requiſites by *Calculation*; but ſhall leave the Operations for the Learner's Practice.

Case 4.

The Bearing and Diſtance of two Head-lands from each other, and the Bearing of one of them from the Ship at one Station, and of the other at another Station, as alſo the Courſe and Diſtance run between theſe two Stations, and the Difference of Latitude by Obſervation, being given; to find the true Courſe and Diſtance ſailed, and alſo the Setting and Drift of the Current.

Examp. A Ship takes her Departure from an *Head-land* in the Latitude of 30° 00′ N, bearing S S W, Diſtance 5 Leagues; and having ſailed N E *b* E 30 Miles in 6 Hours, ſaw another *Head-land* whoſe Bearing from the firſt (by the *Chart*) is N W *b* N 40 Miles, bearing W *b* N, and then the Ship by Obſervation is found to be in the Latitude of 30° 20′; the Ship's true *Courſe* and *Diſtance* ſailed, and alſo the *Setting* and *Drift* of the *Current*, (which is ſuppoſed to be in this Place) is required.

Con-

Fig. 3. *Conſtruction.* 1. Having drawn the *Compaſs* N, E, S, W; let A repreſent the firſt *Head-land*, from whence draw the N N E Line A B = 15 Miles, and alſo the N W *b* N Line A D = 40 Miles; then will B repreſent the Place of the Ship when ſhe took her Departure, and D the ſecond *Head-land.*

2. From D draw the E *b* S Line D C, then lay the Difference of Latitude N^{ly} = 20 Miles from A to F, and draw F C parallel to the Eaſt and Weſt Line until it cuts D C as in C, and join the Lines B C, A C, and D F: then is C the Place the Ship arrived at, B C her true Diſtance *ſailed*, A C her Diſtance from the firſt *Head-land*, D C her Diſtance from the ſecond *Head-land*; and G C the *Setting* and *Drift* of the *Current.* To find which by *Calculation*;

1. In the △ A F D are given the two Sides A D, A F, and the ∠ A, to find the remaining ∠^{s}, and the Side D F; then, 2. In the △ D F C will be given the Side D F, and all the ∠^{s}, to find the Side F C. 3. In the Right ∠^{d} △ A F C will be given the Sides A F, F C, to find the *Hypothenuſe* A C, and the *Acute* ∠^{s}. 4. In the △ A B C will be given the two Sides A B, A C, and the ∠ included at A; to find the ∠ A B C, the true *Courſe* from the S S W Line B A, and the Side B C the true *Diſtance* ſailed: Laſtly, in the △ B C G will be given the Sides B G, B C, and the ∠ included at B; to find the ∠ B G C the *Current's Setting* from the S W *b* S Line G B, and the Side G C the *Current's* Drift in 6 Hours, which being known, the *Drift* per hour is eaſily had.

Case 5.

The Bearing and Diſtance of two Head-lands from each other, and their Bearings from two different Stations of the Ship, and alſo the Courſe and Diſtance ran betwixt theſe two Stations, being given; to find the Ship's true Courſe and Diſtance ſailed, and alſo the Setting and Drift of the Current.

Examp. Being at Sea I ſaw two *Head-lands*, the Southermoſt bearing W N W, the other N *b* W, they bore from each other N N E and S S W, Diſtance 20 Miles; then I ſtood away N E for 6 Hours, and ran 30 Miles, and the firſt bore from me W *b* S and the ſecond W N W; I demand the Setting and Drift of the Current.

Fig. 4. *Conſtruction.* 1. Having drawn the *Compaſs* N, E, S, W; let A repreſent the Southermoſt *Head-land*, and draw the N N E Line A F = 20 Miles, then will F be the Northermoſt *Head-land.* From A draw the E S E Line A B, alſo from F draw the S *b* E Line F B until it interſects the Line A B; then will B be the Place of the Ship when ſhe firſt ſet the *Head-lands.*

2. From A draw the E *b* N Line A D, and alſo from F parallel to the E S E draw the Line F D; then will D be the Place of the Ship the ſecond time.

3. Draw the Line B C parallel to the N E = 30 Miles, and join D C and D B; then is B D the true *Diſtance* ſailed, the ∠ D B C the true *Courſe* from the N E, the ∠ B C D the *Setting* of the *Current* from the S W, and C D the *Current's Drift* in 6 Hours.—To find which by *Calculation*;

1. In the △ A B F are given the Side A F, and all the ∠ˢ, to find the Side B F. 2. In the △ A F D are given the Side A F, and all the ∠ˢ, to find the Side F D. 3. In the △ B F D will be given the Sides F B, F D, and the ∠ F, to find the ∠ B, the true *Courſe* from the N *b* W, and the Side B D, the true *Diſtance* ſailed. Laſtly, In the △ B D C will be given the Sides B C, B D, and the ∠ B, to find the ∠ C, which will be the *Current's Setting* from the S W, and the Side C D the *Current's Drift* in 6 Hours.

PROB. III.

Having the Bearing of the Place departed from, Or, the Latitude by Obſervation, *&c.* and either the Setting or Drift of the Current, given; to find the other.

CASE I.

The Courſe and Diſtance run, and then the true Bearing of the Place departed from, and either the Setting or Drift of the Current, being given; to find the other.

1. Admit a Ship from a certain Place ſails N W *b* W 30 Miles in 5 Hours, in a Current that ſets S W, and then the Place departed from bore E *b* S; I demand the Current's Drift.

Conſtruction. Having drawn the *Compaſs* N, E, S, W; let A be the Place the Ship departed from, and draw the N W *b* W Line A B = 30 Miles, and alſo the W *b* N Line A C; from B draw the Line B C parallel to the S W, until it meets A C in C: then is B C the *Drift* of the *Current* in 5 Hours, and A C the true *Diſtance* ſailed. To find which by *Calculation*; *Fig.* 5

In the △ A B C are given the Side A B=30 Miles, the ∠ A=22° 30′, the ∠ C = 56° 15′, and the ∠ B = 101° 15′; whence, to find C B, it will be,

As the Sine of the ∠ C = 56° 15′	c.080154
To the Sine of the ∠ A = 22° 30′	9.582840
So the Diſtance ſailed A B = 30 *Miles*	1.477121
To the Side B C *the Current's Drift in* 5 *Hours* = 13.8 *m.*	1.140115

And conſequently the *Current's Drift* per hour is 2.76 Miles.

If it be required to find the *Diſtance* ſailed A C, it will be;

As the Sine of the ∠ C = 56° 15′	9.080154
To the Sine of the ∠B=101°15′ (its Sup.78°45′)	9.991574
So A D (the Diſt. given by the Log) =30	1.477121
To A C *the true Diſt. ſailed* = 35.39 *Miles*	1.548849

2. If the *Courſe* and *Diſtance* ſailed, and the *Bearing* of the Place departed from, had been given the ſame as before, and the *Current's Drift* between the South and the Weſt, at the rate of 13.8 Miles, while the Ship is ſailing 30 Miles, to find the *Current's Setting*; the Solution would be much after the ſame manner as above, and the *Setting* of the Current would be found to be either S W, or it might be according to the Direction of the Line B *d*, becauſe the *Drift* of the Current may interſect the Line C A in two Places.

Case 2.

The Courſe ſteered by the Compaſs, and the true Bearing and Diſtance of the Place departed from, and either the Setting or Drift of the Current, being given; to find the other.

Fig. 5. 1. Admit there are two *Ports* bearing W *b* N and E *b* S, diſtant 353.9 Miles, and a Ship from the Eaſtermoſt ſails N W*b*W, and in 50 Hours arrives at the Weſtermoſt, there was all the while a *Current* under foot, *ſetting* S W; required the *Current's Drift* per hour.

The Solution of this *Caſe* is much after the ſame manner with the laſt *Caſe*, for having drawn the E *b* S Line A C = 353.9 Miles, and the N W*b*W Line A B, draw the S W Line C B; then in the △ A BC is given the Side A C, and all the ∠ˢ, to find the Side C B, which will be found to be 138 Miles, and this divided by 50 gives 2.76 Miles, the *Current's Drift* in an hour.

2. If inſtead of the *Current's Setting* the *Drift* had been given = 2.67 Miles per hour, and the other *Data* the ſame as before, to find the *Current's Setting*: The Caſe would have been ambiguous, and the Current's Setting might have been either S W, or according to the Direction of the Line *b* C.

If the *Bearing* and *Diſtance* of the *Ports*, and the *Setting* of the *Current* had been given, the ſame as before, and inſtead of the Courſe, the Diſtance ran by the Ship = 300 Miles; to find the Current's *Drift*, you have two Sides, and the ∠ oppoſite to one of them, given; to find the remaining ∠ˢ, and thence the Side required. Or,

If

If the *Bearing* and *Diſtance* of the *Ports*, the *Diſtance* run by the Ship in a given ſpace of time, and the *Drift* of the Current in the ſame time, be given; you have the three Sides of a △, and the Bearing of one of them given, to find the Bearing of the other Sides. Vide *Caſe* 4. of *Sect.* IV.

CASE 3.

The Difference of Latitude and Departure by dead Reckoning, the Difference of Latitude by Obſervation, and either the Setting or Drift of the Current, being given; to find the other, and the true Courſe and Diſtance ſailed.

Examp. Admit a Ship from the Latitude of 36° 40′ N, ſails 48 Hours on one or more Courſes, till by dead Reckoning ſhe came into the Latitude of 37° 10′ North, and her Departure was 120 Miles E^ly; but by a good Obſervation ſhe is found to be in the Latitude of 37° 50′ N, and all this while there is a Current under foot ſetting N N E: I demand the true *Courſe* and *Diſtance* ſailed, and alſo the *Drift* of the *Current*.

Conſtruction. 1. Having drawn the *Compaſs* N, E, S, W; let A be the Place the Ship departed from, and lay the Difference of Latitude by Account = 30 *m.* from A to B; from B draw B C parallel to the Eaſt and Weſt = 120 *m.* the *Departure*, and join A C: then is C the Place of the Ship by dead *Reckoning*. *Fig.* 6.

2. Lay the Difference of Latitude by Obſervation = 70 Miles from A to D, and draw D G parallel to the Eaſt and Weſt.

3. Draw C G parallel to the N N E, until it meets D G, alſo draw C F parallel to the Meridian A D; then is G the true Place of the Ship, and C G the Current's Drift in 48 Hours. To find which by *Calculation*;

1. In the Right ∠^d △ C F G are given C F = 40 Miles, (the Difference between the Latitude by Account, and the Latitude by Obſervation,) and the ∠ C = 22° 30′ (the Current's *Setting*;) whence,

To find C G the *Current's Drift* in 48 Hours, it will be,

As the Co-Sine of the Current's Setting the ∠ C = 22° 30′	9.965615
Is to the Difference of Latitude C F = 40 *Miles*;	1.602060
So is the Radius	10.
To the Current's Drift in 48 *Hours* C G = 43.3	1.636445

Hence, the Current's *Drift* per hour is $\frac{9}{10}$ Mile.

To find F G the Alteration of *Departure* made by the *Current*, it will be,

As the Radius	10.
To the Sine of the ∠ C=22° 30′	9.582840
So is the Side C G=43.3	1.636445
To the Side required F G=16.57	1.219285

This therefore (becauſe the Ship's *Departure* and the *Setting* of the *Current* are both the ſame way) added to the Departure D F (=B C) = 120 Miles, gives D G = 136.57 Miles, the true *Departure* the Ship made. Whence, to find the true *Courſe* and *Diſtance* ſailed, it will be,

1. *As the Diff. of Lat.* A D = 70 *Miles*	1.845098
Is to the Departure D G = 136.6	2.135451
So is the Radius	10.
To the Tang. of the Courſe 62° 52′	10.290353

Hence, the true Courſe is N 62° 52′ E^{ly}.

2. *As the Co-Sine of the Courſe* = 62° 52′	9.659025
To the Diff. of Lat. A D = 70	1.845098
So is the Radius	10.
To the true Diſt. ſailed A G=153.5 *m.*	2.186073

2. If inſtead of the *Current's Setting*, its *Drift* had been given, and the reſt of the *Data*, the ſame as before, the Solution of the Caſe would have been much after the ſame manner; only if it ſhould not be known whether the Current *ſets* Eaſterly or Weſterly, the *Caſe* would be doubtful, and the true Place of the Ship may be either in G or *g*; and the *Setting* of the *Current* may be either N N E or N N W: for having conſtructed the △ A B C, and drawn the Line D G, as before directed, then taking the Drift of the Current in 48 Hours, betwixt the Compaſſes, and ſetting one Foot in C, with the other interſect D G; then if you know in what Quarter the Current ſets, you have the true Place of the Ship, *viz.* if the Current ſets $\left\{\begin{matrix}\text{Eaſterly,}\\\text{Weſterly,}\end{matrix}\right\}$ the true Place of the Ship is $\left\{\begin{matrix}G;\\g;\end{matrix}\right\}$ otherwiſe the Caſe is doubtful.

If inſtead of the *Difference* of *Latitude* and *Departure* by Account, had been given the *Courſe* and *Diſtance* ſailed, or any other *Data*, whereby the △ A B C might be conſtructed; the Solution of the *Caſe* would have been juſt the ſame.

This

This *Caſe* is very uſeful when the *Setting* of a *Current* is known, for determining the true Place of the Ship; foraſmuch as that the *Difference* of *Latitude* by *Obſervation* is the ſureſt *Data* Mariners have for correcting their Account.

Case 4.

The Bearing and Diſtance of two Places from each other, the Bearing and Diſtance of one of theſe Places from one Station, and the Bearing of the other from another Station, the Courſe and Diſtance run between theſe two Stations, and either the Setting or Drift of the Current, being given; to find the other, and the true Courſe and Diſtance ſailed.

1. Admit being at Sea, I ſaw a *Head-land* bearing S W, Diſtance (by Eſtimation) 4 Leagues; then I ſtood away S E *b* E for 8 Hours and run 52 Miles, and ſaw another *Head-land*, whoſe Bearing from the former I find by the Chart to be S S E, Diſtance 60 Miles; it bore from me W S W; there was all the while a Current under foot ſetting S S W: I demand the Ship's true *Courſe* and *Diſtance* ſailed, and the *Drift* of the *Current*.

Conſtruction. 1. Having drawn the *Compaſs* N, E, S, W; let A repreſent the Northermoſt *Head-land*, from whence draw the N E Line A B = 12 Miles, and the S S E Line A C = 60 Miles; then B repreſents the Place of the Ship when ſhe took her *Departure*, and C the Southermoſt *Head-land*. *Fig.* 7.

2. From B draw the S E *b* E Line B D = 52 Miles, and from D draw the S S W Line D F, alſo from C draw the E N E Line C F until it meets D F, and join the Lines A D, D C, and B F; then is F the true Place of the Ship, and D F the *Drift* of the *Current* in 8 Hours. To find which by *Calculation*;

1. In the △ A B D are given the two Sides B A, B D, and the ∠ included at B, to find the oppoſite ∠ˢ, and the Side A D; then, 2. In the △ A D C will be given the two Sides A D, A C, and the ∠ at A, to find the oppoſite ∠ˢ, and the Side D C; then, 3. In the △ C D F will be given the Side D C, and all the ∠ˢ, to find the Side D F, the Current's *Drift* in 8 Hours; laſtly, In the △ B D F will be given the two Sides B D, D F, and the ∠ at D, to find the ∠ B, the true *Courſe* from the S E *b* E, and the Side B F the true *Diſtance* ſailed.

2. If inſtead of the Current's *Setting*, its *Drift* in 8 Hours had been given = F D, and the reſt of the *Data*, the ſame as before; the Conſtruction, and alſo the *Arithmetical* Solution would be much after the ſame manner as above.

Case

Case 5.

The Latitudes of two Places, the Bearings of theſe Places from two different Stations, the Courſe and Diſtance run between theſe two Stations in any given Space of Time, the Latitude arrived into by Obſervation, and the Setting of the Current, being given; to find the true Courſe and Diſtance ſailed, the Drift of the Current, and the Bearing and Diſtance of the two Places.

Examp. A Ship at Sea eſpies an *Iſland* in the Latitude of 30° 00′ N, bearing S E, diſtant 5 Leagues, and having ſailed W *b* N 60 Miles in 10 Hours, ſaw another *Iſland* in the Latitude of 30° 45′ N, bearing E N E, and by a good Obſervation the Ship is found to be in the *Latitude* of 30° 36′ N; and during the whole time there is a *Current* under foot, ſetting N E *b* N: required the Ship's true *Courſe* and *Diſtance* ſailed, the *Drift* of the Current, and the *Bearing* and *Diſtance* of the two *Iſlands*.

Fig. 8. *Conſtruction.* 1. Having drawn the *Compaſs* N, E, S, W; let A repreſent the firſt *Iſland*, from whence draw the N W Line A B = 15 Miles, and let fall the ⊥ B *a*; then will B be the Place of the Ship when ſhe took her Departure.

2. From B draw the W *b* N Line B C = 60 Miles, alſo draw B *e* parallel to the Meridian, and through C draw C *d* parallel to the Eaſt and Weſt Line; then will C be the Place of the Ship by dead *Reckoning*, A *d* her *Difference* of *Latitude*, and C *d* her *Departure* from the firſt *Iſland*.

3. Lay the Ship's true *Difference* of *Latitude* from the firſt Iſland = 36 min. from A to *f*, and draw *f* D parallel to the Eaſt and Weſt, then draw the Line C D parallel to the N E *b* N, until it interſects the Line *f* D, and let fall the ⊥ D *h*; then is D the true Place of the Ship, B D the true *Diſtance* ſailed, and C D the Current's *Drift* in 10 Hours.

4. Lay the Difference of Latitude of the ſecond *Iſland* from the firſt = 45 min. N^ly from A to G, and draw F G parallel to the Eaſt and Weſt, then draw D F parallel to the E N E until it interſects the Line F G, and join F A; then is F the ſecond *Iſland*, F A the mutual *Diſtance* of the two *Iſlands*, and the ∠ F A G their *Bearing* from the *Meridian*.

To find the ſeveral Requiſites by *Calculation*;

1. In the Right ∠^d △ A B *a* are given the Side A B and the ∠ A, to find B *a*; 2. In the Right ∠^d △ B *b* C are given the Side B C and the ∠ B, to find the Sides B *b* and C *b*; then, 3. In the Right ∠^d △ C *h* D will be given the Side D *h* (= B *e* — B *b*) and the ∠ *h* C D, to find C D the Current's *Drift* in 10 Hours, and C *h* the Alteration of *Departure* made by the Current: then, 4. In the Right ∠^d △ B *e* D will be given the Side B *e* (= *f* A — *a* A,)

—*a* A,) and the Side D*e* (= C*b* — C*h*) to find the ∠ B the true *Courſe*, and the Side BD the true *Diſtance* ſailed. Then, 5. In the Right ∠^{d} △ D F *k* will be given the Side *k* F (= A G — A*f*) and the ∠ D, to find D*k*, and alſo (if required) the Side D F the Diſtance of the Ship from the ſecond *Iſland*. Then, laſtly, In the Right ∠^{d} △ A G F are given the Side A G, and alſo F G (=D *e* + *ef* — D*k*,) to find the ∠ A the *Bearing*, and the Side A F the *Diſtance* between the two *Iſlands*.

There may be other Varieties propoſed concerning *Currents*, but whoever underſtands what has been already delivered on this Head, will eaſily ſolve any other Caſe whatſoever.

PROB. IV.

How a Ship muſt ſhape her Courſe to hit any known Place to *Windward*, the Lee-way ſhe makes, and likewiſe the Current under foot (if any) being known.

CASE I.

The Bearing and Diſtance of any Place to Windward, and the leaſt Diſtance from the Wind that the Ship can make her way good, being given; to find upon what Courſe, and how far the Ship muſt ſail upon each Tack, to fetch the ſaid Place.

Defin. TACKS are *Ropes* (in a Ship) for carrying forwards the *Clews* of the *Sails*, to make them ſtand cloſe by a Wind.

The *Tacks* are always faſtened to that ſide the Ship the *Wind* is of, and if it be to the { Right / Left } Hand ſide when your Face is turned towards the Head of the Ship, they are called the { *Star-board* / *Lar-board* } *Tacks*; becauſe the Sides of the Ship are ſo called; and whenever theſe *Tacks* are faſtened (as above) the Ship is ſaid to ſail upon ſuch a *Tack*.

The Angle which the *Wake* of the Ship makes with the Line of the *Wind*, is the Quantity from the Wind that the Ship makes her way good through the Water; and the Difference betwixt this Angle, and that which the Head of the Ship makes with the Wind, is called the Ship's DRIFT or LEE-WAY.

Examp. A Ship that makes her way good within 6 ½ Points of the Wind at N N E, is bound to a *Port* bearing N E, diſtant 120 Miles; I demand the *Courſe* and *Diſtance* upon each *Tack* to gain the ſaid *Port*.

Con-

Fig. 9. *Conſtruction.* 1. Having drawn the *Compaſs* N, E, S, W; let A repreſent the Place the Ship is in, from whence draw the N E Line A B = 120 Miles; then will B repreſent the *Port* the Ship is bound to.

2. Set off $6\frac{1}{2}$ Points on each Side the N N E or Wind-Line A *d*, to *a* and *b*, and draw A *a* and A *b*; then will A *a* be the N W $\frac{1}{2}$ W Line, or Courſe upon the *Star-board Tack*, and A *b* the E $\frac{1}{2}$ S Line, or Courſe upon the *Lar-board Tack*.

3. Through B draw B C parallel to A *b* till it meets A *a* produced in C, then will A C and C B be the Diſtances upon each *Tack*. To find which by *Calculation*;

In the $\triangle$ A B C are given the Side A B = 120 Miles, the $\angle$ A = 95° 37′ $\frac{1}{2}$ (= 8 $\frac{1}{2}$ Points, the Diſtance between the N E and the N W $\frac{1}{2}$ W) the $\angle$ C = 33° 45′, (= 3 Points, the Diſtance between the E $\frac{1}{2}$ S and the S E $\frac{1}{2}$ E) and the $\angle$ B = 50° 37′ $\frac{1}{2}$ (the Supplement of the other two;) whence,

1. To find the Diſtance upon the *Star-board Tack*, it will be,

As the Sine of the $\angle$ C = 33° 45′	$\bar{0}$.255261
Is to the Sine of the $\angle$ B = 50° 37′ $\frac{1}{2}$	9.888185
So is the Diſtance A B = 120 *Miles*	2.079181
To A C *the Diſt. upon the* Star-board Tack = 167 *m.*	2.222627

2. To find the Diſtance upon the *Lar-board Tack*, it will be,

As the Sine of the $\angle$ C = 33° 45′	$\bar{0}$.255261
To the Sine of the $\angle$ A = 95° 37′ $\frac{1}{2}$ (its Sup. 84° 22′ $\frac{1}{2}$)	9.997903
So the Side A B = 120 *m.*	2.079181
To C B *the Diſt. upon the* Lar-board Tack = 215 *m.*	2.332345

So that the Ship in ſailing N W $\frac{1}{2}$ W 167 Miles, and E $\frac{1}{2}$ S 215 Miles, will arrive at her *Port*.

In the former Conſtruction, the Line B D might have been drawn parallel to the N W $\frac{1}{2}$ W Line, and the Solution would have been the ſame; and in this *Caſe* the Ship in ſailing E $\frac{1}{2}$ S 215 Miles, with her *Lar-board Tacks* on Board, and then N W $\frac{1}{2}$ W 167 Miles, with her *Star-board Tacks* on Board, would have arrived at the *Port* B in the ſame time.

To know how near the Wind a Ship will lie, obſerve what Courſe ſhe makes on each *Tack*, then (generally) the middle between both is the Wind; yet this doth not always hold, for ſome Ships will lie nearer the Wind upon one Tack, than they will upon the other: wherefore, in ſuch a Caſe lay the number of Points the Ship lies from the Wind on each Tack, on each ſide the Wind-Line, each in its proper Side, and you may compute the moſt advantageous Courſes, and how far you muſt ſail upon each Courſe to hit any given Place; as in the *Example* above.

Case 2.

How to hit a Ship under ſail.

Examp. Being at Sea, about four in the Evening, I ſaw a Ship directly to Windward bearing N N E $\frac{1}{2}$ E about 4 Leagues; ſhe ſtood with her *Star-board Tacks* on board, and ſo did we till ſeven, having run at the rate of 5 *Knots* an hour, and then the Ship bore from us N E *b* E; we both made our way good within 6 $\frac{1}{2}$ Points of the Wind, but we fore-reaching of her, and being willing to ſpeak with her at 1 next Morning: I would know when I muſt *Tack*, and how far I muſt ſail on each *Tack* to be with her at that time, ſuppoſing the Wind to continue the ſame, and ſhe to ſail on the ſame Courſe, and at the ſame rate.

Conſtruction. 1. Having drawn the *Compaſs* N, E, S, W; let A repreſent the Place of the firſt Ship, from whence draw the N N E $\frac{1}{2}$ E Line A B = 12 Miles; then will B be the Place of the ſecond Ship at 4 o'-clock. *Fig.* 10.

2. Draw the N W Line A G the Courſe upon the *Star-board Tack*, and parallel to it the Line B F.

3. Make A C = 15 Miles, (the Diſtance run by the Ship A in 3 Hours) then will C be the Place of the Ship A at 7 o'th' Clock; from C draw C D parallel to the N E *b* E, then will D be the Place of the other Ship at the ſame time, and B D her Diſtance ſailed in 3 Hours.

Laſtly, Make D F to B D as 6 to 3, (or as 2 to 1) then will F be the Place of the Ship B at 1 o'Clock next Morning. Through F draw F G parallel to the E *b* S, (the Courſe upon the *Lar-board Tack*) then will C G be the Diſtance you muſt ſail upon the *Star-board Tack*, and G F the Diſtance upon the *Lar-board Tack*, for to hit the Ship B at 1 o'-Clock.

To find the ſeveral Requiſites by *Calculation*.

1. Draw B *a* parallel to C D; then in the $\triangle$ B A *a* are given the Side A B = 12 Miles, the $\angle$ A = 73° 07′ $\frac{1}{2}$, (= 6 $\frac{1}{2}$ *Points*, the Diſtance between the N W and the N N E $\frac{1}{2}$ E) the $\angle$ *a* = 78° 45′, (the Diſtance between the S E and the N E *b* E) and the $\angle$ B = 28° 07′ $\frac{1}{2}$, to find the Sides A *a* and *a* B; which by *Caſe* 8. of *Trigonom.* will be found to be, *viz.* *a* B = 11.71 Miles, and A *a* = 5.76 Miles. Now becauſe *a* B is parallel to C D, and *a* C $\parallel$ B D, A C 15 — A *a* 5.76 = B D 9.24, the *Drift* of the Ship B in 3 Hours, and conſequently is 3.08 Miles per hour, and D F her *Drift* in 6 Hours is 18.48 Miles.

2. Draw F *b* parallel to C D; then in the $\triangle$ *b* F G are given the Side *b* F (= *a* B) = 11.7 Miles, the $\angle$ *b* = 101° 15′, (= 9 *Points*, the Diſtance between the N W and the N E *b* E) the $\angle$ G = 33° 45′, (the Diſtance between the S E and the E *b* S,) and conſequently the $\angle$ F = 45° 00′, to find the Side *b* G, and alſo G F the Diſtance upon the *Lar-board Tack*; which

by *Caſe* 8. of *Trigonom.* will be found to be, *viz.* GF = 20.67 Miles, the Diſtance upon the *Lar-board Tack*, and *b* G = 14.9 *m.* whence C G the Diſtance upon the *Star-board Tack* is = 33.38 Miles (= G *b* 14.9 + F D 18.48 ;) whence the Diſtance upon both *Tacks* is 54.05 Miles, and conſequently the firſt Ship muſt ſail at the rate of 9 Miles an hour from 7 o'-Clock, before ſhe can be up with the other Ship at 1 o'-Clock next Morning. To find when the Ship muſt *Tack*, it will be C G + G F = 54.05 : C G = 33.38 :: 6 Hours : 3^h 42^m; whence the Ship muſt *Tack* at 42 min. after 10 a-Clock.

If it be required how far the Ship A muſt ſail, and when ſhe muſt *Tack* to hit the Ship B, ſuppoſing the two Ships can ſail no faſter than they did at firſt, and that the Ship B ſtill keeps on the ſame Courſe.

1. Having drawn the proper Courſes of the Ships, and found the rate of ſailing of the Ship B in a given ſpace of time, (as in the *Example* above) which let be to the Motion of the Ship A, as B D = 9.24 to A C = 15 Miles, the Motions of the two Ships in 3 Hours ; and conſequently their hourly Motion is, *viz.* A 5 Miles per hour, and B 3.08 Miles per hour.

2. Having determined the Points C and D, the Places of the Ships at 7 o'Clock, ſuppoſe the Ship B to *Anchor* at D, and draw D *d* parallel to the E *b* S, (the Courſe upon the *Lar-board Tack*) then find C *d* and *d* D, the Diſtance upon each *Tack* that the Ship A muſt ſail from C to hit the Point D, which are above determined to be, *viz.* C *d* (= *b* G) = 14.9 Miles, and *d* D (= G F) = 20.67 Miles ; hence, ſuppoſing the ſecond Ship to remain in the ſame Place, the firſt Ship has 35.57 Miles to run before ſhe can come up with her ; then becauſe A ſails 1.92 Miles per hour faſter than B, it will be, 1.92 : 1 hour : : 35.57 : 18^h 31^m ; ſo that the Ship A will come up with the Ship B at half an hour paſt 1 o'-Clock in the Afternoon next Day, after ſhe had ſailed 92 $\frac{1}{2}$ Miles from C: To find when ſhe muſt *Tack*, ſubſtract 4 Hours (the time required to ſail 20.67 Miles the Diſtance upon the *Lar-board Tack*) from 18 $\frac{1}{2}$ Hours, the Remainder 14 $\frac{1}{2}$ Hours ſhews the Ship muſt *Tack* at half an hour paſt 9 o'-Clock next Morning.

In like manner knowing the Advantage that two Ships have of the Wind, and their Velocities through the Water, you may eaſily find what Courſe the ſwifter Ship muſt ſteer for to overtake the ſlower, in the ſhorteſt time poſſible, as alſo at what time ſhe'll be able to come up with her.

Case 3.

How to hit a Port, or double a Cape or Head-land to Windward, in a Current or Tide's Way.

Examp. A Ship that makes her way good within 6 Points of the Wind at N *b* E, is bound to a Port bearing N N E, diſtant 120 Miles, and all the way there is a *Current* under foot, ſetting N E, at the rate of 1 Mile in

in the time the Ship ſails 3 Miles; I would know the *Courſe* and *Diſtance* upon each *Tack*, to fetch the *Port*.

Conſtruction. 1. Having drawn the *Compaſs* N, E, S, W; let A be the Place of the Ship, from whence draw the N N E Line A B = 120 Miles, then will B repreſent the *Port*; next, lay 6 Points on each Side the N *b* E, (the *Wind*-Line) and draw the N W*b*W Line A G, and through B draw the Line B G parallel to the E *b* N; then if there were no *Current*, A G would be the Diſtance upon the *Star-board Tack*, and G B the Diſtance upon the *Lar-board Tack*, to fetch the *Port*. *Fig.* 11.

2. Through B draw B *a* parallel to the N E, (the *Setting* of the *Current*) and make *a b* = $\frac{1}{3}$ of A *a*, and through *b* draw A *b*, which produce to H; then is A H the true Courſe of the Ship upon the *Star-board Tack*.

3. Lay any convenient Diſtance from B to *d*, and draw *d f* parallel to B H, then taking B *d* betwixt the Compaſſes, lay off that Diſtance 3 times from *d* to *e*, and draw B *e*, which continue directly to F; then draw F C parallel to *f d*, alſo draw F D parallel to B *a*: then A D is the Diſtance upon the *Star-board Tack*, and F C the Diſtance upon the *Lar-board Tack*, required to hit the *Port*; A F being the Courſe and Diſtance made good upon the *Star-board Tack*, and F B the true Courſe and Diſtance upon the other *Tack*.

To find A D and F C by *Calculation*.

1. In the △ A D F are given the *Ratio* of the Sides A D, D F, *viz.* A D = 3, and D F = 1, and the ∠ included D = 78° 45′, whence, to find the ∠ A, it will be,

∠ D 78° 45′

101 15

50 37½, ½ Z *op.* ∠ˢ

As $\frac{AD+DF}{2}$	= 2	0.301030
To $\frac{AD-DF}{2}$	= 1	0.
Tang. 50° 37′ ½		10.085827
Tang. 31 21		9.784797

∠ A 19 16

2. In the △ F C B are given the *Ratio* of the Sides F C, C B, *viz.* F C 3, and C B 1, and the ∠ included C = 146° 15′; whence the ∠ F will be found to be 8° 15′.

3. In the △ A B F are given the Side A B = 120 Miles, the ∠ A = 59° 29′, (= D A B 78° 45′ − D A F 19° 16′) the ∠ A F B = 72° 31′, (= A F C 64° 16′ + C F B 8° 15′) and conſequently the ∠ B = 48° 00′; whence the Sides A F, F B will be found to be, *viz.* A F = 93.47 Miles, and F B = 108.3 Miles. Then,

4. In the △ A D F are given the Side A F = 93.47, the ∠ D = 78° 45′, and the ∠ F = 81° 59′; whence the Side A D, the Diſtance required upon the *Lar-board Tack*, will be = 94.4 Miles.

5. In the △ F B C are given the Side F B = 108.3, the ∠ C = 146° 15′, and the ∠ B = 25° 30′, whence the Side F C, the Diſtance required upon

the *Lar-board Tack* will be = 87.95 Miles; ſo that the Ship in ſailing 94.4 Miles upon the *Star-board Tack* from A to D, will by means of the *Current* be carried to the Point F; then in ſailing 87.95 Miles upon the *Lar-board Tack* from F to C, will by means of the *Current* be carried to the Port B.

By this Caſe it may be eaſily found, whether a Ship may be able to weather a *Head-land*, *&c.* in a *Current* or *Tide's* Way: Let us ſuppoſe a Ship that can make her way good within 6 Points of the Wind at N *b* E, to be bound to a *Port* bearing N N E 120 Miles, as before, and that all the way there is a *Current* ſetting S W, at the rate of 1 Mile, while the Ship is ſailing 3 Miles; and it be required how far ſhe muſt ſail upon each *Tack* to fetch the *Port*.

Fig. 12. Having drawn the N N E Line A B = 120 Miles, and the N W *b* N Line A *a*, and alſo the E *b* N Line B *a*, the Courſes upon each *Tack*; thro' *a* draw the S W Line *a b*, then divide the Line A *a* into 3 equal Parts, and lay one of thoſe Parts from *a* to *b*, and draw the Line A *b*; next draw the S W Line B C, and lay ſome convenient Diſtance from B to *d*, thro' *d* draw *d e* parallel to B *a*, then taking B *d* betwixt the Compaſſes, lay that Extent 3 times from *d* to *e*, and draw the Line B *e* until it meets A *b* produced in F: Through F draw F D parallel to the N E, until it meets A *a* produced in D, then will A D be the Diſtance required upon the *Star-board Tack*, and F the true Place of the Ship at the end of that Courſe: Through F draw F C parallel to *a* B, until it meets B *d* produced in C, then will F C be the Diſtance required upon the *Lar-board Tack*.

The ſeveral Requiſites may be eaſily found by *Calculation*, by the Directions given in the former *Example:* For, 1. In the △ A *a b* are given the Proportion of the Sides A *a*, *a b*, and the ∠ A *a b*, to find the ∠ *a* A *b*. Alſo, 2. In the △ B *d e* are given the Proportion of the Sides B *d*, *d e*, and the ∠ *d*, to find the ∠ *d* B *e*; then, 3. In the △ A B F will be given the Side A B, and all the ∠ˢ, to find the Sides A F and B F. Then, 4. In the △ A D F will be given the Side A F, and all the ∠ˢ, to find the Side A D, the Diſtance required upon the *Lar-board Tack*, which will be found to be = 434.5 Miles. Laſtly, In the △ B F C will be given the Side B F, and all the ∠ˢ, to find the Side F C, the Diſtance required upon the *Lar-board Tack*, which you'll find to be = 683 Miles; ſo that ſuppoſing the Ship can make good 4 Miles an hour, ſhe'll ſpend 11 Days and 15 Hours upon her Voyage; a very conſiderable time for ſo ſmall a Diſtance. And if the *Velocity* of the Ship had been but double that of the Current, ſhe would not have been able ever to fetch the Port.

CHAP.

Plate 9

Fig. I

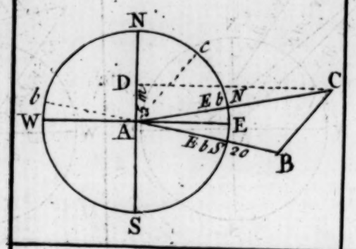


Fig. II

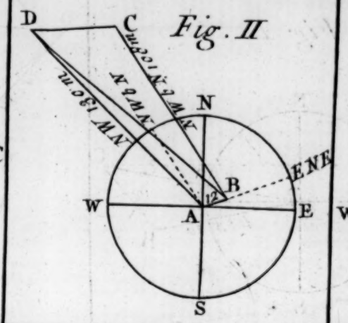


Fig. III

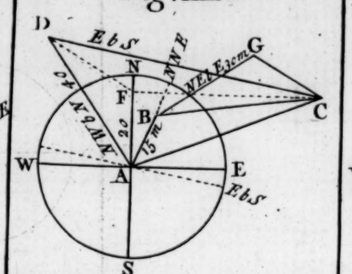


Fig. IV

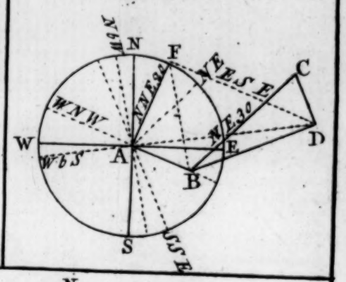


Fig. V

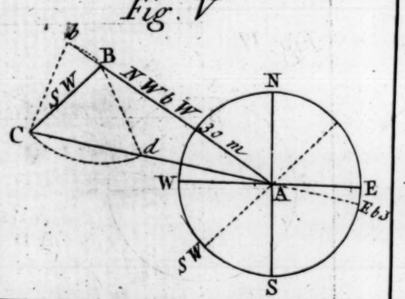


Fig. VI

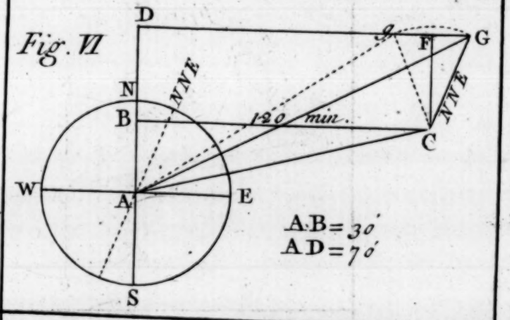


Fig. VII

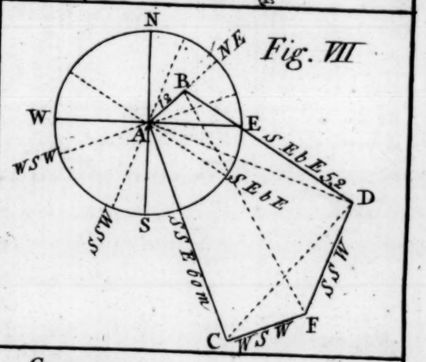


Fig. VIII

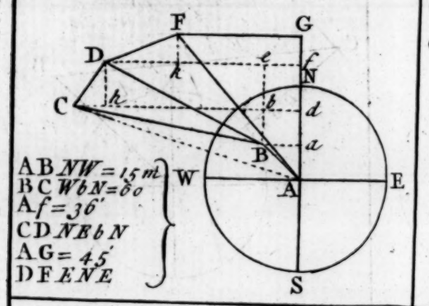


Fig. IX

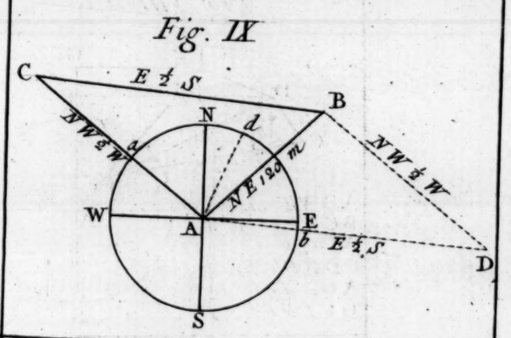


Fig. X

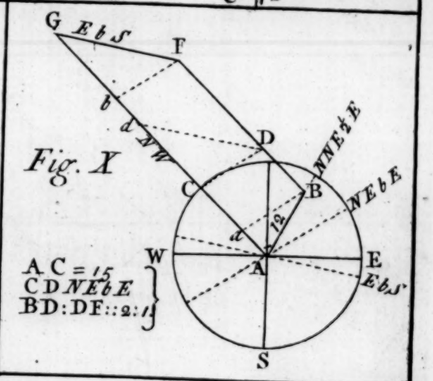


Fig. XI

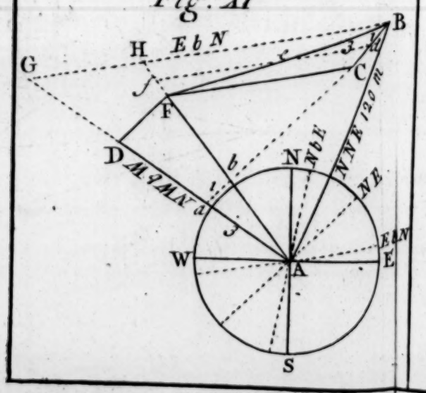
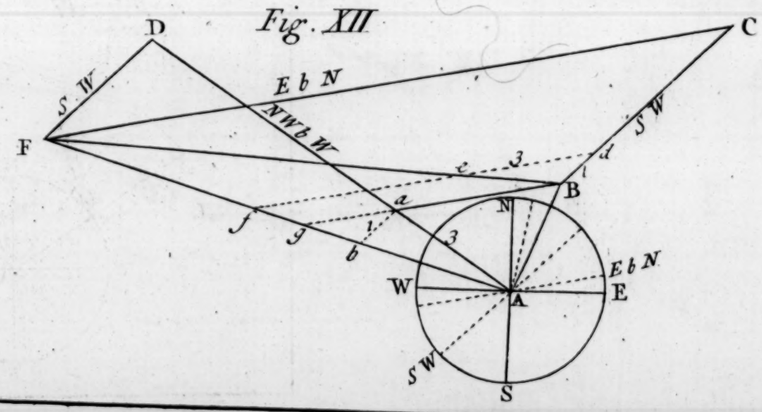


Fig. XII



CHAP. III.

Containing the Solution of ſome few Aſtronomical Problems, uſeful in Navigation.

SECT. I.

Aſtronomical Definitions.

THE *Equinoctial*, and its *Poles*; The *Horizon*, and its *Poles*; The *Latitude* and *Longitude* of Places upon the Earth, have been already defined in *Sect.* 2. of the firſt *Chapter:* Wherefore, now there remains to be explained,

1. The ECLIPTICK is a Great Circle, in which the *Sun* appears to move in its *annual Motion* round the Earth: This *Circle* makes an Angle with the *Equinoctial* of 23° 29′, and interſects it in two oppoſite Points, which are called the EQUINOCTIAL POINTS; and the two Points in the *Ecliptick* that are at the greateſt Diſtance from the *Equinoctial*, are called the SOLSTITIAL POINTS: The two *Meridians* paſſing through thoſe Points, are called the COLURES; whereof that which paſſeth through the *Equinoctial Points*, is called the EQUINOCTIAL COLURE; and that which is at right Angles to it, paſſing through the *Solſtitial Points*, is called the SOLSTITIAL COLURE.

The *Ecliptick* is uſually divided into 12 equal Parts called SIGNS, (and conſequently each *Sign* muſt contain 30 Degrees) beginning from the Interſection of the *Equator*, and numbered from *Weſt* to *Eaſt*: The *Names* and *Characters* of the 12 *Signs* are as follow:

viz.	*Aries.*	*Taurus.*	*Gemini.*	*Cancer.*	*Leo.*	*Virgo.*
	1. ♈.	2. ♉.	3. ♊.	4. ♋	5. ♌.	6. ♍.
	Libra.	*Scorpio.*	*Sagittarius.*	*Capricornus.*	*Aquarius.*	*Piſces.*
	7. ♎.	8. ♏.	9. ♐.	10. ♑.	11. ♒.	12. ♓.

The first six of these are called the *Northern Signs*, and possess that half of the *Ecliptick* which is to the Northward of the *Equator*; beginning at the first Point of ♈, and ending with the last Point of ♍.

The latter six are called the *Southern Signs*, because they possess the Southern half of the *Ecliptick*; beginning at the first Point of ♎, and ending with the last Point of ♓.

The Poles of the *Ecliptick* are both situated in the *Solstitial Colure* at 23° 29′ Distance from the *Poles* of the World; and they take their Denomination from that Hemisphere wherein they are placed, *viz.* that Pole which lies in the Northern Hemisphere is called the *North Pole*, and the other is called the *South Pole* of the *Ecliptick*.

2. All Great Circles passing through the *Poles* of the *Ecliptick*, (and consequently intersecting it at right Angles) are called *Circles* of *Longitude*: So that,

3. The Longitude of any Point in the Heavens (as a *Star* or *Planet*, &c.) is an Arch of the *Ecliptick*, contained between the Circle of *Longitude* passing through that Point, and the *Equinoctial* Point ♈. And that *Degree* of any *Sign* which lies under the Circle of Longitude passing through any *Star* or *Planet*, is called the *Place* of the *Star* or *Planet*.

It may be here noted, that the *Sun* never goes out of the *Ecliptick*, and therefore the *Sun's* Place is that *Sign*, *Degree*, *Minute*, &c. of the *Ecliptick*, which he at any time possesses; and his *Longitude* is the Arch of the *Ecliptick*, intercepted betwixt this Place and the first Point of ♈.

4. All Circles parallel to the *Ecliptick*, are called *Parallels of Latitude*: So that,

5. The Latitude of any Point in the Heavens, (as of a fixed *Star* or *Planet*, &c.) is an Arch of the Circle of Longitude passing through that Point, and intercepted betwixt it and the *Ecliptick*: and if the said Point be to the *Northward* of the *Ecliptick*, it is called *North Latitude*; but if it be to the *Southward*, it is called *South Latitude*.

6. All Circles parallel to the *Equinoctial*, are with respect to any Point in the Heavens, called *Parallels* of *Declination*: So that,

7. The Declination of any Point in the Heavens, is an Arch of the Meridian passing through that Point, and intercepted betwixt it and the Equator; and if the said Point be to the {Northward / Southward} of the *Equator*, it is called {*North* / *South*} *Declination*.

Of the Parallels of Declination, there are four that have particular Denominations, *viz.* the two *Tropicks*, and the two *Polar Circles*.

8. The Tropicks are on different Sides of the *Equator*, each 23° 29′ distant from it; that which lies in the *Northern Hemisphere*, touches the *Ecliptick* in the Point ♋, and for that reason is called the Tropick *of* Cancer: And the Southern *Tropick* touches the *Ecliptick* in the Point ♑, and from thence is called the Tropick *of* Capricorn.

These

Theſe Circles are the Limits of the Sun's greateſt Declination, and are called *Tropicks*, becauſe whenever the Sun arrives to them, he ſeems to return back again towards the *Equator*.

9. The POLAR CIRCLES are each of them at the ſame Diſtance from the *Poles* of the *World*, that the *Tropicks* are from the *Equator*, viz. 23° 29′; that which lies near the *North Pole* is called the ARCTICK CIRCLE, from *Arctos*, a Conſtellation ſituated in the Heavens near that Place; whence alſo this Pole is ſometimes called the *Arctick Pole*. The other *Polar* Circle which is ſituated near the *South Pole*, is called the ANTARCTICK CIRCLE; becauſe its Poſition is contrary to the other. And the *South Pole* is ſometimes called the *Antarctick Pole*.

It ſeems theſe two Circles are called *Polar*, becauſe they are deſcribed by the *Poles* of the *Ecliptick*, in the Diurnal Motion of the Earth round its Axis.

10. All Great Circles paſſing thro' the *Zenith* and *Nadir*, (and conſequently interſecting the Horizon at right ∠ˢ) are called either VERTICAL CIRCLES, or AZIMUTHS; whereof that which paſſeth thro' the *North* and *South* Points of the *Horizon*, (and conſequently through the *Poles* of the World) is (as before defined) called the *Meridian*; and that *Vertical Circle* which paſſeth through the *Eaſt* and *Weſt* Points of the Horizon (croſſing the Meridian at right Angles) is called the PRIME VERTICAL, or *Circle* of *Eaſt* and *Weſt*.

11. The AZIMUTH of any *Point* in the Heavens (above the Horizon) is an *Arch* of the *Horizon* contained between the *Meridian*, and the *Vertical Circle* paſſing through the ſaid Point; and is the ſame with the Angle at the *Zenith*, made by the *Meridian* and the ſaid *Vertical Circle*.

12. The AMPLITUDE is an *Arch* of the *Horizon* intercepted between the *Sun*, *Moon*, or *Stars*, &c. at their Riſing or Setting, and the Eaſt or Weſt Points of the *Horizon*: Or, the Angle made at the *Zenith*, between the *Prime Vertical*, and the *Vertical Circle* paſſing through the *Sun*, *Moon*, or *Star*, &c. at their Riſing or Setting. For Riſing, it is called *Amplitude Ortive*, and *Occaſive* for Setting.

13. When any Point of the Heavens is upon the *Meridian*, it is then ſaid to CULMINATE.

14. All Circles parallel to the *Horizon* are call'd ALMICANTERS, or PARALLELS of *Altitude*; becauſe they ſhew how far the *Sun*, *Moon*, or *Stars*, &c. are aſcended above the *Horizon*: So that,

15. The ALTITUDE of any Point in the Heavens is an *Arch* of the *Vertical Circle* paſſing through that Point, and intercepted betwixt it and the *Horizon*; which can never exceed a *Quadrant* or 90 *Degrees*.

16. ZENITH DISTANCE, is an *Arch* of the *Vertical Circle* paſſing through any Point in the Heavens, and intercepted betwixt it and the *Zenith*; and therefore is equal to the Complement of the *Altitude* to 90 Degrees.

17. MERIDIAN ALTITUDE, is the Altitude of the *Sun*, *Moon*, or *Stars*, &c. when they are upon the *Meridian*; and is the greateſt Altitude they can have on that Day.

18. Of

18. Of the Circles parallel to the *Horizon*, that which is 18 Degrees below it, is called the CREPUSCULUM, or the Circle *terminating Light* and *Darkneſs*: becauſe it is found by Experience, that when the Sun comes to this Circle in the Morning, it is then the Day breaks; alſo, when the Sun comes to the ſame in the Afternoon, it grows quite *dark*.

19. All the Space contained betwixt this Circle and the *Horizon*; or rather the Time elapſed between the Sun's Riſing or Setting, and the time of its being upon the *Crepuſculum*, is called the TWILIGHT; becauſe it is then between Day and Night.

The *Horizon* and the *Equator* may be either *Parallel*, *Perpendicular*, or *Oblique* to each other.

20. A PARALLEL SPHERE is that Poſition where the *Equator* coincides with the *Horizon*, and conſequently the *Poles* of the World are in the *Zenith* and *Nadir*. The Inhabitants of this *Sphere* (if any) are thoſe who live under the *Poles* of the *World*; here is but one Day and one Night throughout the whole Year.

21. A RIGHT SPHERE is that Poſition where the *Equator* is perpendicular to the *Horizon*; the Inhabitants of this *Sphere* are thoſe who live under the *Equinoctial*, where the Days and Nights are always equal throughout the whole Year.

22. An OBLIQUE SPHERE is that Poſition where the *Equator* and the *Horizon* make Oblique Angles with each other; which every where happens but under the *Equinoctial* and the *Poles*: Here one of the Poles is always elevated above the Horizon, and the other is equally depreſſed below it, and this Elevation or Depreſſion is always equal to the *Latitude* of the Place. The *Equinoctial* is here alſo cut into two equal Parts by the *Horizon*; and conſequently when the Sun is in the *Equinoctial*, the Days and Nights will be equal: But all the *Parallels* of *Declination* are unequally cut by the *Horizon*, and conſequently when the Sun is in either of them, it will make the Days and Nights unequal, and this Inequality will be more or leſs according to the *Latitude* of the Place (from the *Equator*.)

23. The *Arch* of any Parallel of Declination which ſtands above the *Horizon*, is called the DIURNAL ARCH; and the remaining Part of it which is below the *Horizon*, is called the NOCTURNAL ARCH.

24. It may be here obſerved, that that Parallel of Declination which is at the ſame Diſtance from the *Equinoctial*, that the Place is from the *Pole*, juſt touches the Horizon without cutting of it; and all Points in the Heavens that are betwixt this and the elevated Pole, conſtantly appear above the Horizon; but thoſe that are betwixt the oppoſite Circle and the depreſſed Pole, are always hid below it: whence the former of theſe two *Parallels* is called the *Circle* of PERPETUAL APPARITION, and the latter the *Circle* of PERPETUAL OCCULTATION.

25. That Point of the *Equinoctial* which comes to the {Eaſtern / Weſtern} Part of

of the *Horizon* with any Point in the Heavens, is called the {*Aſcenſion* / *Deſcenſion*} of that Point, counted from the beginning of ♈; and if it be in a *Right Sphere*, the *Aſcenſion* or *Deſcenſion* is called *Right*; but if it be in an *Oblique Sphere*, it is called *Oblique Aſcenſion* or *Deſcenſion*. So that,

26. The RIGHT ASCENSION of the *Sun*, *Moon*, or any *Star*, &c. is an *Arch* of the *Equinoctial* contained between the beginning of ♈, and that Point of the *Equinoctial* which riſes with them in a *Right Sphere*; or which comes to the *Meridian* with them in an *Oblique Sphere*.

27. OBLIQUE ASCENSION or DESCENSION, is an *Arch* of the *Equinoctial* intercepted between the beginning of ♈, and that *Point* of the *Equator* which riſes or ſets with any *Point* in the Heavens in an *Oblique Sphere*.

28. ASCENSIONAL DIFFERENCE, is the Difference betwixt the *Right* and *Oblique Aſcenſion* or *Deſcenſion*, and ſhews how long the Sun riſes or ſets before or after the Hour of ſix; or, half the time that the *Moon* or any *Star* continuing above or below the Horizon, exceed or want of ſix Hours.

29. The *Right Aſcenſion* of that *Point* of the *Ecliptick* which at any time is upon the *Meridian*, is called the *Right Aſcenſion* of MID-HEAVEN at that time.

If a *Sphere* or *Globe*, having all the fore-mentioned Circles delineated upon its Surface, be conceived to be tranſparent, and the Eye of a Spectator be placed in one of the *Equinoctial Points*; all the ſaid *Circles* will be projected upon the Plane of the *Solſtitial Colure*; and the *Equator*, the *Ecliptick*, and the *Horizon*, will be repreſented by right Lines, and their *Secundaries* and *Parallels* will be all Circles in this *Projection*.

The Circle H, Z, O, N repreſents the *Solſtitial Colure*; the Circle Æ⊙Q the *Equinoctial*; P the *North Pole* of the *Equator*, and S the *South Pole*: All Great Circles conceived to be drawn from P to S, (and conſequently cutting the *Equinoctial* at right Angles) are *Meridians*; and a right Line conceived to be drawn from P to S through the Center of the *Sphere*, is the *Axis* of the *Equator*, or the *Earth's Axis*; alſo the great Circle P⊙S is the *Equinoctial Colure*: All Circles conceived to be drawn from the *Poles* P and S, parallel to the *Equinoctial*, are the *Parallels* of *Declination*, whereof thoſe expreſſed by double Lines are the *Tropicks* and *Polar Circles*; viz. the double Line marked {♋ ♋ / ♑ ♑} is the *Tropick* of {*Cancer*, / *Capricorn*,} and the double Line marked {*a b* / *c d*} is the {*Arctick* / *Antarctick*} *Circle*. *Plate* 10. *Fig.* 1, 2, 3, 4, & 5.

2. The Great Circle ♋ ⊙ ♑ making an Angle of 23° 29′ with the *Equator*, is the *Ecliptick*; the right Line *b* ⊙ *c* paſſing through the *Center* of the *Sphere*, and cutting the *Ecliptick* at right Angles, is the *Axis* of the *Ecliptick*, *b* the *North Pole* thereof, and *c* its *South Pole*. The Point ⊙ where the *Ecliptick* croſſes the *Equator*, is one of the *Equinoctial Points*;

and the Points ♋ and ♑, where the *Ecliptick* touches the *Tropicks*, are the *Solſtitial Points*: All Circles conceived to be drawn from *b* to *c* (cutting the *Ecliptick* at Right Angles) are the *Circles* of *Longitude*; and all Circles deſcribed from the *Poles b* and *c* parallel to the *Ecliptick*, are the *Parallels* of *Latitude*.

3. The Great Circle H ⊙ O repreſents the *Horizon*; the two Points Z, N (which are 90° diſtant from the *Horizon*) the *Zenith* and *Nadir*: All Great Circles conceived to be drawn from Z to N, (and conſequently croſſing the *Horizon* at right Angles) are the *Azimuths*; whereof HZON is the *Meridian*, and the Great Circle Z ⊙ N (croſſing the Meridian at right Angles) is the *Prime Vertical*, or *Circle* of *Eaſt* and *Weſt*; alſo the Circle P ⊙ S (likewiſe croſſing the Meridian at right Angles) is the *Six o'-Clock Hour-Circle*: All ſmall Circles conceived to be drawn from the *Zenith* Z, parallel to the *Horizon*, are the *Almicanters* or *Parallels* of *Altitude*: The Circle T W drawn 18 Degrees below the *Horizon* (and parallel to it) is the *Crepuſculum*, or the Circle terminating *Light* and *Darkneſs*; and the time that the Sun is betwixt this Circle and the Horizon, is called the *Twilight*.

In *Fig.* 1. the *Equator* and its *Parallels* cut the *Horizon* at right Angles, whence this Poſition is called a *Right Sphere*: Here the *Equator* Æ Q paſſes through the *Zenith* and *Nadir*; the *Poles* of the *World* are exactly in the *Horizon*; and all the *Parallels* of *Declination* are cut into two equal Parts, whence the *Days* and *Nights* are here always equal; alſo all the *Stars* riſe and ſet, and their *Diurnal* and *Nocturnal* Arches, or, the time that they appear above the *Horizon*, and are hid below it, is always equal.

The *Fig.* 2. repreſents a *parallel Poſition* of the *Sphere*; here the *Equinoctial* co-incides with the *Horizon*, and the *Poles* of the *World* are one in the *Zenith*, and the other in the *Nadir*: All the *Parallels of Declination* are parallel to the *Horizon*, (and conſequently none of them interſect it;) wherefore all the *Stars*, whoſe Declinations are towards the *elevated Pole*, always appear above the *Horizon*, and at the ſame Height; and thoſe whoſe Declinations are towards the contrary *Pole*, are always hid below it: but the *Sun* and the *Planets*, by reaſon of the Obliquity of their *Annual* Motion, to the *Diurnal* Motion of the *Earth*, are during one half of their *Periods* above the *Horizon*, and for the other half below it; their greateſt Altitude and Depreſſion being equal to the Angle which their reſpective *Orbits* make with the *Equator*.

The *Figures* 3, 4, and 5, repreſent the *Sphere* projected in different Latitudes, and in all theſe the *Equator* and its *Parallels* make Oblique Angles with the *Horizon*; whence theſe Poſitions are called *Oblique Spheres*.

The *Fig.* 3. repreſents the *Sphere* projected for the Latitude of 23° 29′ North, (equal to the Sun's greateſt Declination:) here the *Parallels* of *Declination* are all unequally cut by the *Horizon*, the *Diurnal* Arches of

thoſe towards the elevated Pole continually increaſing, until the Declination becomes equal to the Complement of the Latitude, (=66° 31′) equal to the Diſtance of the *Arctick Circle* from the *Equator*; which Circle touches the *Horizon* in the Point O, and is therefore the *Circle* of *Perpetual Apparition* in this Latitude, (becauſe all the Parallels betwixt it and the *Pole* are wholly above the *Horizon*:) The *Diurnal Arches* of thoſe Parallels that are towards the depreſſed Pole, do likewiſe decreaſe in the ſame Proportion that thoſe Arches of the oppoſite Parallels increaſe, until the Declination becomes equal to the Complement of the Latitude, where that Parallel will juſt touch the *Horizon*; which Parallel is here the *Antarctick Circle*, and therefore is the *Circle* of *Perpetual Occultation*. Here it may be obſerved, that by the Earth's Rotation round its Axis, the *Solſtitial* Point ♋ paſſes through the *Zenith*, and ♑ through the *Nadir*.

The farther any Place is from the *Equator*, the *Diurnal Arches* of the Parallels towards the elevated Pole increaſe, and of thoſe towards the depreſſed Pole decreaſe, the faſter; for the *Circles* of *Perpetual Apparitions* and *Occultations* are always at the ſame Diſtance from the *Equator*, that the Place is from the *Pole*: Thus, in *Fig.* 5. which is conſtructed for the Latitude of 51° 30′ N, the *Circles* of *Perpetual Apparitions* and *Occultations* are the Parallels D O, H D of 38° 30′ of Declination, North and South. Again, in *Fig.* 4. which is projected for the Latitude of 66° 31′ N, the two *Tropicks* juſt touch the Horizon; ſo that when the Sun is in the *Summer Solſtice*, he juſt touches the Horizon without Setting; and again, when he is in the *Winter Solſtice* he once appears in the *Horizon* without Riſing.

From the preceding *Definitions* are deduced the following *Corollaries*.

1. *All Places of the Earth do equally enjoy the benefit of the Sun in reſpect of Time, and are equally deprived of it.*

2. *In all Places of the Earth, ſave exactly under the Poles, the Days and Nights are of equal Length,* (viz. 12 *Hours each*) *when the Sun is in the Equinoctial.*

3. *In all Places between the Equinoctial and the Poles, the Days and Nights are never equal but when the Sun is in the Equinoctial Points* ♈ *and* ♎.

4. *The nearer any Place is to the Equator, the leſs is the Difference betwixt the Length of the artificial Days and Nights in the ſaid Place; and the more remote the greater.*

5. *To all the Inhabitants lying under the ſame Parallel of Latitude, the Days and Nights are of equal Lengths, and that at all times of the Year.*

6. *The Sun is Vertical twice a Year to all Places between the Tropicks; to thoſe under the Tropicks once a Year; but never any where elſe.*

7. *In all Places between the Polar Circles and the Poles, the Sun appeareth ſome number of Days without Setting, and at the oppoſite time of the Year he is for the ſame length of time without Riſing; and the nearer unto, or farther*

from the Pole thoſe Places are, the longer or ſhorter is his continued Preſence in, or Abſence from, the ſame. And in all Places lying exactly under the Polar Circles, the Sun at his greateſt Declination appears for one Day without Setting, and diſappeareth at another: but at all other times of the Year he riſes and ſets there as in other Places.

8. *In all Places lying in the* {*Northern* / *Southern*} *Hemiſphere, the longeſt Day and ſhorteſt Night, is when the Sun hath the greateſt* {*Northern* / *Southern*} *Declination*; and the contrary.

SECT. II.

Aſtronomical Problems.

Plate 10. *Fig.* 5

LET B be the *Sun's Place* in (the firſt Quadrant of) the *Ecliptick* at any time; through which let there be drawn the *Meridian* P B S, and alſo the *Parallel* of *Declination* d B e; then will ⊙A be his *Right Aſcenſion*, and A B (=Æ *d*) his *Declination*, at the ſame time. Again, ſuppoſing the *Latitude* to be fixed as before, *viz.* 51° 30′ N: the Arch *h m* will be the *Sun's Altitude* at the *Hour* of *ſix*, and ⊙*h* his *Azimuth* from the *Prime Vertical* at the ſame time; ⊙ *c* will be the *Sun's Altitude* when he is upon the *Prime Vertical*, and the ∠ ⊙P *b*, which the *Meridian* P *b* S makes with the *Prime Vertical*, the time from *Six*, when he appears due *Eaſt* or *Weſt*; (the Complement of which to ſix Hours will be the time from *Noon*.) Again, ſuppoſing the *Latitude* and the *Sun's Declination* to be the ſame as before, ⊙*n* will be the *Sun's Amplitude*; the ∠ *n* P O the *Time* from *Mid-night* when he riſes and ſets, the Complement whereof the Arch ⊙*a* will be the *Aſcenſional Difference*. Theſe things being premiſed, we proceed to,

PROB. I.

Of theſe four, *viz.* 1. The Sun's greateſt Declination, 2. The Longitude of the Sun from the next Equinoctial Point, 3. The Right Aſcenſion of the Sun from the ſame Equinoctial Point, and 4. The Declination of the Sun in that Place; any two being given, we may find the reſt.

CASE

CASE I.

The Sun's greateſt Declination, and his Place in the Ecliptick, being given; to find his Preſent Declination, and Right Aſcenſion.

Examp. In the Spherical Triangle ☉AB Right ∠ᵈ at A, are given; the Side ☉B = 50° 00′, (the *Sun's Place* in 20° of ♉) and the ∠A☉B, (the Sun's *greateſt Declination*) = 23° 29′; to find ☉A his *Right Aſcenſion*, from the *Equinoctial Point* ♈, and the Side AB his *Preſent Declination*.

1. To find the *Right Aſcenſion*, it will be,

	10.
As the Radius	
To the Co-Sine of the Sun's greateſt Declination = 23° 29′	9.962453
So the Tangent of the Sun's Longitude from ♈ = 50° 00′	10.076186
To the Tangent of the Right Aſcenſion = 47° 32′	10.038639

N. B. When the Sun is in the {1, 2, 3, 4} Quadrant of the *Ecliptick*, the *Right Aſcenſion* found as above {without Alteration; ſubſtracted from 180°; added to 180°; ſubſtracted from 360} is the true *Right Aſcenſion* from the beginning of ♈.

That is, if the Sun had been in {10, 20, 10} Deg. of {♌, ♎, ♐} his Right Aſcenſion would have been {132° 28′, 227 32, 312 28} from the beginning of ♈.

2. To find the *Preſent Declination*, it will be,

	10.
As the Radius	
To the Sine of the Sun's Longitude from ♈ = 50° 00′	9.884254
So the Sine of the Greateſt Declination = 23° 29′	9.600409
To the Sine of the Preſent Declination = 17° 46′ 24″.6	9.484663

Which is *North*, becauſe the Sun is in the *Northern Signs*.

N. B.

N. B. Whenever the Sun is at the ſame Diſtance from the *Equinoctial Points* ♈ or ♎, the Quantity of *Declination* will be the ſame, but may be of different Denominations, according as the Sun is either in the *Northern or Southern Signs*; ſo that the *Declination* before found 17° 46′ 24″.6 is common in the { *Northern Signs*, to ♉ 20° 00′, and ♌ 10° 00′. / *Southern Signs*, to ♍ 20° 00′, and ♒ 10° 00′.

The *Sun's Declination* may be found to a *Minute* of a *Degree*, by the *Table* of the Sun's Declination annexed to this Book; wherefore, I ſhall next ſhew how from the Sun's Declination, to find his *Place* and *Right Aſcenſion.*

CASE 2.

The Sun's Greateſt Declination or Obliquity of the Ecliptick, and his Preſent Declination, being given; to find his Longitude, and Right Aſcenſion.

Examp. Let the Sun's *greateſt Declination* be (as before fixed) 23° 29′, and his *preſent Declination* 17° 46′ 24″.6 N, and increaſing, and let it be required to find his *Longitude* and *Right Aſcenſion* from the firſt Point of ♈.

1. To find the *Sun's Place*, the Proportion will be,

S. ∠ A ⊙ B (23° 29′) : S. A B (17° 46′ .41) :: R : S. ⊙ B (50° 00′) } the Long. from ♈, or ♉ 20°.

It has been ſhewed before, that the ſame Quantity of *Declination* correſponds to four Points in the *Ecliptick*; wherefore, if the Sun's *Declination* had been 17° 46′ .41 { *North*, decreaſing, / *South*, increaſing, / *South*, decreaſing, } the Sun's Place would have been { ♌ 10° / ♍ 20° / ♒ 10. }

2. To find the *Right Aſcenſion*, it will be,

T. ∠ A ⊙ B (23° 29′) : T. A B (17° 46′ .41) :: R : S. ⊙ A (47° 32′) } the Right Aſcenſion.

Now becauſe the Declination is North, and increaſing, the Sun is in the firſt Quadrant of the *Ecliptick*, and therefore the Arch laſt found 47° 32′, is his *Right Aſcenſion* from ♈.

If

If the Sun's Declination had been 17° 46′.41 { *North*, decreaſing, / *South*, increaſing, / *South*, decreaſing, } the Sun's *Right Aſcenſion* would have been { 132° 28′ / 227 32 / 312 28 } from the beginning of ♈.

If the ∠ A B ⊙ which the *Ecliptick* makes with the *Meridian*, (or Circle of Right Aſcenſion paſſing through the Sun) be made a part of the *Data*, this Problem will admit of 12 more *Caſes*; all which, as alſo any Caſe in Right-Angled Spherical Triangles, may be ſolved by the following *Table* of Proportions.

Given.	Reqd.	Proportions.	Caſe.	Prob.
A ⊙ B, ⊙ B	⊙ A	R : C.S.⊙ :: T.⊙B : T.⊙A.	1	I
	A B	R : S.⊙B :: S.⊙ : S.AB.	2	
	B	R : C.S.⊙B :: T.⊙ : C.T.B.	3	
⊙ B, A B	⊙	S.⊙B : S.AB :: R : S.⊙.	4	II
	⊙ A	C.S.AB : R :: C.S.⊙B : C.S.⊙A.	5	
	B	T.⊙B : R :: T.AB : C.S.B.	6	
A B, ⊙	⊙ B	S.⊙ : S.AB :: R : S.⊙B.	7	III
	⊙ A	T.⊙ : R :: T.AB : S.⊙A.	8	
	B	C.S.AB : R :: C.S.⊙ : S.B.	9	

Given.	Reqd.	Proportions.	Caſe.	Prob.
⊙ A, ⊙	⊙ B	R : C.S.⊙ :: C.T.⊙A : C.T.⊙B	10	IV
	A B	R : S.⊙A :: T.⊙ : T.AB.	11	
	B	R : S.⊙ :: C.S.⊙A : C.S.B.	12	
⊙ A, A B	⊙ B	R : C.S.⊙A :: C.S.AC : C.S.⊙B	13	V
	⊙ B	S.⊙A : R :: T.AB : T.⊙.	14	
		S.AB : R : T.⊙A :: T.B.		
⊙, B	⊙ B	T.⊙ : R :: C.T.B : C.S.⊙B.	15	VI
	⊙ A A B	S.⊙ : R :: C.S.B : C.S.⊙A. S.B : R :: C.S.⊙ : C.S.AB.	16	

PROB. II.

In the Right ∠ᵈ Spherical △ P O *n*, where, 1. P O is the *Poles* Height, or *Latitude* of the Place; 2. *n* O the Complement of the *Amplitude*; 3. P *n* the Complement of the *Sun's Declination*, 4. The ∠ *n* P O the Complement of the *Aſcenſional Difference*; and, 5. P *n* O the ∠ which the *Meridian* P *n* makes with the *Horizon*; any two of theſe being given, the reſt may be found.

CASE I.

The Latitude of the Place, and the Sun's Declination, being given; to find his Amplitude and Aſcenſional Difference, and thence the time of the Sun's Riſing and Setting, and conſequently the Length of the Day and Night.

Examp. In the Latitude of 51° 30′ N, the Sun having 17° 46′.41 of North Declination; I demand his *Amplitude*, *Aſcenſional Difference*, Time of *Riſing* and *Setting*, &c.

1. To

1. To find the *Amplitude*, it will be,

As the Co-Sine of the Latitude 51° 30′	9.794150
To the Radius:	10.
So the Sine of the Declination 17° 46′.41	9.484663
To the Sine of the Amplitude 29° 22′ 50″	9.690513

Which is Northerly, becauſe the Sun's Declination is North, (for the *Sun's Declination* and *Amplitude* are always the ſame way, and when the Sun is in the *Equinoctial* he riſes due Eaſt, and ſets due Weſt) and conſequently the Sun riſes 29° 23′ to the *Northward* of the *Eaſt* Point of the *Horizon*, or E N E $\frac{1}{2}$ N 1° $\frac{1}{4}$ N^{ly}, and ſets 29° 23′ to the *Northward* of the *Weſt*, or W N W $\frac{1}{2}$ W 1° $\frac{1}{4}$ N^{ly}. If the Declination had been 17° 46′.41 S, (the Latitude being the ſame as before) the *Amplitude* at Sun-riſing would have been E 29° 23′ S^{ly}, and at Sun-ſetting W 29° 23′ S^{ly}.

After the ſame manner may the *Amplitude* of any *Star*, (that may riſe and ſet) whoſe Declination is known, be found.

2. To find the *Aſcenſional Difference*, it will be,

As the Co-Tangent of the Latitude 51° 30′	9.900605
To the Radius:	10.
So the Tangent of the Declination 17° 46′ 24″.6	9.505924
To the Sine of the Aſcenſional Difference 23° 46′	9.605319

After the ſame manner may the Aſcenſional Difference of any *Star* that may riſe and ſet, be found; and to find what Stars do not riſe or ſet, obſerve the following Rule.

If the *Latitude* {North / South} be greater than the Complement of the *Star's Declination* {North, / South,} the Star never ſetteth, but appears conſtantly above the *Horizon*: On the contrary, if the Latitude {North / South} be greater than the Complement of the *Star's Declination* {South / North} the Star never riſeth, but is always hid below the *Horizon*.

When the Sun's Aſcenſional Difference is known, the time of the Sun's Riſing and Setting, and conſequently the Length of the Day and Night may be found as followeth:

1. If the *Latitude*, and the *Sun's Declination* are of {the ſame / contrary} Denomination,

mination, the *Aſcenſional Diff.* converted into *Time*, and {ſubſtracted from / added to} ſix o'th' Clock, gives the Time of the *Sun's Riſing*.

2. If the *Latitude* and *Declination* are of {one / diff.} kind, the *Aſcenſional Difference* {added to / ſubſtracted from} ſix o'th' Clock, gives the Time of the *Sun's Setting*.

Thus, in the Example above, becauſe the Latitude and Declination are both North, if the Aſcenſional Difference 23° 46′ be converted into Time, allowing 15 deg. to an Hour, and 15 min. of a Degree to a min. of Time, *&c.* the Aſcenſional Difference will be 1 Hour, 35 Min. and 4 Seconds; wherefore this being {ſubſtracted from / added to} ſix, gives {$4^h\ 24^m\ 56^f$ / $7\ 35\ 04$} the Time of the Sun's {Riſing / Setting}.

And if the Declination had been 17° 46′.41 South, (the Latitude being the ſame as before,) the Time of the Sun's Riſing would have been at 35 Min. and 4 Seconds paſt *Seven*, and Setting at 24 Min. and 56 Seconds after *Four*.

Now, becauſe the Hour of Sun-ſetting ſhews the Quantity of Time elapſed from Noon, and the end of the Day, and which is therefore equal to half the length of the Day; if the Hour of Sun-ſetting be doubled, it gives the length of the Day, or the number of Hours the Sun is viſible above the Horizon.

Thus, in the *Example* above, where the Hour of Sun-ſetting was $7^h\ 35^m\ 04^f$, if this be doubled, the Product $15^h\ 10^m\ 08^f$ is the length of the Day.

Again, the time of the Sun's Riſing being doubled, gives the *Nocturnal Arch*, or the length of the Night. Thus, in the *Example* above, the length of the Night is $8^h\ 49^m\ 52^f$.

When the *Right Aſcenſion* and *Aſcenſional Difference* are both known, the *Oblique Aſcenſion* and *Deſcenſion* may be found as followeth:

1. If the Elevation of the Pole and Declination be both North, or both South, the {Differ. / Sum} of the *Right Aſcenſion*, and the *Aſcenſional Difference*, is the *Oblique* {*Aſcenſion* / *Deſcenſion*} required.

2. If the Elevation of the Pole and Declination be one North and the other South, the {Sum / Differ.} of the *Right Aſcenſion*, and *Aſcenſional Difference*, is the *Oblique* {*Aſcenſion* / *Deſcenſion*} required.

CASE 2.

Having the Sun's Amplitude and Declination, given; to find the Latitude of the Place, and the Aſcenſional Difference.

1. To find the *Latitude*, it will be,

As the Sine of the Amplitude 29° 21′ 50″	9.690513
Is to the Radius:	10.
So is the Sine of the Declination 17° 46′.41 N	9.484663
To the Co-Sine of the Latitude 51° 30′	9.794150

Which is *North*, if the Sun riſes before, (or ſets after,) the Hour of *Six*; otherwiſe the contrary.

2. To find the *Aſcenſional Difference*, it will be,

As the Co-Sine of the Declination 17° 46′.41	9.978763
To the Co-Sine of the Amplitude 29° 21′ 50″	9.940279
So is the Radius	10.
To the Co-Sine of the Aſcenſional Diff. 23° 46′	9.961516

Whence the Time of the Sun's Riſing and Setting, *&c.* may be found, as above.

CASE 3.

The Sun's Declination, and the Time of his Riſing or Setting, being given; to find the Latitude of the Place.

Note, The Time of Sun-riſing or ſetting being given, his *Aſcenſional Difference* may be found by reducing the Time from *ſix* into *Degrees* and *Minutes*. Thus, admitting the *Sun* riſes at $4^h\ 24^m\ 56^f$, this ſubſtracted from 6 Hours, gives $1^h\ 35^m\ 04^f$ the *Aſcenſional Difference* in *Time*; which being reduced is 23° 46′.

Examp. Admit the *Sun's Declination* to be 17° 46′ N, and I obſerve the Sun to riſe at $24^m\ 56^f$ after *Four*; and it be required to find the *Latitude*.

1. By the Time of the *Sun's Riſing* find the *Aſcenſional Difference* as above, *viz.* 23° 46′, then it will be,

As the Sine of the Aſcenſional Diff. 23° 46′	9.605319
To the Radius:	10.
So the Tangent of the Declination 17° 46′.41	9.505924
To the Co-Tangent of the Latitude 51° 30′	9.900605

Which,

Which, becauſe the Sun riſes before ſix, and the Declination is North, the *Latitude* is alſo North.

If the *Hypothenuſe* P *n* of this △ (*n* P O) be called ⊙ B, and the *Legs* O P, O *n* be called ⊙ A, A B; the remaining *Caſes* of this *Problem* may be all wrought by the *Table* of Proportions exhibited under *Prob.* I. And in the right ∠ᵈ △ ⊙ *h m*, right ∠ᵈ at *h*; (where ⊙ *m* is the Sun's *Declination*, *h m* the Sun's *Altitude* at *Six*, ⊙ *h* his *Azimuth* from the *Prime Vertical* at the ſame time, the ∠ *h* ⊙ *m* the *Latitude*, and ⊙ *m h* the ∠ which the *Vertical Circle* paſſing thro' the Sun makes with the *Six o'-Clock* Hour-Circle; if the *Hypothenuſe* ⊙ *m* be called ⊙ B, and the *Legs* ⊙ *h*, *h m* be called ⊙ A, A B;) any two of the fore-mentioned Parts being given, the reſt may be found by the foreſaid *Table* of *Proportions:* Alſo in the right ∠ᵈ Sph. △ ⊙ *b c*, right ∠ᵈ at *b*; (where *b c* is the Sun's *Declination*, ⊙ *c* his *Altitude* when he is upon the *Prime Vertical*, ⊙ *b* the *Time* then from *Six*, the ∠ *b* ⊙ *c* the *Latitude*, and ⊙ *c b* the ∠ which the *Hour-Circle* paſſing through the *Sun* makes with the *Prime Vertical*;) if the *Hypothenuſe* ⊙ *c* be called ⊙ B, and the *Legs* ⊙ *b*, *b c* be called ⊙ A, A B; any two of the *Parts* of this △ being given, the reſt may be found by the Table of Proportions in Page 159: But theſe two laſt *Problems* being of no immediate Uſe in *Navigation*, we ſhall here paſs them by without any Examples.

PROB. III.

In the Oblique ∠ᵈ Spherical △ P B Z, where, 1. P Z is the Complement of the *Latitude*, 2. P B the Complement of the *Declination*, 3. Z B the Complement of the *Altitude*, 4. The ∠ Z P B the *Hour* from the *Meridian*, 5. The ∠ P Z B the *Azimuth* from the North-part of the Meridian, and 6. Z B P the ∠ which the *Vertical* and *Hour-Circles* paſſing through the Sun (or Star) make with each other: any three of theſe being given, the reſt may be found.

CASE I.

The Latitude of the Place, the Declination of the Sun, and his Altitude, being given; to find the Hour of the Day, and Azimuth.

Examp. Admit in the *Latitude* of 51° 30′ *North*, the Sun's *Declination* being 17° 46′ N, his *Altitude* was found to be 42° 33′; I demand the *Hour* of the *Day*, and the *Azimuth* of the *Sun*.

1. To find the *Hour* of the *Day*.

In the Δ PZB are given ZP = 38° 30′, (the *Co-Lat.*) PB = 72° 14′, (the *Co-Declination*) and ZB = 47° 27′, (the *Comp. of the Altitude*) to find the ∠ P.

The moſt compendious way of ſolving this *Caſe*, is as followeth;

Let the Side ZB oppoſite to the required ∠ P, be called the *Baſe*, and the two containing Sides PZ, PB be called the Legs; then, *To the Arithmetical Complements of the Logarithms of the Sines of the Legs, add the Logarithm-Sines of the Differences of the ſaid Legs from the half Sum of the three Sides; half the Sum of theſe 4 Numbers will be the Logarithm of the Sine of half the Angle ſought.*

EXAMPLE,

Baſe ZB = 47° 27′

Legs { ZP = 38 30 —— *Arith. Compl. Sine* 0̄.205850
PB = 72 14 —— *Arith. Compl. Sine* 0̄.021223

Sum 158.11 { *Diff. of the Legs* { 40° 35′½ — *Sine* 9.813356
½ *Sum* 79.05½ { *from the* ½ *Sum* { 06 51½ — *Sine* 9.077058

Sum 19.117487
Sine of 21° 14′ —— *half* 9.558743
Doubled = 42° 28′ = 2ʰ 49ᵐ 52ˢ.

Whence, if it be in the Forenoon, the Time is 10ᵐ 08ˢ paſt IX. but if it be in the Afternoon, it is 49ᵐ 52ˢ after H. o'th' Clock.

2. To find the *Azimuth*, the Solution is after the ſame manner, only you are to note, that PB is now the *Baſe*; wherefore,

Baſe PB = 72° 14′

Legs { ZB = 47 27 —— *Arith. Comp. Sine* 0̄.132717
ZP = 38 30 —— *Arith. Comp. Sine* 0.205850

Sum 158 11 } *Diff. of the Legs* { 31° 38′½ — *Sine* 9.719832
½ *Sum* 79 05½ } *from the* ½ *Sum* { 40 35½ — *Sine* 9.813356

Sum 19.871755
Sine of 59° 37′ —— *half* 9.935877

The Double thereof = 119° 14′ is the *Azimuth* from the *North*, the Supplement of which = 60° 46′ is the *Azimuth* from the *South*-part of the *Meridian*.

After the ſame manner may the *Azimuth* of any *Star*, whoſe Declination is known, and the time it wants of coming upon, or how much it is paſt the *Meridian*, be found.

Note, 1. If the Latitude and Declination had been both South, and of the ſame Quantity as above, the Hour and Azimuth would have been juſt the ſame.

Note,

Note, 2. When the Latitude is {North, South,} and the Declination {South, North,} you muſt add 90 Degrees to the Declination, and call that Sum the Complement of the Declination, then you are to proceed in all reſpects as above.

Case 2.

The Altitude, Declination, and Azimuth of the Sun, or any known Star, being given; to find the Latitude of the Place, and the Hour of the Day.

Examp. In the *Oblique* ∠ᵈ *Spherical* △ Z P B are given Z B (the Complement of the *Altitude*) = 47° 27′, P B (the Complement of the *Declination*) = 72° 14′, and the ∠ B Z P (the *Azimuth* from the *North*-part of the *Meridian*) = 119° 14′; to find Z P the Complement of the *Latitude*, and the ∠ Z P B the *Hour* from *Noon*.

1. To find the *Latitude*, it will be,

1. *As the Radius*	10.
To the Co-S. of the Azimuth (PZB) = 119° 14′	9.688747
So is the Tang. of Z B (*the Co-Alt.*) = 47° 27′	10.037187
To the Tang. of (Z ☋) *a 4th Arch* 28° 01′	9.725954
2. *As the Co-Sine of* Z B = 47° 27′	$\bar{0}$.169904
Is to the Co-Sine of P B = 72° 14′	9.484501
So is the Co-S. of the 4th Arch (Z ☋) = 28° 01′	9.945808
To the Co-S. of a 5th Arch (P ☋) = 66° 31′	9.600213

But P ☋ 66° 31′ — Z ☋ 28° 01′ = P Z 38° 30′, the Complement of the *Latitude* required.

2. To find the ∠ P the *Hour* from *Noon*, it will be,

S. P B : S. Z B : : S. P Z B : Z P B; that is,

As the Co-Sine of the Declination 17° 46′	$\bar{0}$.021223
To the Co-Sine of the Altitude 42 33	9.867283
So is the Sine of the Azimuth 119° 14′	9.940835
To the Sine of the Hour from Noon 42° 28′	9.829341

Which

Which being reduced into *Time* is $2^h\ 49^m\ 52^s$; whence the *Time* is either $10^m\ 8^s$ paſt *Nine o'-Clock* in the *Morning*, or $49^m\ 52^s$ paſt *Two* in the *Afternoon*.

Case 3.

If the Declination, Altitude, and the Hour of the Day, were given; to find the Latitude and Azimuth.

1. To find the *Latitude*, it will be,

R : C.S.Z P B :: T.P B : T.P ♋; then again,
C.S.P B : C.S.Z B :: C.S.P ♋ : C.S.Z ♋ : then P ♋ — Z ♋ = P Z the Complement of the *Latitude* required.

2. To find the *Azimuth*, it will be,

S.Z B : S.P B :: S.Z P B : S.P Z B, the *Azimuth* from the *North*-part of the *Meridian*.

There may be more *Caſes* propoſed under this *Problem*, but thoſe that have been here exhibited are the moſt uſeful.

If it be required to find when the *Twilight* begins and ends, (which by Obſervation is found to be when the *Sun* is 18 Deg. below the *Horizon*) the *Caſe* will be directly the ſame with *Caſe* 1. of the foregoing *Problem*.

Examp. Let it be required to find when the *Twilight* begins and ends in the *Latitude* of 51° 30′ N, when the *Sun's Declination* is 17° 46′ N.

In the △ Z P *t* are given Z P = 38° 30′, (the Complement of the *Latitude*) P *t* = 72° 14′, (the Complement of the *Declination*) and Z *t* = 108° 00′, (= 90° + 18° the Depreſſion below the *Horizon*;) to find the ∠ P the *Hour* from *Noon*.

Baſe Z *t* = 108° 00′

Legs { P *t* = 72 14 ——— *Arith. Comp. Sine* 0.021223
{ Z P = 38 30 ——— *Arith. Comp. Sine* 0.205850

Sum = 218 44 } *Diff. of Legs* { 37° 08′ — *Sine* 9.780801
½ *Sum* = 109 22 } *from the* ½ *Sum* { 70 52 — *Sine* 9.975321

Sum 19.983195
The Sine of 78° 46′ ½ *Sum* 9.991597

The Double thereof = 157 32 is the ∠ P = $10^h\ 30^m\ 08^s$, the *Time* when *Twilight* ends in the *Evening*, the Complement of which = $1^h\ 29^m\ 52^s$ is the *Time* when *Twilight* begins in the *Morning*.

Here

Here you may obſerve, that when the Sum of the Complements of the Latitude and Declination is leſs than the *Baſe* Z *t* (108°,) there is no total *Darkneſs*; and it will be more or leſs *dark*, in proportion as the *Baſe* Z *t* is greater or leſs than the Sum of the *Legs* Z P, P *t*.

I ſhall next ſhew the Uſe of the *Tables* of the *Sun's Declination*, *Right Aſcenſion*, and the *Catalogue* of the *Fixed Stars*, which are annexed to this Book.

I. *The Uſe of the Table of the Sun's Declination.*

This Table ſhews the Sun's Declination every Day at *Noon* under the Meridian of *London*, for the firſt Year mentioned at the top, and without ſenſible Error, will ſerve for each of the other two; and indeed for common uſe it may indifferently ſerve for 8 or 12 Years longer, by adding 4, 8, or 12 Years to the laſt Year mentioned in the reſpective Pages.

To find the Sun's Declination, look the Year and Month in the Head of the Table, and the Day of the Month in the left or right-hand Column, and in the common Angle of Meeting you'll have the Sun's Declination that *Day* at *Noon*, under the *Meridian* of *London*.

Examp. 1. Admit it be required to find the Sun's Declination the 10th of *October* at *Noon*, Anno 1729, under the *Meridian* of *London*.

I find the Year 1729 in the head of the Table, and right under *Octob.* and againſt 10 *Days* in the left-hand Column, I find 10° 48′ *South*, the *Sun's Declination* required. This Number 10° 48′ S. may be alſo taken for the Sun's Declination on the ſame Month and Day in the Years 1733, 1737, 1741; the Error not amounting to 2 Minutes.

Examp. 2. Let it be required to find the *Sun's Declination* at *Noon Auguſt* 20, *Anno* 1732, under the Meridian of *London*.

Having turned to the Year 1732 in the head of the Table, right under *Aug.* and againſt 20 *Days* in the left-hand Column, I find 8° 31′ *North*, the *Sun's Declination* for the *Time* and *Place* propoſed. The ſaid Number 8° 31′ N may be alſo taken for the Sun's Declination on the ſame Month and Day, in the Years 1736, 1740, for the reaſons given above.

But if you are far Eaſtward or Weſtward from the *Meridian* of *London*, (for which theſe *Tables* are calculated,) you muſt make a proportional Allowance, making the Declination ſomewhat greater or leſs than what is given in the *Tables*; for if you be to the *Weſtward*, the Sun coming to your Meridian after he has paſt the Meridian of the Tables, the Sun's Declination will be ſomewhat increaſed or diminiſhed, according as he then either recedes from, or approaches nearer to, the *Equator*; and if you be to the *Eaſtward*, the contrary.

In order to find the Sun's Declination in any Place, either to the Eaſtward or Weſtward from *London*, obſerve the two following Rules:

1. *If the Longitude be Eaſterly, and the Declination increaſing, the proportional Parts (found in the little Table at the bottom*

bottom of the right-hand Page of the Table of the Sun's Declination) muſt be ſubſtracted; but the Declination decreaſing, it muſt be added.

2. *If the Longitude be Weſterly, and the Declination increaſing, the proportional Minutes above-mentioned muſt be added to the Declination found in the Table, but the Declination decreaſing it muſt be ſubſtracted.*

Examp. 1. Admit it be required to find the *Sun's Declination* at *Noon January* 20*th*, 1730, at a Place lying 105 Deg. of Longitude to the *Eaſtward* of *London*.

The Declination for the time given under the *Meridian* of *London*, is 17° 22′ South *decreaſing*, and the daily Difference of Declination at that time, I find (in the little Col. marked *Diff.*) to be 17′; wherefore, I look in the Head of the Table for adjuſting the Sun's Declination, for 105 Deg. and in that Column right againſt 18′ (which is the neareſt Numb. to 17′) I find 5′; which becauſe the Longitude is *Eaſterly*, and the Declination *decreaſes*, I add to the Number before found 17° 22′, and the *Sum* 17° 27′ S. is the true *Declination* ſought: But if the Declination *increaſed*, the Number before found muſt have been ſubſtracted, and the Reſidue 17° 17′ would have been the true *Declination* ſought.

Examp. 2. *March* 20th, 1731, being 175 Deg. to the Weſtward of the Meridian of *London*, I would know the Sun's true *Declination* then at *Noon*.

The *Declination* for the time given at *Noon* under the *Meridian* of *London*, I find to be 4° 04′ N, (increaſing) and the daily Diff. to be 23′; then looking in the fore-mentioned little *Table*, for the neareſt *Longitude* to 175°, which is 180′, and for the neareſt daily Diff. to 23′, which is 24′, I find the proportional Part to be 12′, which becauſe the Longitude is *Weſterly*, and the Declination *increaſing*, muſt be added to 4° 04′, and the Sum 4° 16′ N is the true *Declination* ſought: But if the *Longitude* had been 175° W^{ly}, as before, and the *Declination* 4° 4′ *decreaſing*, then the proportional Parts muſt have been ſubſtracted, and the Reſidue 3° 52′ N would have been the true Declination required.

Examp. 3.	*The Declin. in the Meridian of* London	20° 20′ N	*increaſing*
	The proportional Minutes	00 4	
	The Declin. for the Longitude 120 E^{ly}	20 16 N	
	The Declin. for the Longitude 120 W^{ly}	20 24 N	
Examp. 4.	*The Declin. for the Meridian of* London	20° 20′	*decreaſing.*
	The proportional Minutes	00 4	
	The Declin. for the Longitude 120 E^{ly}	20 24	
	The Declin. for the Longitude 120 W^{ly}	20 16	

N. B.

N. B. If the *Longitude*, either Eaſterly or Weſterly, exceed 180 Deg. it muſt be ſubſtracted from 360 Deg. and the Reſidue will be the neareſt Longitude from the given Place, of a contrary Denomination to what it was before; and then the proportional Parts being found, muſt be either added or ſubſtracted, according to which of the preceding Rules the Caſe falls under: Thus, if the Longitude from *London* be reckoned 220 Deg. W^ly, the neareſt Longitude from *London* will be 140 Deg. E^ly, and the proportional Parts muſt accordingly be managed by the preceding Rules.

What has been here ſaid concerning finding the Sun's Declination at *Noon*, under a different Meridian from that of *London*, the ſame muſt be underſtood if the Sun's Declination be required at any other time of the Day, than when the Sun is upon the Meridian.

Having found the Sun's Declination at *Noon* under the *Meridian* of the given Place; obſerve the two following Rules:

1. *If the Time be in the Forenoon, and the Declination increaſing, ſubſtract the proportional Parts found as before in the Table, (allowing 15° of Longitude for every Hour from Noon) and the Reſidue will be the true Declination ſought: but the Declination decreaſing, add the proportional Parts, and the Sum will be the Declination required.*

2. *If the Time be in the Afternoon, and the Declination increaſing, add the proportional Parts: but if the Declination be decreaſing, ſubſtract, and the Sum or Difference will be the Declination ſought.*

This being in effect illuſtrated before, needs no Example here.

Note, If for every Hour the Sun wants of coming to the Meridian, you reckon 15° Eaſterly, and for every Hour that he is paſt the Meridian, you reckon 15° Weſterly; the Rules for finding the Sun's Declination at any Hour of the Day, (the Declination at *Noon* in the given Place being firſt found) will be the ſame with thoſe delivered, for finding the Sun's Declination at Noon in any Longitude from *London*.

Note alſo, that the Time counted from Noon muſt not exceed 12 Hours; thus, if the Time given be 1 o'th' Clock in the Morning, you are to reckon 11 Hours before the Noon of the following Day, and make your Allowance accordingly; and not reckon 13 Hours paſt the Noon of the preceding Day.

* *That the common Neglect of not allowing for the daily Diff. of the Sun's Decl. may make an Error of 12 Min. in the Lat. and alſo 24 Minutes difference in the Lat. of the ſame Place taken at different times.*

* From what has been ſhewed above, it appears, that the Neglect of allowing for the Difference of Longitude, and the daily Difference of the Sun's Declination, may make an Error of 12 Minutes in the *Latitude*, ſuppoſing the Sun's Meridian Altitude to be exactly taken: Alſo, the ſame Neglect may ſometimes make 24 Minutes difference in the Latitude of the ſame Place. For *Examp.* In the Latitude of 51° 30′ North, when the Sun's Declination is 3° 00′ $\left\{\begin{matrix}North,\\South,\end{matrix}\right\}$ the *Meridional Zenith Diſtance* is $\left\{\begin{matrix}48° 30′\\54\ \ 30.\end{matrix}\right\}$ But if a Perſon ſhould be in the ſame Latitude as before, and 180 Deg. to the Weſtward of the former Place, when the Sun's Declination

tion is 3° 00′ {North / South} *increaſing*, and the *daily Difference* is 24′, the *Meridional Zenith Diſtance* will be {48° 18′ / 54 42,} and conſequently, (without making a proportional Allowance for the Sun's Declination,) in the former Caſe he'll make his *Latitude* to be 51° 18′, and in the latter 51° 42′; thereby making 24 Min. difference in the *Latitude* of the ſame Place, though we ſuppoſe the *Obſervation* to be accurately made at both times. This I ſhewed as a Caution for *Mariners*, becauſe theſe things are not commonly (if ever) regarded: and altho' an Obſervation cannot be taken at Sea, to any great Exactneſs; yet in all Caſes we ought to endeavour to approach the Truth as near as poſſible, by correcting all Errors that we know to be ſuch.

II. *The Uſe of the Table of the Sun's Place and Right Aſcenſion.*

The chief Deſign of this *Table* being in order to find the time of *Culmination* of the *Fixed Stars*, it is here only ſet down for one *Year*, being a Mean between the Extremes of the Difference ariſing on the account of the 6 Hours, which is continually to be added at the end of every Year; that being ſufficient for the Purpoſe intended: The Knowledge of the *Sun's Place* is (at preſent) but of little uſe in *Navigation*, yet becauſe both *Tables* take no more room than one alone would have done, the Table of the Sun's Place hath been likewiſe inſerted. Thoſe that deſire to find the Sun's Place to a greater Exactneſs, muſt have recourſe to *Aſtronomical* Tables calculated for that Purpoſe.

To find the Sun's Place and Right Aſcenſion; look for the Month in the Head of the Table, and the Day of the Month in the left-hand Column, and in the common *Area*, under the reſpective Titles, you'll have the *Sun's Place*, and *Right Aſcenſion* required.

Thus, for *Example*, let it be required to find the *Sun's Place* and *Right Aſcenſion*, *Auguſt* 20, 1730, at Noon, under the Meridian of *London*: By the Directions above, I look for the Month and Day, and under *Auguſt*, and right againſt 20 Days, I find in the Col. ☉ˢ *Pl.* ♍ 7° 41′ the *Sun's Place*, and in Column ☉ˢ *R. Aſc.* 10 Hours 37 Min. the *Sun's Right Aſcenſion*; and this for common Uſe may be taken for the Sun's Right Aſcenſion on the ſame Month and Day of any Year for 100 Years to come. Neither need there be any Allowance made for different Meridians or different Times of the Day, the Error here being of no Importance.

III. *The Deſcription and Uſe of the Table of the Right Aſcenſions, and Declinations of the Fixed Stars.*

This *Table* contains the *Right Aſcenſions* and *Declinations* of ſo many of the principal *Fixed Stars*, as are of a convenient Altitude for Obſervation in any

any *Latitude* from 60 Degrees *North*, to 60 Deg. *South*: And whereas the Altitude of a *Star* can't well be taken at Sea (by any Inſtrument hitherto contrived) when it is above 40 Degrees high, (becauſe the *Star* and the *Horizon* ought to be both diſtinctly ſeen at the ſame time) I have ſet down an Eſtimate of the Stars Meridian Altitudes in whole Degrees, where the ſame does not exceed 40 Deg. in every 10° of Latitude, from 60° N to 60° S, whereby may be readily found which Star to chuſe for Obſervation, in any *Latitude* not exceeding the Limits of the *Table*. And for the readier finding which *Stars* come to the *Meridian* at a proper time for Obſervation, at all Times of the Year; I have ſubjoined a *Table*, ſhewing the *Right Aſcenſion* of thoſe Points of the Heavens which *culminate* on the 1ſt and 16th Day of each Month, at every two Hours of the *Night*, from 8 o'th' Clock in the *Evening* till 4 o'th' Clock in the *Morning*.

The firſt Column in the left-hand Page has the Names of the ſeveral *Conſtellations*; and the ſecond Column ſhews the *Parts* of the reſpective *Conſtellations* wherein the *Stars* are placed, and thoſe Stars that have proper Names given them, are here expreſſed in Roman Capitals: the third Column is the *Right Aſcenſion* of the *Stars* in Hours and Minutes; and the 4th Column is their *Declination* from the *Equator*, both fitted to the Year 1726, and for common uſe will very well ſerve without any Alteration, as far as the Year 1740: The fifth and ſixth Columns ſhew the Denomination of the Stars Declinations, and their Magnitudes. The right-hand Page is divided into Columns for every 10 Degrees of Lat. from 60 Deg. North to 60 Deg. South; in which Columns are ſet down the *Meridian Altitudes* of the reſpective *Stars*, where the ſame does not exceed 40 Deg. or is leſs than 3 Degrees.

The Table being thus explained, its Uſes as to find the *Right Aſcenſions* and *Declinations* of the *Stars*, &c. are ſelf-evident. I ſhall only add an *Example* or two, of finding which *Star* is propereſt for Obſervation in any *Latitude*, and at all Times of the *Year*.

Admit *January 25th* being ſomewhere near the *Latitude* of 50 Degrees *North*, it be required to make an Obſervation by ſome *Fixed Star*; I would know which *Star* is the propereſt for that Purpoſe.

1. In the *Table*, entitled, *A Table ſhewing the Right Aſcenſion of that Point of the Heavens*, &c. I look for *Jan. 25th*, and finding it not, I take the neareſt Number to it, which is *Feb.* 1. and I find that any *Star* whoſe *Right Aſcenſion* is not leſs than 5 Hours, or greater than 13 Hours, will be upon the upper Part of the Meridian ſome time between 8 o'th' Clock in the Evening, and 4^h in the Morning. Then turning to the *Catalogue* of the *Stars*, I carry my Eye along the Column 50° North, until I come againſt a Star whoſe Meridian Altitude is noted in that Column, and whoſe Right Aſcenſion is betwixt 5 and 13 Hours, and I find Syrius to be a proper Star for my Purpoſe. Alſo, by the ſame Directions ſeveral other Stars will be found which will indifferently ſerve.

Note, In finding the *Time* when any Star comes to the *lower* Part of the *Meridian*, you muſt count 12 Hours from its *Right Aſcenſion*, and then proceed according to the Rules above: Thus, if in the *Example* above it be required to find which of the Stars that come to the *Meridian* under the *Pole*, are convenient for Obſervation; by the Directions above, I find that the right *Aſcenſion* of ſuch Stars muſt be betwixt 17^h and 1^h, then looking in the Column 50° N for ſome *Star* that has a ∾ over it, and whoſe *Right Aſcenſion* is either greater than 17^h, or leſs than 1^h; I find that the Stars marked β, α, ε in *Caſſiopea*, alſo FOMALHAUT, with ſome others, are fit for my Purpoſe.

Note, For the better diſtinguiſhing of the Fixed Stars, and knowing their Situation one from another, I would recommend to the Mariner a *Pair of Hemiſpheres* publiſhed and ſold by Mr. *Senex*; wherein all the Stars contained in the *Britiſh Catalogue*, alſo the Southern Stars obſerved by Dr. *Halley*, are carefully laid down by their Right Aſcenſions and Declinations; and alſo noted with *Characters*, as in the preceding *Table*, whereby any Star may be eaſily expreſſed by the Letter of the Alphabet which is annexed to it.

How to find the exact Time when any *Star* comes to the *Meridian*, ſhall be ſhewed in the next *Problem:* alſo how to find the *Latitude* by the *Meridian Altitude* and *Declination* of the *Sun*, or of any *Fixed Star*, ſhall be ſhewed in the next *Section*.

PROB. IV.

Of theſe three, *viz.* 1. The Right Aſcenſion of the *Sun*, 2. The *Hour* of the Day, and 3. The Right Aſcenſion of *Mid-Heaven*; any two being given, to find out the third.

CASE I.

If the Right Aſcenſion of the Sun, and the Time ſince Noon, be given; their Sum will be the Right Aſcenſion of Mid-Heaven.

Examp. Admit the *Sun's* Right Aſcenſion to be $13^h\ 40^m$, and the *Time* ſince *Noon* to be $6^h\ 15^m$; the Sum of theſe two $= 19^h\ 55^m$, is the Right Aſcenſion of *Mid-Heaven*; or the Right Aſcenſion of that *Star* or *Point* of the *Heavens*, which at that very Point of Time is upon the *Meridian*. Hence, knowing the Right Aſcenſion of the *Sun*, and the *Hour* of the *Day* (or *Night*,) we may find which *Stars* are then upon the *Meridian*; then comparing thoſe Stars with the ſame laid down upon a *Pair* of *Hemiſpheres*, or a *Celeſtial Globe*, we may eaſily learn to diſtinguiſh the *Conſtellations* in the *Heavens*.

Note,

Note, If the Sum of the *Sun's Right Aſcenſion*, and the *Time* ſince *Noon*, ſhould exceed 24 Hours, you muſt ſubſtract 24^h therefrom, and the Remainder will be the Right Aſcenſion of *Mid-Heaven*: Thus, if in the *Example* above, where the *Sun's* Right Aſcenſion was $13^h\ 40^m$, the *Time* ſince *Noon* had been 15 Hours, (or 3 o'-Clock in the *Morning*) the Right Aſcenſion of *Mid-Heaven* would have been $4^h\ 40^m$.

This *Caſe* being thus explained, if the Right Aſcenſion of the *Sun*, and of *Mid-Heaven*, be given; the *Hour* of the *Day* or *Night* may be eaſily found: Or, if the Right Aſcenſion of *Mid-Heaven*, and the *Time* ſince *Noon*, be given; the Right Aſcenſion of the *Sun* is eaſily had.

Case 2.

The Right Aſcenſion of the Sun at any time, and the Right Aſcenſion of a Fixed Star, being given; to find at what Time (of the Day propoſed) that Star will be upon the Meridian.

Subſtract the Right Aſcenſion of the *Sun* from the Right Aſcenſion of the *Star*, (borrowing 24 Hours if the *Sun's* Right Aſcenſion be greater than the *Star's*) the Remainder, if under 12 Hours, is the *Time* of the *Star's Culminating* after Noon; but if the Remainder be more than 12 Hours, ſubſtract 12 therefrom, and the Reſidue will be the *Time* after Mid-night.

Examp. Admit it be required to find the *Time* when Aldebaran comes to the Meridian, *November* 10, 1729.

The *Star's* Right Aſcenſion I find to be $4^h\ 20^m$; and the Right Aſcenſion of the *Sun* at that Time is $15^h\ 48^m$, which being ſubſtracted from $4^h\ 20^m + 24^h = 28^h\ 20^m$, gives $12^h\ 32^m$ after *Noon*, or 32 Minutes after *Mid-night*, the Time of the *Star's* coming upon the *Meridian*.

Case 3.

The Right Aſcenſion of the Sun, the Right Aſcenſion of any Fixed Star, and the Quantity of Time that the ſaid Star is diſtant from the Meridian, being given; to find the Hour of the Night.

The *Latitude* of the Place, the *Declination* and *Altitude* of any *Fixed Star*, being given; the Quantity of *Time* that the ſaid Star is diſtant from the *Meridian*, (or the *Hour-Circle* paſſing through the *Star*) may be found by the laſt *Problem*, after the ſame manner as the *Hour* of the *Day* was found by the *Sun*. Then,

Having found the time of the *Star's* coming to the *Meridian*, (by the laſt *Caſe*) the Sum or Difference of that, and the Diſtance of the *Star* from the *Meridian*, according as the *Caſe* requires, gives the *Hour* of the *Night*. Thus, if on the 10th of *November*, 1729, ALDEBARAN be obſerved to be $2^h\ 30^m$ paſt the *Meridian*, the *Time* of the *Night* in this Caſe will be $3^h\ 02^m$ after *Midnight*; but if *Aldebaran* had been $2^h\ 30^m$ to the *Eaſtward* of the *Meridian*, the Time would have been $10^h\ 02^m$ after *Noon*.

PROB. V.

To find the Time of High-Water in any Place, the Age of the Moon, and the Time when it flows at the New or Full Moon in the ſaid Place, being given.

The Time of *High-Water* in any Place muſt be obtained by a *Tide-Table*, and the *Age* of the *Moon* is readieſt found by an *Ephemerides*; but for want thereof, it may be found ſomewhat nearly as followeth: but firſt, you muſt find the *Golden Number*, and then the *Epact* for the given *Year*.

1. *To find the Golden Number:* Add 1 to the given *Year*, and divide the *Sum* by 19, the *Remainder* (if any) will be the *Golden Number* ſought; but if there is no Remainder, the *Golden Number* is 19.

Examp. What is the *Golden Number* for the Year 1729? To 1729 I add 1, and divide the Sum = 1730 by 19, the Remainder = 1 is the *Golden Number* ſought.

2. *To find the Epact:* Divide the *Golden Number* by 3, and multiply the Remainder (if any) by 10, the *Product* being added to the *Golden Number*, the Sum of both, if it be leſs than 30, otherwiſe the Exceſs above 30 is the *Epact* ſought. Or, according to the following *Verſe*.

Divide by 3, *for each one left add* 10,
30 *reject, the Prime makes Epact then.*

Examp. For the Year 1729, the *Prime* or *Golden Number* has been found above to be 1, but becauſe 1 cannot be divided by 3, I add 10 to it, and the Sum = 11 is the *Epact* required.

3. *To find the Age of the Moon:* To the *Epact* add the *Day* of the *Month*, and alſo the Number in the Margin correſponding to the ſaid Month; the *Sum*, if it be leſs than 30, is the *Moon's Age*; but if the ſaid Sum exceed 30, ſubſtract 30 therefrom ſo oft as you can, and the Remainder will be the *Age* of the *Moon* that Day.

Examp. To find the *Age* of the *Moon Novemb.* 10*th*, 1729. The Numb. for the Month is 10, which added to the *Epact* (for this Year) = 11 makes 21, and this added to 10 (the Day propoſed) makes 31; rejecting 30, there remains 1, which is the *Age* of the *Moon* the 10th of *Nov.* 1729. The *Age* of the *Moon* being thus found, the Day of the *Change* or *Full* may be eaſily had.

January	0
February	2
March	1
April	2
May	3
June	4
July	5
Auguſt	6
September	8
October	8
November	10
December	10

4. *To find the Time of the Moon's coming to the Meridian:* Multiply her *Age* by 4, and divide the *Product* by 5, and the *Quotient* gives *Hours*, and every *Unit* that remains is 12 *Minutes* more.

Examp. Let it be required to find when the *Moon* comes to the *Meridian*, *Novemb.* 10*th*, 1729; the ☽'s Age at that time has been found above to be 1 Day, but becauſe I cannot divide the Product of 1 into 4 by 5, I multiply 4 by 60, (the Minutes in an Hour) and divide the Product ($=240^m$) by 5, the Quotient = 48 Minutes ſhews that the ☽ is upon the *Meridian* at 48 Min. *Afternoon.*

The reaſon why you are directed to multiply the Moon's Age by 4, and divide by 5, is, becauſe allowing the Moon 30 Days between New and New, ſhe departs 48 *Minutes*, or $\frac{4}{5}$ of an Hour each Day from the Sun.

To find the Time of High-Water in any Place.

Having found the *Moon's* Age in the firſt Column of the annexed Table, the Number right againſt it in the ſecond, being added to the Hour of *High-Water* on the Change Day, makes the Time of *High-Water*, at the Place and Time propoſed.

Novemb. 20*th*, 1729. Let it be required to find the Time of High-Water at *London*, where it always flows till 3 o'-Clock on the Days of the *New* and *Full Moon.*

The *Moon's Age* at that time will be found (by the Directions above) to be 11 Days; wherefore I look for 11 Days in the firſt Col. and right againſt in the ſecond, I find $7^h\ 59^m$, this being added to 3^h gives 11 o'-Clock the Time of *High-Water* at *London* on the Day propoſed. Or, otherwiſe thus: To the Time of the *Moon's* coming to the *Meridian* on the given Day, (found as in the 4th *Art.* aforegoing) add the Time of *High-Water* in the given Place on the Day of the *Change* or *Full*, and take the Sum for the Time of High-Water on the Day propoſed. But this Method is not ſo accurate as by the Table.

The ☽'s Age.		*Tides.*	
Days.		H.	M.
1	16	0	43
2	17	1	20
3	18	1	52
4	19	2	22
5	20	2	52
6	21	3	26
7	22	4	07
8	23	4	55
9	24	5	50
10	25	6	53
11	26	7	59
12	27	9	04
13	28	10	08
14	29	11	05
15	30	12	00

SECT.

SECT. III.

Of Mathematical Inſtruments uſed at Sea for taking of Altitudes; and how to find the Latitude by the Meridian Altitude of the Sun or Fixed Stars.

Plate 10. *Fig.* 8. CEleſtial Obſervations are founded on theſe two Principles.

1. That when many Circles are concentrick, if right Lines be drawn from the Center through their Circumferences, they ſhall divide them all after the ſame manner: Thus, if the Circles A B *a*, D E *b*, F G *e* be deſcribed from the ſame Center C, and the right Lines C A, C B be drawn; the Arches A B, D E, F G ſhall be like Parts of their reſpective Circumferences, and contain a like Number of Degrees.

2. That the Terraqueous *Globe* whereon we live, compared with the vaſt Diſtance of the *Sun*, is no more than a mere Point, (or rather the *Earth* ſeen from the Sun appears no bigger than a *Point*) and conſequently with regard to *Solar* Obſervations, any Point on the *Earth's* Surface may very well be accepted for its *Center*, as alſo for that of the *Sphere* of the *Fixed Stars*; whence, and from the preceding, the Center of every *Inſtrument* may be ſubſtituted as Center of the *Heavens*, and the Arch of any Inſtrument does inſpect the like Arch in the Heavens.

You may farther obſerve, (ſince all Angles are meaſured by Arches of Circles) that all *Inſtruments* for meaſuring of *Angles*, however they are framed or compoſed, muſt be ſo graduated as to ſhew the quantity of the *Arch*, or the number of *Degrees* contained between any two Lines drawn from the *Center* through the ſaid Graduations.

The Antients uſed divers Sorts of Inſtruments for taking the Altitudes of Celeſtial Objects at Sea; as the *Aſtrolabe*, the *Croſs* or *Fore-Staff*, the *Demi-Croſs*, the *Plow*, and the *Bow*, *&c.* but theſe (excepting the *Fore-Staff*) are now moſtly (if not wholly) laid aſide, and therefore there is no need of giving any Deſcription of them here: The Inſtruments that are now chiefly uſed at Sea, are theſe two, *viz.* the QUADRANT, and the FORE-STAFF.

I. *Of the* SEA-QUADRANT.

This Inſtrument was contrived by one Captain *Davis*, an *Engliſh* Man, in Queen *Elizabeth*'s Time; (whence by us it is often called *Davis*'s Qua-

Quadrant, but by moſt Foreigners the *Engliſh* Quadrant) and is the principal Inſtrument now uſed at Sea: It conſiſts of two *Arches a b, d e,* and three *Vanes* A, B and C. *Fig. 6.*

I. *Of the two Arches.*

The *Radius* of the *Arch a b* is commonly about 24 Inches, and formerly this Arch was made to contain 30 Deg. but now it is ſeldom made to contain above 25 Deg. Each of theſe Deg. are uſually ſubdivided either into 6 or 12 equal Parts, each Part being accordingly either 10 or 5 *Minutes:* Alſo upon the *Plane* of this *Arch* are uſually deſcribed ſeveral *concentrick Circles* with *Diagonal Lines*, intended for dividing a *Degree* into leſſer Parts than the Diviſions upon the *Limb*; but this is a ſuperfluous Trouble, (and indeed theſe *Diagonals* are ſeldom regarded by Mariners,) for even in moving the *Vane* 10′ there is ſcarce any ſenſible Difference made in the Obſervation. The *Arch d e* is generally of about $\frac{1}{3}$ the *Radius* of the other, and is now commonly made to contain 65 Degrees, both Arches together making 90 Deg. whence the Inſtrument is called a *Quadrant:* this Arch is divided only to whole *Degrees*, and ſometimes only to every 5 ~~or 10~~ *Deg.* theſe Diviſions being numbered from *e* towards *d*; and upon the other *Arch* the Diviſions are numbered from *a* towards *b*; the *Sum* of the Diviſions upon both *Arches*, ſhewing the *Diſtance* of the *Object* from the *Zenith*.

II. *Of the Vanes.*

The *Vane* A is called the SIGHT-VANE, and is made to ſlide upon the back of the *Arch a b*, ſo as always to ſtand perpendicular to the Plane thereof; in this *Vane* there is made a ſmall round *Hole*, to look at the *Horizon*, through a Slit made in the *Vane* C. The *Vane* B ſerves to caſt the *Shade* of the *Sun* upon the *Vane* C, and is therefore called the SHADE-VANE; it is made to ſlide upon the *Arch d e*, after the ſame manner that the *Sight-Vane* ſlides upon the *Arch a b*. The *Vane* C, called the HORIZON-VANE, is made to ſtand upright upon the end of the *Staff* C *b*, ſo that the flat Side next to *b*, juſt touches the common *Center* of the two *Arches a b, d e.*

The *Slit* in the *Horizon-Vane* is always cut ſloping, ſo that the upper Edge of the Slit on the Fore-ſide of the *Vane*, (or that Side next to *b*) and the lower Edge of the Back-ſide, are directly in a right Line, paſſing through the Center, near the middle of the *Quadrant*; ſo that when the *Sight-Vane* happens to be in *b*, the Opening in the *Horizon-Vane* appears ſo contracted, that you cannot ſee the *Horizon* ſo diſtinct as you ought to do; and the ſaid Opening will appear wider or narrower, according as the *Sight-Vane* is nearer to, or farther from *a*. This Method of ſlitting the *Vane* is evidently very inconvenient and abſurd, for the Reaſons given above:

The common Method of ſlitting the Horizon-Vane is abſurd, and leads the Obſerver into Error.

above: And what could be the Meaning of this cunning Artifice, is what I cannot conceive, except it be for an imaginary Conveniency of having two Centers to the ſame Inſtrument, that ſo the Obſerver might make uſe of either Edges, or obſerve any where between them; at leaſt I know that that is the common Notion of Mariners, and this leads them into Error more or leſs, according as they obſerve more or leſs open, (as they call it) or the Poſition of the *Sight-Vane* is nearer to, or farther from *a*.

The Manner that the Slit in the Horizon-Vane ought to be cut.

Inſtead of cutting the *Slit* in the *Horizon-Vane*, after the manner above deſcribed, the upper Edge thereof on the Fore-ſide of the *Vane*, muſt be made the *fiducial Edge*, (becauſe the Sky is quicker diſcerned than the Water) and ſo that it may be exactly perpendicular to the Center of the Inſtrument, and the upper Part of the *Vane*, on the Back-ſide, ſloped down quite to this Edge; then the lower Part of the *Slit* muſt be made ſufficiently wide from the upper, (for the lower Edge is never to be regarded) and cut either ſquare-wiſe or ſloping downwards, ſo that it may at all times appear wide enough from the upper Edge, (which is perpendicular to the Center:) Or, the lower Part of the *Vane* may be cut quite off, ſo as not to have any ſlit at all. I have uſed another ſort of *Horizon-Vane* with good ſucceſs; this *Vane* had a wide Opening in the middle, and a ſmall String therein fixed perpendicular to the Center of the *Quadrant*, ſo that in time of Obſervation the String was made to cut the *Horizon*, *i. e.* ſo that nothing but Water appeared below the *String*, and Sky above it.

A Caution in uſing the Shade-Vane.

Note, When you obſerve by the upper Edge of the *Shade-Vane*, (as it is cuſtomary to do) the *Shadow* is terminated by a Ray proceeding from the upper Part of the *Sun's Diſk*, (and not from its Center) and therefore the *Sun's Altitude* is repreſented more than it really is, by the Quantity of his apparent Semi-diameter, which generally is about 16′, and conſequently the *Zenith Diſtance* is 16′ too ſmall; wherefore to the *Zenith Diſtance* upon the *Quadrant* you muſt add 16′, and the Sum will be the true *Zenith Diſtance* of the *Sun's Center*. On the contrary, if you obſerve by the lower Edge of the *Shade-Vane*, (or the Edge next to *d*) then it is the lower Part of the Sun that determines the Shadow; therefore (in that Caſe) you muſt ſubſtract 16′ from the *Zenith Diſtance* upon the *Quadrant*, and the Reſidue will be the true *Zenith Diſtance* required. The reaſon of this may be eaſily made appear, thus:

Fig. 10.

Let *a b c* be the *Sun*, and A B a *Gnomon* erected perpendicular to the Horizon, (H O;) (the end whereof B may repreſent the upper Edge of the *Shade-Vane*) then is the ∠ A *c* B the Altitude of the *Sun's* Center, and the ∠ A *a* B the Altitude of his upper Limb; but becauſe the Space between the Lines *c* B, *a* B, is ſomewhat enlightned by the upper Part of the Sun, the dark Shade of the *Gnomon* A B reaches no farther than the Point *a*, where it is defined by the *Ray a a* proceeding from the upper Limb of the Sun. Whence, the Converſe, or that the Shadow of the lower Edge of the *Shade-Vane* is defined by a *Ray* proceeding from the lower Edge of the Sun, is likewiſe evident.

There

There is alſo another *Vane* commonly fitted to the Quadrant, called the *Glaſs-Vane*, having a ſmall double Convex-*Glaſs* fixed in it, which contracts the Sun's *Rays*, and caſts them in a ſmall bright *Spot* upon a little Circle made upon the *Horizon-Vane*; this Spot being made by all the *Solar Rays* contracted together, correſponds to the Center of the Sun, and is ſo clear and conſpicuous, that even in *hazy* Weather the Sun's Altitude may be taken; and in clear Weather the *Spot* is more conſpicuous than the *Shadow*, which at beſt is hardly defined, (as plainly appears from a due Conſideration of the *Figure* above deſcribed;) ſo that it ſeems ſtrange to me (ſince ſo good an Improvement has been made in the Inſtrument) that People ſhould regard the Shadow at all. The Edge of this *Vane* is likewiſe ſo cut, that it may be uſed as a *Shadow-Vane*, ſo that there is no neceſſity for that plain Piece of Wood called the *Shade-Vane*; but if inſtead thereof, there ſhould be fitted another *Glaſs-Vane*, having its *Glaſs* of a larger Size than ordinary, the Inſtrument would be very much improved; for then an Obſervation might be taken in ſuch *hazy* Weather, that the common *Glaſſes* prove of no effect, as I have myſelf experienced.

That the Glaſs-Vane is preferable to the Shade-Vane.

Note, Upon the back of the *Arch de*, there is commonly drawn Lines for every 5 Degrees, about $\frac{1}{4}$ Deg. higher than their correſpondent Lines upon the Side, intended for placing the *Shade-Vane* thereto, in order to ſave the trouble of allowing for the *Sun's Semi-diameter:* But I never knew any allowance made, or theſe Lines regarded by Mariners when they uſe the *Shade-Vane*; but they generally place the *Shade* and *Glaſs-Vane* in the ſame Place; and ſome I have known to place the *Glaſs-Vane* where they ought to place the *Shade-Vane*, (inſiſting on their being right, becauſe they always uſed to do ſo, and that they had been at Sea ſeveral Voyages) thereby making $\frac{1}{4}$ deg. Error in each Caſe, the one making the Latitude $\frac{1}{4}$ deg. more than the Truth, and the other $\frac{1}{4}$ deg. leſs. Whence we may judge of the Exactneſs of Obſervations made at Sea, and how much in ſuch Caſes People are improved by their Experience. I preſume none will judge the above Remarks to be trifling, at leaſt they do not appear ſo to me, ſince they are concerned in one of the moſt important Articles in the Art of Navigation.

To obſerve the Sun's Altitude by the common Sea-Quadrant.

Having put the *Horizon-Vane* upon the upper End or Center of the *Quadrant* at C, the *Shade* or *Glaſs-Vane* upon the *Arch de*, (to a number of Degrees leſs than you judge the *Zenith Diſtance* will be, by 10 or 15 Deg.) and the *Sight-Vane* upon the *Arch ab*; then turning your Back towards the Sun, lift up the *Quadrant* and look through the ſmall *Hole* in the *Sight-Vane*, and ſo raiſe or lower the Inſtrument till the bright *Spot* coming from the *Glaſs-Vane*, falls upon the ſmall Circle correſponding thereto upon the *Horizon-Vane*; if then, at the ſame time, the upper

Edge of the *Slit* in the *Horizon-Vane* every where cuts the *Horizon*, the *Quadrant* is perpendicular and duly elevated, and the Obſervation is finiſhed. But if the *Sea* appears inſtead of the *Horizon*, then remove the *Sight-Vane* lower towards *a*; and if the *Sky* appears inſtead of the *Horizon*, you muſt ſlide your *Sight-Vane* a little higher; and ſo continue removing the *Sight-Vane* till the upper Edge of the Slit of the *Horizon-Vane* cuts the *Horizon*, and the bright *Spot* proceeding from the *Glaſs-Vane* falls on its proper Place: Then look how many Degrees and Minutes are cut by that Edge of the *Sight-Vane* which correſponds to the *Sight-Hole*, and to them add the Degrees that are cut by the upper Edge of the *Glaſs-Vane*; and the Sum is the *Zenith Diſtance* or Complement of the *Altitude* required.

But to find the *Sun's Meridian* or greateſt *Altitude*, you muſt be ſure to begin your Obſervation ſome time before *Noon*; and then continue obſerving ſo long as you ſee the *Altitude* increaſe, which you'll perceive by having the *Sea* appear (at the repeating of the Obſervation) inſtead of the *Horizon*, removing the *Sight-Vane* lower; but when you ſee the *Sky* appear inſtead of the *Horizon*, the Altitude is diminiſhed, therefore deſiſt from obſerving farther at that time, and add the *Degrees* and *Minutes* given upon the two *Arches* together, and the *Sum* will be the *Meridional Zenith Diſtance*, or the Complement of the *Sun's Meridian Altitude* required.

But if you regard the *Shade* and not the *Spot* coming through the *Glaſs*, (which I would not adviſe any body to do, for the Reaſons given above) you muſt hold the *Quadrant* ſo that the ſaid *Shade* may fall along the upper Edge of the *Slit* in the *Horizon-Vane*, and alſo that the *Shade* coming from the top of the *Vane* may fall exactly upon the Croſs-Line upon the *Horizon-Vane:* then if you ſee the *Horizon* cut by the upper Edge of the *Slit* in the *Horizon-Vane*, the Obſervation is rightly made; if not, you muſt move the *Sight-Vane* as before directed. And the Sum of the Degrees and Minutes given upon both *Arches*, will be the *Zenith Diſtance* or *Co-Altitude* of the *Sun's* upper Limb, to which you muſt add 15 or 16 Min. and the Sum will be the *Co-Altitude* of the *Sun's* Center. If you have a *Horizon-Vane* with a *Thread* in the middle of the *Slit*, and perpendicular to the *Center* of the *Quadrant*, that Thread muſt cut the *Horizon*, and the *Shade* fall along the middle of it; or the *Spot* upon a little Circle deſcribed upon the *Horizon-Vane*, as before.

I take it, that it would be a good Improvement upon this *Inſtrument*, if there ſhould be ſmall *Bars* of *Braſs* fixed on the *Arch d e*, at 10 or 15 Degrees diſtance from each other, and a Groove made in the bottom of the *Glaſs-Vane*, ſo as to exactly fit the ſaid Bars; then ſlipping the Groove upon theſe Bars, (according as you judge the *Co-Altitude* to be) you'll be ſure that the *Vane* is rightly fixed, and not liable to move out of its Place.

Note,

Height of the Eye.	*Allowance.*
Feet.	*Min.*
3	2
6	3
12	4
20	5
30	6
40	7
50	8

Note, 1. Becauſe the Eye of the Obſerver is always above the Surface of the Sea, the *Apparent Horizon* muſt be lower than the *true Horizon*, (paſſing thro' the Eye) and conſequently the *Altitude* muſt be made more than it really is, and the *Zenith Diſtance* ſo much leſs; and this Error will be greater or leſs, according to the Height of the Eye above the Water, which being known, the *Dip* may be found by the annexed *Table:* thus, if the Height of the Eye above the Surface of the Water be 20 Feet, the *Dip* of the *Horizon* will be 5′, and ſo much muſt be added to the *Zenith Diſtance* found by the *Quadrant*, in order to obtain the true *Zenith Diſtance*. In common Practice indeed it is not very material whether this Allowance be made or not, yet is it ſtill better to keep as near the Truth as we can.

The fore-mentioned *Table* may be calculated as followeth;

Let D E be the *Diameter* of the *Earth*, C D = C A its Semidiameter, and B D the Height of the *Eye* above the Surface of the *Water*, then will A be the Point where the *Sky* and the *Water* ſeem to meet; let B *b* be parallel to D A (which becauſe of the Smallneſs of the Arch, may in this Caſe be taken for a right Line,) then is the ∠ A B *b* the *Dip* of the *Horizon*; to find which by *Calculation*, in the right ∠ᵈ △ C A B, right ∠ᵈ at A, are given the Side C A equal to the Semidiameter of the Earth, and the Side C B (= C D + D B) equal to the Semidiameter of the Earth, together with the Height of the Eye above its Surface; whence to find the ∠ B, it will be, C B : C A :: R : S.B; the Complement whereof is A B *b* the *Dip* required. After the ſame manner B A the *Semidiameter* of the *Apparent Horizon* may be eaſily found, if required; or, B A may be otherwiſe found by this *Theorem*, *viz.* B E × B D = B A *q* (*Euclid* 36. III.) therefore √ BE × B D = B A. *Fig.* 9.

Zenith Diſt.	*Refraction.*
Deg	*Min.*
90	33
89	23 1/3
88	17 1/2
87	13 1/3
86	10 2/3
85	8 3/4
84	7 1/2
83	6 1/2
82	5 2/3
81	5
80	4 1/2
79	4
77	3 1/2
75	3
70	2 1/4
60	1 1/3
50	1

Note, 2. The *Refraction* of all heavenly Bodies (occaſioned by the Denſity of the *Atmoſphere*, or thick *Vapours* of the *Air*) makes them appear higher than they really are, eſpecially when they are near the Horizon; therefore, when you have found the *Zenith Diſtance*, add thereto the *Refraction*, (or ſubſtract it from the *Altitude*) which according to the Obſervations of Mr. *Flamſtead* is (in our Latitude) as in the annexed *Table:* thus, if the *Zenith Diſtance* be 75 Deg. the *Refraction* is 3 Min. which added to 75° makes the true *Zenith Diſtance* 75° 03′. Again, ſuppoſing the Obſerver's Eye to be 20 Feet above the Sur-

Surface of the Sea, and the *Zenith Diſtance* obſerved by the *Quadrant* to be 75 Deg. the true *Zenith Diſtance* (allowing for *Refraction* and *Dip* of the *Horizon*) will be 75° 08′. And after the ſame manner muſt be allowed for *Refraction* and *Dip* in all other Caſes, according to what is given in theſe two little *Tables*; only, you may note, that when the *Zenith Diſtance* is leſs than 50 Deg. (or the *Altitude* greater than 40 Deg.) the *Refraction* in a manner ceaſes, and there needs no allowance be made.

It is upon the account of the *Refraction*, that a *Stick* when one end is plunged into Water, the other remaining in the Air appears crooked; the Part which is in the Water appearing higher than it really is, becauſe the *Rays* proceeding from that Part meet with a greater Reſiſtance, and therefore are more bent towards the Perpendicular to the *Plane* upon which they fall; and for the ſame reaſon the *Sun* (in our Latitude) appears totally above the *Horizon*, when his upper Limb is really below it.

II. *Of the Common* Fore-Staff.

The *Fore-Staff* (as commonly made) conſiſts of a ſtrait Square graduated *Staff*, of about 3 Feet in length, having four *Croſſes* or *Vanes*; the firſt and ſhorteſt is called the *Ten Croſs* or *Vane*, and belongs to that Side of the *Staff* where the Diviſions begin at about 3 Deg. proceeding towards the *Eye* or *Flat* end to 10 Degrees. The next longer is called the *Thirty Croſs* or *Vane*, and belongs to that Side of the *Staff* where the Diviſions begin at 10 Deg. and end at 30 Degrees: The next to this is called the *Sixty Croſs* or *Vane*, and belongs to the *Sixty-Side* of the *Staff*, or where the Diviſions begin at 20 Deg. and end at 60 Degrees: The longeſt *Croſs* is called the *Ninety Croſs* or *Vane*, and belongs to that Side where the Diviſions begin at 30 and end at 90 Degrees, called the *Ninety Side* of the *Staff*.

The chief Deſign of this *Inſtrument* is for taking the *Altitudes* of the *Stars*, and the ſeveral *Croſſes* are to be uſed according as the *Altitude* is greater or leſs; that is, if it be leſs than 10 Deg. the *Ten Croſs* muſt be uſed; but if it be greater than 10 Deg. and leſs than 30 Degrees, uſe the *Thrity Croſs*, &c.

The Manner of obſerving an Altitude with the Fore-Staff.

Having placed a proper *Vane* upon the *Staff* (according as you judge the Altitude to be,) put the Square-end thereof to the Corner of your Eye, as near as you can without hindering your Sight, and look at the upper End of the *Croſs* for the Center of the *Sun* or *Star*, and at the lower End for the *Horizon*, (holding the *Staff* as near as you can, ſo that the

Croſs

Croſs may lie in the Vertical Circle, paſſing thro' the Eye and the Object.) If you ſee the *Sky* inſtead of the *Horizon*, you muſt draw the *Croſs* a little nearer to your Eye; but if you ſee the *Sea* inſtead of the *Horizon*, put the *Croſs* a little farther from your Eye; and ſo continue moving or removing the *Croſs*, till you ſee the *Sun* or *Star's* Center exactly over the upper End of the *Croſs*, and the *Horizon* by the lower End thereof: then the Deg. and Min. cut by the inner Edge of the *Croſs* upon the Side of the *Staff* peculiar to that *Croſs*, is the preſent *Altitude* of the *Sun* or *Star*. But if you are to find the *Meridian Altitude*, you muſt continue your Obſervation ſo long as you find the *Altitude* increaſe, ſtill drawing the *Croſs* nearer to your Eye; but when you find the *Altitude* is diminiſhed, (which you'll diſcover by ſeeing, at repeating the Obſervation, the *Sea* appear inſtead of the *Horizon*,) you muſt leave off obſerving for that time, and the *Degrees* and *Minutes* on the Side proper to the *Croſs*, is the *Meridian Altitude* required.

That the common Fore-Staff is incommodious, and liable to Error.

The Inſtrument above deſcribed is very incommodious, and alſo liable to Error: For, 1. becauſe the *Vanes* are made to ſlide towards or from the Eye, the *Radius* is continually altered, and this Difference is ſo great, that the Breadth of a *Degree* in ſome Places upon the *Staff* is not ſo much as the Breadth of 1 *Min.* in ſome other Places; and this continual Alteration in the *Radius* from about 30 Inches till about 8 or 9 Inches, is certainly a very great Defect and Inconveniency in the *Inſtrument*, and more eſpecially for obſerving at Sea, where it often proves an advantage to be well acquainted with the *Radius* of the Inſtrument, becauſe then we are better able to judge how much (or what Part of a Degree) the *Star* or the *Horizon* is diſtant from the Edge of the *Vane*, when (by reaſon of the toſſing of the Ship) it is very difficult to bring the *Inſtrument* into its due Poſition. 2. The manner we are obliged to hold this Inſtrument being ſideways from the Eye, and then look (as it were) ſlanting over the *Croſſes*, is very aukward and unnatural; for certainly in all Obſervations, the *Inſtrument*, or that Part of it which carries the *Sights* ought to be ſo contrived, as to be held directly betwixt the *Eye* and the *Object*. Laſtly, it is very difficult to place the *Staff* ſo that the Eye may anſwer exactly to the *Center*, (for the Eye cannot be brought into the Center, which ſhould be done;) but to prevent this Error as much as can be, you muſt place two *Vanes* to the ſame number of *Degrees* upon their correſpondent *Sides*, then move the *Staff* higher or lower about the Eye, until you ſee the upper Ends of both *Croſſes* in the ſame right Line, and alſo their lower; and that muſt be the Poſition of the *Staff* in all Obſervations: but this is very difficult to be done, as well as uncertain when you uſe only one of the *Croſſes*. I need not here take notice of the Inconveniency of having ſo many *Croſſes*, (and ſo many Setts of Diviſions upon the *Staff*,) and of the difficulty of making them to ſlide ſo as to keep always perpendicular to the Plane of the *Staff*; theſe things indeed being not of ſo much Conſequence, (though ſtill a Defect) as thoſe above-mentioned.

Having thus ſhewed the Defect and Imperfection of the common *Fore-Staff*, I ſhall next give the Deſcription of a new one, which is liable to none of the Inconveniencies above noted.

The Deſcription of a new FORE-STAFF.

Fig. 7. This Inſtrument conſiſts of a flat *Staff* A, and one *Croſs* or *Tranſum* B; the *Staff* may be made of about 2 Foot long, 1 $\frac{1}{4}$ Inch wide, and about $\frac{1}{2}$ Inch thick; that Part which is *ſhaded* repreſents the *ſolid* Part of the *Staff*, and the *light* Part reaching from *a* to *b*, a Piece taken out, or rather bevelled down on both Sides to a fine Edge near to *a*. The Part *a* is a *Sight-Vane*, the light Part in the middle repreſenting a ſmall Hole to look through in time of Obſervation; *d* is that Part of the Staff, whereon the *Tranſum* (B) is fixed; *e* the *Handle* to hold the Inſtrument by, which is alſo made to ſlide upon the Staff, ſo as to be brought to ſuch a Poſition as may be judged moſt convenient. The *Tranſum* B conſiſts of two ſquare Rulers (of about the thickneſs of the Staff) faſtened together at both Ends, and likewiſe at the Middle, (here repreſented by that Part which is ſhaded;) the white Part in the Middle repreſents a void or an empty Space, wherein are two ſliding *Vanes c* and *d*, each guarded on the back by ſteel *Springs* to keep them ſteady. The *Vane c* (being in the upper Part of the *Tranſum*) may be called the *Object Vane*, and the *Vane d* the *Horizon-Vane*, (becauſe the lower Edge thereof is to cut the Horizon,) one of the Rulers is divided into Deg. and the *Tranſum* is made ſo long as to contain 40 Deg. (being the greateſt Altitude that can well be taken by an Inſtrument of this kind, where both the *Object* and the *Horizon* ought to be diſtinctly ſeen at the ſame time;) and theſe Degrees are numbered upwards and downwards from the Middle; the lower Part of the Ruler (below *a*) being divided only to whole *Degrees*, but the upper Part is divided to every 10 *Min.* (or it may very well be divided to every 5′ if required.) You may obſerve, that the *Vane c* has an oblong Hole in the Middle, wherein is fixed a *Braſs* Wire, for cuting the Object in time of Obſervation, or the Object may be cut by either of the Edges of the *Vane*, as you judge moſt convenient; *b* repreſents a ſmall *Slip* of *Braſs* intended for an *Horizon-Vane*, when the Altitude is leſs than 6 Degrees, (becauſe no leſs an Altitude than 6° can be taken by the two *Vanes c* and *d*;) by which an *Altitude* may be taken of a ſingle Degree, or leſs, (which cannot be done by any Inſtrument hitherto contrived,) and which may be of good uſe for taking the *Sun's* Altitude when he is near the *Horizon*.

In time of Obſervation the *Tranſum* is fixed on (the Part *d* of) the Staff, (ſlipping into the white Part oppoſite to *a*, in the *Tranſum*) in ſuch a manner that the *Staff* and the *Tranſum* are exactly at right Angles to each other, (being forced into that Poſition by a Guard placed on the back

back of the *Tranſum*) the flat Sides of the *Staff*, exactly facing the flat or broad Sides of the *Tranſum*.

There needs no Directions to be given here for graduating this *Inſtrument*, the Line of *Degrees* being evidently a double *Line* of *Tangents*, one on each ſide the Middle (*a*,) the *Radius* being the Length of the *Staff*: but it may be here noted, that there is ſome nicety required in placing the *Sight-Vane* in its due *Center* or *Focus*; but of that I ſhall give proper Directions to the *Inſtrument-Maker*, whom I ſhall employ for making this Inſtrument.

To obſerve an Altitude with this Inſtrument.

Having placed the lower Edge of the *Horizon-Vane* (*d*) to ſome even *Degree*, (according as you judge convenient) take hold of the *Handle* (*e*) and place the *Center* (*a*) of the *Staff* cloſe to your Eye; then turn your Face towards the *Object*, (holding the *Inſtrument* as upright as you can) and look either over the upper Edge, or under the lower Edge of the *Object-Vane*, (as you think fit) for the *Object*; then if you ſee the *Sky* inſtead of the *Horizon*, move the *Object-Vane* a little higher, but if you ſee the *Sea* inſtead of the *Horizon*, move the *Object-Vane* a little lower, and ſo continue moving this *Vane* upwards or downwards, till one Edge of it cuts the *Object*, and the lower Edge of the *Horizon-Vane* cuts the *Horizon*: Then the number of Degrees correſponding to the lower Edge of the *Horizon-Vane*, being added to the Degrees and Minutes correſponding to that Edge of the other *Vane*, which was made to cut the Object, (having allowed for *Refraction*, &c. as before directed in uſing the *Quadrant*) is the Altitude required. Or, you may make the *Wire* in the Middle of the *Object-Vane* to cut the *Star*, then (upon the leaſt Motion of the Ship) you'll have the advantage of ſeeing the *Star* (as it were) playing both above and below the *Wire*, which is a conſiderable help for rectifying the Inſtrument; but this indeed cannot be done but in a very clear Night. To find the *Meridian Altitude*, you muſt continue your Obſervation ſo long as you find the *Altitude* increaſe, which you'll diſcover by ſeeing (at the repeating of the Obſervation) the *Sky* appear inſtead of the *Horizon*. If the *Altitude* be leſs than 7 *Deg.* you muſt make the lower Edge of the *Croſs-bar b* to cut the *Horizon*, and the lower Edge of the *Vane c* to cut the *Object*; then the number of *Degrees* and *Minutes* cut by the ſaid Edge (of the *Vane c*) will be the *Altitude* required.

When the *Sun's Altitude* exceeds 12 or 15 *Deg.* the beſt way for taking it (if he ſhines out bright) is by the *Sea-Quadrant*; but when the Sun is lower than 12 or 15 *Deg.* this *Fore-Staff* is a very good Inſtrument for taking his Altitude; (and I believe much better than any other hitherto contrived) thus, making the lower Edge of the *Croſs-Bar b* to cut the *Horizon*, let the lower Edge of the *Object-Vane* (*c*) cut the lower Edge of the *Sun*, (for ſo you'll preſerve your *Sight* from the Splendour of his *Rays*;)

then add 15′ to the Number corresponding to the lower Edge of the *Object-Vane*, and the Sum (after you have allowed for Refraction and the Height of the Eye above the Surface of the Water,) will be the *Altitude* of the *Sun's Center*. Thus the Sun's Altitude may be taken when his Center is not above 40′ above the *Horizon*.

Having thus explained the Uſe of this Inſtrument, and having before ſhewed the Defect and Inconveniency of the common *Fore-Staff*; I preſume that the Advantage of the one above the other, appears very evident.

The Almacanter's Staff not a fit Inſtrument for Obſervation.

For taking the Sun's Altitude forwards, ſome uſe an Inſtrument which they call an *Almacanter's Staff:* This Inſtrument is nothing elſe but the common *Sea-Quadrant*, omitting the *Arch d e*, having on the *Arch a b* two *Vanes*, and on the Center is fixed a *Vane* having a *ſmoaky* or coloured *Glaſs* in the Middle, intended to preſerve the Sight from the *Rays* of the *Sun*; but the ſaid *Glaſs* being put in the *Sight-Vane*, makes the *Horizon* appear ſo dull, and (as it were) *hazy*, that you can hardly perceive it, which renders the Inſtrument very unfit for Obſervation: And indeed the common *Sea-Quadrant* would be much more handy for this Purpoſe than the *Almacanter's Staff*, if there was but a *Sight-Vane* fitted to the Center, which would be ſtill an Improvement upon this Inſtrument. But, I think, for finding the *Sun's* Altitude forwards, the *Fore-Staff* above deſcribed is much preferable to either the *Quadrant* or the *Almacanter's Staff*.

That the Nocturnal is imperfect and erroneous.

Being I am about deſcribing of the *Mathematical Inſtruments* now uſed at Sea, it may be expected that I ſhould ſay ſomething of the *Nocturnal:* This Inſtrument is commonly uſed by Mariners, and is deſigned for finding the *Hour* of the *Night*, and the *Altitude* or *Depreſſion* of the *Pole-Star*, above or below the *Pole*; but it happens to be ſo imperfect as not to anſwer either of theſe Particulars. In order to find the *Hour* of the *Night* thereby, it muſt be held in the *Plane* of the *Equinoctial*, and the *Index* brought to the ſame *Hour-Circle* with ſome known *Star* near the *Pole*; or as the Inſtrument is commonly made, the *Index* muſt be brought to the ſame Hour-Circle with either the *Pointers* of the *great Bear*, or the foremoſt of the *Guards* of the *little Bear*. 1. For holding the Inſtrument in the *Plane* of the *Equinoctial*, you have no other Guide than to look through a Hole made in the Middle of a thin Plane, and in this Caſe you may err even half a ſcore Degrees, both in the Direction towards the *North*, and in the Elevation above the *Horizon*; and again, you are liable to err as much in not holding the *Handle* and *Top* of your Inſtrument in the *Plane* of the *Meridian*, and alſo in not putting the *Index* to the ſame *Hour-Circle* with the *Star*; for in all theſe Caſes you have in reality no other Guide than your own Gueſs, and conſequently your Obſervation muſt needs be very uncertain: and I cannot help thinking, but that a better gueſs might be made without the Inſtrument, than with it. The next thing propoſed by the *Nocturnal*, is, to find the Height or Depreſſion of the *Pole Star* above or below the *Pole*; in order to which, the *Hour-Circle* wherein the fore-

fore-mentioned *Stars* are at that time, muſt be found; or, in the *Sea-Phraſe*, their *Bearings*: but that we have already ſhewed to be impracticable by the *Nocturnal*. But even granting the Hour to be known, the Mariner will be ſtill vaſtly deceived by the *Nocturnals* now in uſe; for upon theſe Inſtruments (if they may be called ſuch) the Diſtance of the *Pole-Star* from the *Pole*, is made 20 Minutes more than it really is at preſent, and conſequently thoſe that depend upon them muſt be vaſtly deceived in finding their *Latitude*; ſuppoſing the *Altitude* of the *Pole-Star* to be rightly taken. Again, ſuppoſing the Numbers upon the *Nocturnal* were rightly calculated for the preſent time, they'll not anſwer but for one *Latitude*, and in ſome Places they may deceive the Obſerver very conſiderably; and therefore thoſe that deſire to find the *Latitude* by the *Pole-Star*, when it is upon different *Hour-Circles*, ought to have not only ſome better Contrivance for a *Nocturnal*, but alſo *Tables* calculated for the ſeveral *Latitudes* wherein they deſign to obſerve, with *Variations* to make them ſerve for time to come, (for the *Pole-Star* continually approaches the *Pole*;) which would be too great a trouble for ſo little a Purpoſe, eſpecially becauſe the *Pole-Star* is not of a convenient Altitude for Obſervation but in few *Latitudes*.

The beſt way for finding the *Latitude*, (eſpecially at Sea where Obſervations are very imperfect) is by the *Meridian Altitude* and *Declination* of the *Sun* or *Fixed Stars*; which how to do, comes next to be ſhewed.

How to find the *Latitude* by the *Meridian Altitude* (or *Zenith Diſtance*,) and the *Declination* of the *Sun* or *Fixed Stars*.

That the reaſon of the following Rules may the more eaſily be underſtood, the Marginal Schemes are added; in each of which, the *Circle* ÆPQS repreſents the *Meridian*, HO the *Horizon*, Z the *Zenith*, and N the *Nadir*, ÆQ the *Equinoctial*, P the *North Pole*, S the *South Pole*, and *de* a *Parallel* of *Declination*; H the *South* Point of the *Horizon*, O the *North* Point, H*d* or O*d* the *Meridian Altitude*, Z*d* the Meridional *Zenith Diſtance*, ÆZ the *Latitude* = OP or HS the *Pole's Elevation*. Theſe things being thus explained, the three firſt *Rules* are ſelf-evident, and need no Examples. *Fig.* 11, 12, 13, 14, 15, 16, & 17.

I. *If the Sun or Star have no Declination, the Meridional Zenith Diſtance is the Latitude; and if they come to the Meridian due South, the Latitude is Northerly; if due North, the Latitude is Southerly.*

II. *If the Sun or Star be in the Zenith, the Latitude is equal to the Declination, and on the ſame Side.*

III. *If the Declination and Zenith Distance are both equal, and of the same Denomination, the Latitude vanishes; and the Place is situated under the Equator.*

IV. *If the Meridional Zenith Distance and the Declination, are both the same way, their Difference is the Latitude; and if the Declination be* {*greater* / *less*} *than the Zenith Distance, the Latitude is on the* {*same* / *contrary*} *Side, that the Declination is of.*

Fig. 11. *Examp.* 1. The Declination North 22° 30′ Æ *d*
The Zenith Distance North 6 15 Z *d*
The *Latitude* North 16 15 Z Æ = QP

Fig. 12. *Examp.* 2. The Declination South 46° 40′ Æ *d*
The Zenith Distance South 12 50 Z *d*
The *Latitude* South 33 50 Z Æ = HS

Fig. 13. *Examp.* 3. The Zenith Distance South 62° 40′ Z *d*
The Declination South 11 10 Æ *d*
The *Latitude* North 51 30 Z Æ = QP

Fig. 14. *Examp.* 4. The Zenith Distance North 40° 25′ Z *d*
The Declination North 18 30 Æ *d*
The *Latitude* South 21 55 Z Æ = HS

V. *If the Declination of the Sun or Star be North or South, and the Zenith Distance be on the contrary Side, the Sum of both will be the Height of that Pole, that the Declination is towards.*

Fig. 15. *Examp.* 1. The Zenith Distance South 30° 50′ Z *d*
The Declination North 20 40 Æ D
The *Latitude* North 51 30 Z Æ = QP

Fig. 16. *Examp.* 2. The Zenith Distance North 16° 50′ Z *d*
The Declination South 30 45 Æ *d*
The *Latitude* South 47 35 Z Æ = HS

N. B.

Plate 10

Fig. I

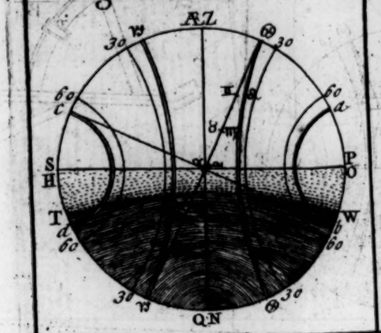


Fig. VI

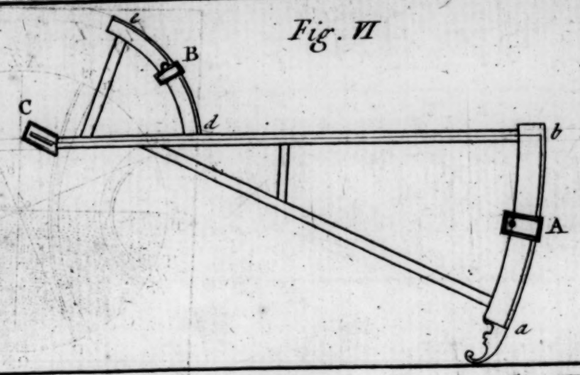


Fig. II

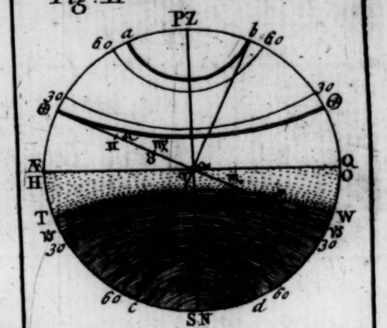


Fig. XI

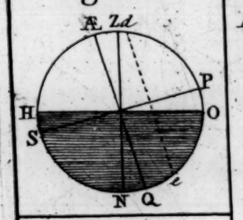


Fig. VIII.



Fig. V

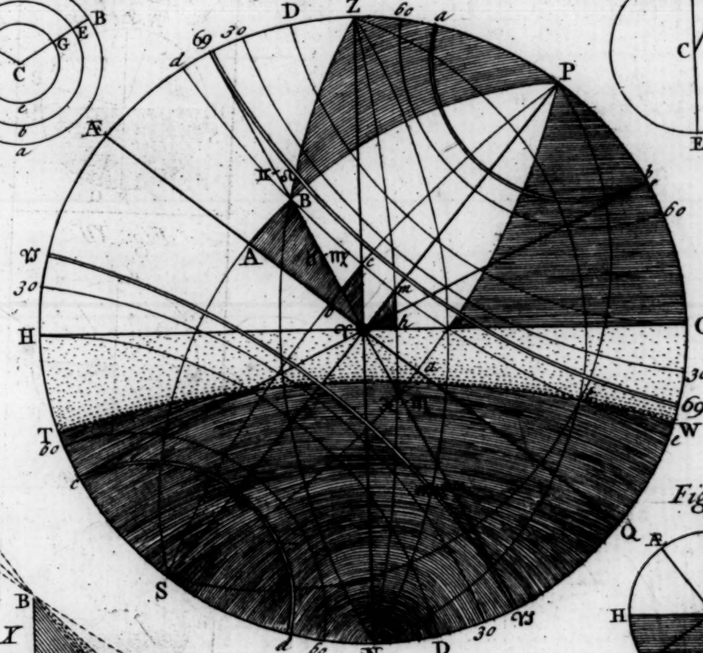


Fig. IX



Fig. XIV

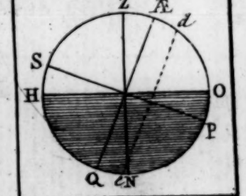


Fig. XII

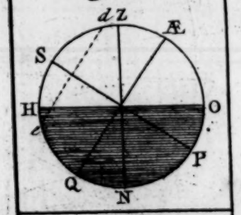


Fig. XV



Fig. XIII

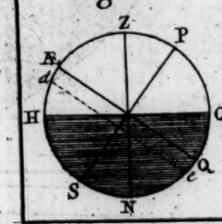


Fig. XVII



Fig. XVI

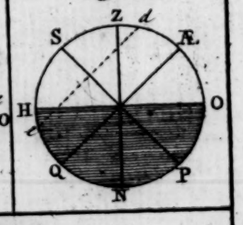


Fig. X

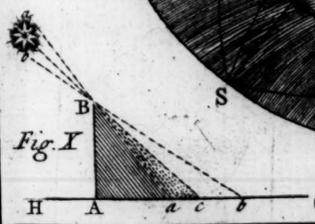


Fig. III

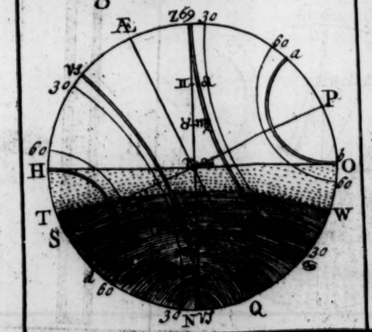


Fig. VII

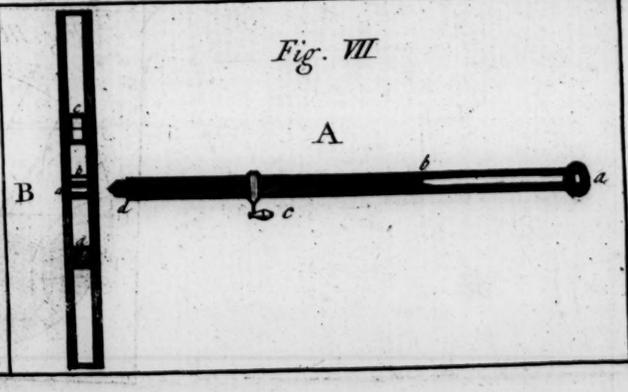
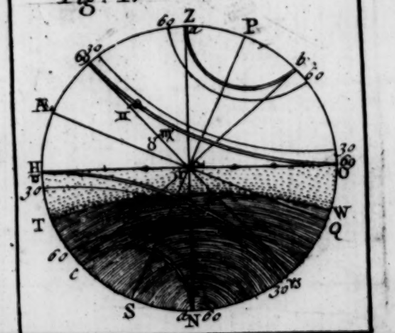


Fig. IV



N. B. If the *Declination* be greater than the *Meridian Altitude*, and (both) on the ſame Side, then the *Sun* or *Star* may have two *Meridian Altitudes* in 24 *Hours*, the one above the *Pole*, and the other below it. Or, let the *Meridian Altitude* and *Declination* be which way ſoever, if the *Meridian Altitude* and twice the *Co-Declination* be leſs than 180 *Deg.* the *Sun* or *Star* will tranſit the *Meridian* above the *Horizon* twice in 24 *Hours.*

VI. *When the obſerved Object hath two Meridian Altitudes, if the ſaid Object be upon the Meridian* {*below* / *above*} *the Pole, the* {*Sum* / *Diff.*} *of the Meridian Altitude and the Co-Declination, is the Height of the Pole towards which the Declination is.*

Examp. 1. The Meridian Altitude below the Pole 12° 20′ O *e* — *Fig.* 17.
The Declin. 65° 30′ N. its Complement 24 30 P *e*
The *Latitude* North 36 50 O P

Examp. 2. The Merid. Alt. betwixt the Pole and the Zen. 61° 20′ O *d*
The Declination 65° 30′ its Complement 24 30 P *d*
The *Latitude* North 36 50 O P

N. B. If the greateſt *Meridian Altitude* be of a contrary Side to the *Declination*, the *Latitude* is found as in the V*th Precept.*

CHAP.

CHAP. IV.

Practical Navigation.

Of the Methods uſed at Sea for eſtimating the Courſe ſteered and the Diſtance run; Of Finding the Variation of the Compaſs; the Manner of keeping and correcting a Sea-Journal.

SECT. I.

Of the Mariner's Compaſs, and the Log-Line, (being the Inſtruments uſed at Sea for eſtimating the Courſe and the Diſtance run) of finding the Ship's Drift or Lee-Way.

I. *Of the* MARINER'S COMPASS.

THE *Mariner's Compaſs* is an Inſtrument conſiſting of a *Card* and two *Boxes*; the *Card* repreſents the *ſenſible Horizon*, and is divided into 32 equal Parts called *Points*, as has been ſhewed in *Sect.* 3. of *Chap.* I. Underneath the *Card* is fixed a *Steel Needle* having a *Braſs Cupola* or hollow Center in the middle, which being placed upon the end of a fine *Pin*, the *Card* may eaſily traverſe round about; the *Needle* is touched with a *Loadſtone*, which gives it a Property of keeping itſelf in the *Meridian*, (the Deviation from it not to be here conſidered.)

This *Card* or *Fly* is put into a round *Box* (fitted for it,) having a *Pin* erected in the middle, upon which the *Center* of the *Needle* or the *Hollow* of the *Cupola* is placed; alſo the *Card* is ſo poized that it may lie *horizontal* in

in the ſaid *Box*, and *vibrate* freely according as the Vertue of the Touch ſhall lead it.

The *Box* wherein the *Card* is placed is covered over with a *ſmooth Glaſs*, to preſerve the *Card* from being diſturbed by the Air, which Glaſs is faſten'd round with a Cement; and alſo the ſaid Box is hung in a *Circle* or *Hoop* of *Braſs* upon two *cylindrical Pins* fixed diametrically oppoſite in the Outſide thereof, and the ſaid *Circle* is hung within another Braſs *Circle*, upon two *Axes* or *cylindrical Pins* placed at right Angles to the former; theſe two *Circles*, and the *Box*, are placed within another *ſquare* wooden *Box*, ſo that the innermoſt *Box*, and conſequently the *Card*, may ſtand horizontal which way ſoever the Ship heels.

A Caution in framing and placing the Bittacle.

The *Compaſs* (thus fitted) is placed near the *Helm* in a Caſe of Wood faſtened to the Deck, called the *Bittacle:* In making this Bittacle there ought to be great care taken that the *Hole* wherein the *Compaſs* is placed, be exactly *ſquare*, (or a right $\angle^{d}$ *Parallelogram*;) and afterwards the Bittacle ought to be ſo fixed, that two of the Sides of this ſquare Hole may be exactly parallel to the true Direction of the Ship, (which is generally ſhewed by the *Creviſe* between the two middle Planks upon the *Deck*,) ſo that by putting one Side of the *Compaſs Box* (which muſt be likewiſe exactly ſquare) cloſe to one of the Sides of the ſquare Hole or Box in the *Bittacle*, you'll be ſure that the *Compaſs* is rightly placed; then the *Point* on the *Card*, which is cut by a right Line drawn in the fore-part of the innermoſt Box, (which Line, and its oppoſite, ought to be exactly in the ſame Direction with two Sides of the outward Box) is the *Courſe ſteered*, or the *Point* of the *Compaſs* caped at by the Ship, (allowing for the *magnetick Variation*, if there be any.) I have been thus particular concerning the *Bittacle*, becauſe an Error either in making or fixing it, will cauſe a continual Error in the Courſe.

That two Compaſſes in the ſame Bittacle attract each other.

In large Ships where there are commonly two Men at the *Helm*, there is alſo placed two *Compaſſes* in the *Bittacle:* but I often obſerved that theſe two *Compaſſes* (when rightly placed) would differ from one another about $\frac{1}{2}$ *Point*; which Difference the Sailors attributed to one being touched with a better *Load-ſtone* than the other. But it is well known, that all *Needles* being rightly touched upon any *Load-ſtone* (however different theſe Load-ſtones may be as to the Power of Attraction) will point exactly the ſame way; therefore it plainly appeared, that theſe two *Compaſſes* were placed too near together, and ſo attracted each other out of their true Direction: and ſo I found it upon trial, for having placed them at a conſiderable Diſtance from each other, (ſo that the Sides of the *Boxes* lay in the ſame right Line) I found the *Cards* then pointed exactly the ſame way. And in trying of other *Compaſſes*, I found the like Circumſtances.

* *That to uſe a ſlow Card (as the Phraſe is) in bad Weather, is abſurd.*

* In ſhort tumbling Seas, when the Ship *yaws* or falls off very ſuddenly, a good *Card* will neceſſarily ſeem to *vibrate* very faſt; but when the *Card* (by reaſon of ſome Accident) will not *traverſe*, it will be carried along with the Ship without having hardly any apparent Motion. And in bad Wea-

Weather the Sailors generally make uſe of the worſe *Card* they have, alledging, that a good one will *traverſe* ſo faſt, that they cannot *ſteer* by it. But this is undoubtedly wrong, for if the *Card* will not traverſe, it may as well be laid aſide; and although it is true, that they cannot keep the Ship very ſteady at ſome times, and conſequently not ſo as to agree with the *Card*, yet if they would but conſider that the *apparent Motion* of the *Card* ſhews the *real Motion* of the Ship, and not of itſelf; which Motion of the Ship being thus ſhewed by the *Card*, they have the advantage of knowing how and when to check her, and conſequently of bringing her ſteady much ſooner than otherwiſe they could.

Some Experiments proving the Compaſs to be faulty, and how theſe Faults may be diſcovered.

Being upon Deck in a Star-light Night, I obſerved that the Ship *yawed* very much, and at the ſame time looking at the *Compaſs*, I found little or no Alteration in the *Courſe* pointed thereby; then I took an Inſtrument, and obſerved the greateſt and leaſt Angle made by two Lines paſſing from my Eye, one by a certain *Rope*, and the other to a *fixed Star*, and by taking the Difference of theſe two Angles, I found that the Ship *yawed* no leſs than 3 *Points*, or 1 $\frac{1}{2}$ *Point* on each Side; though, at the ſame time, according to the *Compaſs*, ſhe ſeemed to go forwards almoſt in a right Line. From this Experiment it plainly appeared, that the *Card* did not *traverſe* or keep towards its reſpective *Pole*, but was carried along with the Ship without being able to keep in its own *Meridian*; for if it traverſed as it ought to have done, it would have ſhewed the ſame Irregularity in the Motion of the Ship, as I obſerved by looking at the Stars; which being known to the *Steerſman*, (or Man at the *Helm*) undoubtedly he would have kept the Ship more ſteady, without ſuffering her to deviate ſo much from her true Courſe. Another time being at *anchor*, the Water very ſmooth and ſtill, I held a Knife a little on one Side of the North Point of the *Card*, and ſo drew it almoſt a *Point* out of its Place, (having firſt taken notice what Degree it pointed at before, and alſo how another *Compaſs* ſtood, which was hard by) then finding it not inclined to go back to the Place I drew it from, I went away and came again in about half an Hour, when I found the two *Cards* exactly in the ſame Situation wherein I had left them; then I drew the former (by my Knife) to the Place it was at firſt, afterwards a *Point* on the other Side, in all which Poſitions I found it would ſtand, as I left it: whereby it plainly appeared, that there was ſome Fault in the *Card*. Theſe Experiments I here mention (which may be often eaſily made at Sea) as a Caution for Mariners to look that their *Compaſs* be in good Order, and traverſe as it ought to do, and not implicitely rely upon the *Card*, without examining into the Goodneſs of it.

The Cauſe of the Faults afore-mentioned in the Sea-Compaſs.

The chief Cauſe of the fore-mentioned Defects in the *Sea-Compaſs* muſt undoubtedly be owing to ſome Fault either in the *Center Pin*, (whereon the *Needle* is placed) or in the *Cap* of the *Needle*, or both; as the Head of the *Pin* being either ruſty, or blunt, or the *Cap* of the *Needle* being ruſty *&c.* For as the attractive Power of the *Magnet* is but ſmall in proportion

tion to that of *Gravity*, the leaſt Reſiſtance or Friction about the *Center* of the *Needle* will very much hinder its *Vibration*, as is well known to thoſe who have made Experiments by *Magnetic Needles:* it is no wonder then if this ſhould have a greater Influence upon a *Sea-Compaſs*, where the *Needle* is clogged with a heavy *Card*, and the *Center-Pin* very liable to be wore and blunted by the continual jolting of the Box wherein the Card is placed, and alſo becauſe the Sea-Air is very apt to ruſt *Braſs* and *Iron*.

How the Sea-Compaſs may be improved, ſo that the Needle, &c. may be eaſily kept in good order.

The common Method of faſtening the *Glaſs* to the top of the Box, is certainly very inconvenient and abſurd; for then the *Card* is as it were locked up, ſo that you cannot eaſily come at the *Needle* to clean it, and beſides the common Method of jolting the Card off and on, muſt very much ſpoil the Pin in the Center. Wherefore, to prevent this Inconveniency, the *Glaſs* ought to be faſtened in a Braſs *Ring* fitted to the Head of the *Box*, and made either to turn upon a Hinge, or otherwiſe, as may be thought convenient; ſo that at any time it may be eaſily lifted up or taken off. The Inſtrument being thus prepared, you may eaſily clean the *Center* of the *Needle*, and alſo the *Head* of the *Pin* whereon it is placed, (which ought to be often done) with a hard Piece of Wood; and when the Head of the *Pin* is wore blunt, it ought to be ſharpened with a ſmooth *File*. Then when the Inſtrument is not uſed, you may eaſily take off the *Card*, (without jumbling the Box, which is the Cuſtom) and ſo put it on again as occaſion requires; and beſides, you may have the advantage of having ſeveral *Cards* (which ought to be as light as poſſible, and their Needles of ſufficient weight to govern them) fitted to the ſame *Box*; ſo that at any time you might uſe that which you find beſt, which would be much better, and leſs chargeable than the Lumber of *Compaſs-Boxes* carried in moſt Ships for the ſake only of their Cards. This Addition to the *Sea-Compaſs*, as little as it may ſeem to be, I am ſure (for the reaſons given above) would be a very great Improvement in that Inſtrument; and conſequently of great Benefit in the *Practical Part* of *Navigation*.

I need not here take notice, that a heavy *Braſs Compaſs* is far preferable to a light *Wooden One*, that, I ſhould think, being ſelf-evident: Yet notwithſtanding the later is moſtly uſed at Sea, which by reaſon of its being light is more affected by the Motion of the Ship, thereby keeping the Card in a greater Motion; which is the reaſon given by Sailors for uſing it: but that is certainly a wrong Argument, for the leſs the *Card* is diſturbed, the nearer will it keep to its true *Meridian*.

II. *Of the* Log, *and* Log-Line.

The moſt approved way that has hitherto been uſed, and which is now chiefly followed for eſtimating the *Diſtance run* by a *Ship* at *Sea*, is by the Log and Log-Line.

The Log is only a flat Piece of Wood made ſomewhat like the Shape of a Flounder, having a Piece of *Lead* faſtened to the Bottom, in order to make it ſtand or ſwim upright in the Water; to this *Log* is tied or faſtened a *long Line*, which is called the Log-Line. This *Line* is divided into certain Spaces, called Knots, which Spaces ought to be ſuch a part of a *Nautical Mile* (60 of which make a Degree of a Great Circle) as *half a Minute* (the Time commonly allowed for the Experiment,) is of an *Hour*.

It has been ſhewed in *Chap.* I. *Sect.* 1. that a *Degree* of a great Circle upon the Superficies of the Earth conſiſts of above 360000 *Engliſh Feet*; but for the ſake of a round Number we'll ſuppoſe 360000 *Feet* to be the exact Meaſure of a *Degree*: then a *Nautical Mile* (being $\frac{1}{60}$ Part of a Degree) will contain 6000 *Feet*; which Number being divided by 120 (the number of $\frac{1}{2}$ *Minutes* in an *Hour*) gives 50 *Feet*, the length of the *Spaces* between each *Knot* upon the *Log-Line*.

Theſe *Spaces* commonly commence at about 10 Fathom from the *Log*, (that ſo the *Log* when it is hove over board may be out of the Eddy of the Ship's *Wake* before they begin to count) and are numbered by a Piece of Twine with *Knots* inreeved between the Strands of the *Line*, which ſhew how many *Knots* are run out during the *Half Minute*: And it would be very convenient to have another Line equal in length to the Space between *Knot* and *Knot*, divided into 10 equal Parts, (each Part being of conſequence 5 Feet) for the ready meaſuring the *Decimal Parts* of a *Knot* upon the *Log-Line*, or theſe Parts may be marked upon one of the *Rails* of the *Ship*; ſo that by applying thereto the Space run out more than a certain number of whole *Knots*, you might eaſily find how many Parts of a Knot this Space contains.

The *Log* thus prepared being hove over board from the Poop, and the Line veered out (by the help of a *Reel* about which it is wound) as faſt as the *Log* will carry it, or rather as the Ship ſails from it; will ſhew according to the time of Veering the *Diſtance* ſailed, and conſequently the Rate of ſailing.

Hence knowing how many of theſe *Knots* or *Spaces* the Ship runs in *half a Minute*, we know how many *Miles* ſhe ſails in an *Hour*, ſuppoſing her to move with the ſame Velocity during that time: But if the *Gale* has not been the ſame during the whole Hour, or if there has been more Sail ſet, or any Sail handed, that ſo the Ship has run more or leſs in any Part of the Hour, than ſhe did at the time of the Experiment; there muſt be allowance made for it according to the Diſcretion of the Artiſt.

The Quantity above aſſigned for the length between each *Knot*, (*viz.* 50 *Feet*) is found by Experience to be very near the Truth; and undoubtedly ought to be uſed till we have a greater Certainty of the Meaſure of the *Circumference* of the *Earth*. But it ſeems the firſt that divided the *Log-Line* happened to make the Diſtance between the *Knots* but 42 Feet; and this Diviſion is ſtill uſed by ſeveral, though they know it to be

be falſe, and therefore are obliged to ſhorten their *Half-Minute Glaſſes* by 5 or 6 *Seconds*; the Abſurdity of which having been ſo often ſhewed by ſeveral Writers, I ſhall inſiſt no farther upon it in this Place.

The *Log-Line* being duly divided, (*viz.* by making 50 *Feet* between each *Knot*) if the *Half-Minute Glaſs* ſhould happen to run out either too faſt or too ſlow; having meaſured the Time the Glaſs requires to run out, it will be,

As the Number of Seconds the Glaſs requires to run out, Is to half a Minute or 30 *Seconds; So is the Diſtance given by the Log, To the true Diſtance run in that time.*

Thus, if a Ship ſails at the rate of 4 ½ *Knots* in 24 *Seconds* (or while a Glaſs of that length is running out) to find the *Rate* of *ſailing*, it will be,

24 Sec. : 30 Sec. :: 4.5 Knots : 5.6 Miles; the true Diſtance ſailed by the Ship in an Hour.

Again, If the *Glaſs* be of a due Length, and there be an Error in the Diviſions upon the *Log-Line*, or if there ſhould be an Error in both, the true Quantity ſailed in an Hour may be found by making proportional Allowances; but it would be needleſs to have Examples of that kind in this Place, ſince it is juſt as eaſy to divide the *Log-Line* true, as otherwiſe: and if at any time the *Line* ſhould be found to be lengthened or ſhortened, by reaſon of its ſwelling or ſinking, the *Knots* may be eaſily moved to their proper Diſtances.

In order to find the Time that the *Glaſs* requires to run out, ſome take a *String* having a *Plummet* at the end of it, and fix it upon a *Peg* at 38.2 *Inches* diſtance from the *Center* of the *Plummet*; then giving it a ſmall ſwing, they reckon the number of *Vibrations* made during the Time that the *Glaſs* is running out, to be the length of the *Glaſs* in *Seconds* of Time. This Method on *Terra Firma*, where the *Peg* that holds the *Pendulum* is at reſt, will do very well, (if the deſcribing Arches are but ſmall) but at Sea it muſt be very uncertain, becauſe the Motion of the Ship muſt needs ſome way or other diſturb the Motion of the *Pendulum*: Wherefore ſome of our Writers of Navigation have preſcribed to ſhorten the Pendulum juſt 7 Inches, then giving it a larger Swing, we are to count the *Vibrations* to be ſo many *Seconds* as before; but this is a Myſtery in *Mechanicks*, which I do not (nor I believe any body elſe) underſtand; and it is very abſurd to ſuppoſe, that at Sea the Motion of the *Ship* is always ſo regular, as to cauſe juſt the ſame Alteration in the Motion of the *Pendulum*.

The Error either in the *Log-Line*, or the *Half-Minute Glaſs*, or both, is ſometimes ſo great, that I have known two Ships in company toge-

ther, when one out-ſailed the other by her Reckoning, about 1 Mile in ſix. The Space between each *Knot* upon the *Log-Line* on board the Ship which reckoned herſelf to ſail faſter, was 42 Feet; and if the *Log-Line* on board the other Ship was rightly divided, it is evident the Difference was owing to the Error in the former *Log*, (for we afterwards found that the later Ship kept the beſt Reckoning;) but if the two *Log-Lines* were divided alike, the Difference muſt ariſe from the *Half-Minute Glaſſes* (ſo called) being not rightly adjuſted of a due Length: Alſo the Uncertainty of the *Log* is ſuch, that two People can ſeldom veer it alike, but will always make ſome Difference in the Diſtance.

III. *Of Finding the Ship's* DRIFT *or* LEE-WAY.

Plate 11. *Fig.* 1. *Lee* or *Lee-ward* Way, is the Angle which the *Keel* of the Ship makes with her *Wake*, or the Way ſhe really makes good through the Water: Thus for *Example*, Admit the *Wind* to be at *North*, and the Head of the Ship (by help of the *Rudder*) is kept up E NE or W NW towards *a* or *e*, within 6 *Points* of the *Wind*; but by reaſon of the Force of the *Wind* and *Surge* of the *Sea* upon her, ſhe is drove to *Lee-ward*, we'll ſuppoſe upon the *Rhumb-Line* C *b* or C *d*, then the ∠ *a* C *b* or *e* C *d* formed between the *Rhumb* upon which ſhe endeavours to ſail, and that upon which ſhe really makes her way good, is called the *Lee-Way*; and the *Lee-Way* is ſaid to be ſo much, according to the Quantity of the ∠ *a* C *b* or *e* C *d*.

The common Method among *Mariners* for determining the *Lee-Way*, is by the Sails that are then abroad, or according to what Sails the Ship then carries: The uſual Allowances are as followeth;

1. *When a Ship is cloſe hauled, if all her Sails be ſet, the Water ſmooth, and a moderate Gale of Wind, ſhe is then ſuppoſed to make little or no Lee-Way.*

2. *If it blow ſo freſh as to cauſe the ſmall Sails to be handed, 'tis uſual to allow one Point for Lee-Way.*

3. *If it blow ſo hard as to cauſe the Top-ſails to be cloſe reeft, the uſual allowance is two Points for Lee-Way.*

4. *When both Top-ſails are taken in, and the Sea runs high, it is ſuppoſed the Ship makes between* 3 *and* 4 *Points Lee-Way.*

5. *When the Fore-ſail is furled, and the Ship trys under a Main-ſail and Mizen, there is allowed* 4 *Points for Lee-Way.*

6. *If the Ship trys under a Main-ſail only, they allow* 5 *Points for Lee-Way.*

7. *But if ſhe trys under a Mizen only, ſhe is allowed* 6 *Points for Lee-Way.*

8. *When the Mizen is handed, and the Ship is trying a Hull, ſhe is then ſuppoſed to make* 7 *Points Lee-Way.*

Though

Though theſe *Rules* in ſome *Caſes*, and in ſome *Ships* (poſſibly) may be tolerably exact, yet they muſt needs be very uncertain, ſince it is well known, that ſome Ships with the ſame quantity of Sail, and with the ſame *Gale*, will make more or leſs *Lee-Way* than others: and alſo the ſame Ship when ſhe is out of her *Trim*, (or differently *Loaded*) will make different *Lee-Ways*; for it is obſervable, that the more Water a Ship draws, the leſs *Lee-Way* ſhe makes, becauſe ſhe then meets with a greater Reſiſtance in ſplitting the Water with her Side than otherwiſe ſhe would. Wherefore, Mariners would have us believe, that a Man muſt go abroad two or three *Voyages* in order to gain Experience, before he is capable of making proper Allowances for the *Lee-Way* of a Ship. We muſt allow Experience to be neceſſary in order to qualify Perſons for working the Ship, and managing her Sails, (without which, no Man can be a complete *Navigator*;) but I ſhall take upon me to ſhew how any Man the firſt time of his ever being on board, may determine the *Lee-Way* to a greater Certainty and Exactneſs than any one elſe can do by a pretended Experience, by judging of the Quantity of the Sails abroad, tho' he has been half a dozen Voyages to the *Indies*.

How to find the Lee-Way by Obſervation.

The *Wake* of the Ship, or the Way ſhe paſſed through the Water, may be eaſily diſcerned in the Day-time for a conſiderable Diſtance, and that let the Sea be ever ſo rough; wherefore we have nothing to do but take a *Compaſs* upon the *Poop*, (or the *Aft*-part of the Ship) and ſet her *Wake* as far as we can diſcern it, (for then we are leſs liable to Error) and the oppoſite *Rhumb* is the true *Courſe* made good by the Ship, then the Difference betwixt this and the Courſe given by the *Compaſs* in the *Bittacle*, is the *Lee-Way* required; which ought to be accordingly entered upon the *Log-board*. Or the *Lee-Way* may be eaſier found by the new *Azimuth Compaſs*, deſcribed in the next *Section*: thus, turn the Inſtrument about until you ſee the *Wake* of the Ship either over the *Sights*, or parallel to them, then the *Point* of the *Card* which is cut by the *Vertical Line* in the *Box* which is neareſt to you, is the true *Courſe*; whence the *Lee-Way* may be found as before. The *Lee-Way* being thus found in the Day-time, you may obſerve the Quality of the Sea, and the *Trim* of the Ship at that time; and by that means you'll be able to make a good Eſtimate of the *Lee-Way* in the Night.

If the *Lee-Way* be obſerved according to the above Directions, (as often as may be needful,) and theſe Obſervations punctually ſet down in the *Log-Book*, by the Officer of the reſpective *Watch*; I am ſure that *Mariners* would be able to keep better *Reckonings* than they commonly do: For how is it poſſible that a Perſon when he is either aſleep in his *Cabin*, or elſewhere from the *Deck*, ſhould be able to make allowance for *Lee-Way*, and other Accidents which happened in another's *Watch*; undoubtedly ſuch an Allowance muſt be very imperfect.

The Method above ſhewed for finding the *Lee-Way*, although it may not be ſo exact as might be wiſhed for, yet would it be of great help

to

to *Mariners* if they would but put it in practice; and if they would be but ingenuous enough to make uſe of what Helps are afforded them in the *Practical Part*, and conform themſelves to Truth in the *Theoretick Part* of *Navigation*, I am ſure no reaſonable Perſon, (who has any Notion of the ſeveral Accidents that a Ship is liable to at Sea) would blame them for bad *Artiſts*, though they acknowledge themſelves to be out ſome ſcores of *Leagues* in their *Reckonings*. And if we do but conſider the Nature of Things, we ſhall have reaſon enough to ſuſpect, that thoſe who pretend to the greateſt Exactneſs in their *Sea-Journals*, are in reality the worſt *Artiſts*; for nothing but Ignorance can make People pretend to Impoſſibilities.

When the Ship is within ſight of Land, the Lee-way may be found without ſetting her Wake, as by the Example following:

Fig. 2. Suppoſe a Ship from A bear up NNE towards B, then as ſhe ſails along, obſerve carefully what Point on the *Shore* keeps ſtill on the ſame *Point* of the *Compaſs* without any Alteration, which let be C bearing NE; I ſay the true *Courſe* made by the Ship is the *Rhumb* AC: for it is evident, that no other Point can keep ſtill on the ſame *Point* of the *Compaſs*, but that which lies directly in the Line of the Ship's Way; wherefore the Difference between the NNE Line AB (the Point of the Compaſs the Ship *capes* at) and the NE Line AC in which ſhe is carried, or the ∠BAC, equal in this Caſe to two *Points*, is the Ship's *Drift* or *Lee-Way*.

After the ſame manner the *Lee-Way* might be found, if the *Courſe* ſteered by the Ship had been from the *Land*. But this Example is here laid down more for the Reader's Amuſement, than for any real Uſe.

For other Methods of determining the *Lee-Way*, Vide *Caſe* 2. and *Caſe* 4. of *Sect.* 4. of *Chap.* II.

SECT. II.

Of Finding the Variation of the Compaſs.

THE *Magnetical Needle* in moſt Places deviates from the true *Meridian*, either to the Eaſtward or to the Weſtward; and this Deviation of the *Needle* is called the *Magnetick Variation*, or the *Variation* of the *Compaſs*: So that the *Variation* of the *Compaſs* is an *Arch* of the *Horizon*, contained between the *Magnetical* and *true Meridian*, or between the *true North* and *South* Points, and the *North* and *South* Points pointed out by the *Needle*.

This *Variation* is found by comparing the *true Amplitudes* or *Azimuths* of the *Sun* or *Star*, with their *Amplitude* or *Azimuth* obtained by Obſervation with the *Needle*.

The true *Amplitude* of the *Sun* or any *Star* may be obtained, from proper *Data* by Calculation, by *Caſe* 1. of *Prob.* II. of *Sect.* 2. of *Chap.* III. and the true *Azimuth* may be obtained (from a proper *Data*) by *Caſe* 1. of *Prob.* III. of the ſame *Section*; and the *Magnetick Amplitude* or *Azimuth* is commonly obſerved at Sea, by an Inſtrument called the *Azimuth Compaſs*.

Of the common AZIMUTH COMPASS.

This Inſtrument conſiſts of the ſame Parts, and is alſo hung after the ſame manner with the common *Sea-Compaſs*, deſcribed in the laſt *Section*, only to the innermoſt *Box* of this Inſtrument (wherein the *Card* is placed) is faſtened a broad Circle of Braſs; one half of this Circle is divided into 90 *Degrees* by Lines drawn from a Point taken as a Center in the Periphery of the other Semi-circle: theſe *Degrees* are again ſubdivided into *Minutes* by concentrick Circles and diagonal Lines, and theſe Degrees and Min. are numbered from the Middle of the Diviſions on both Sides to 45 Degrees.

Upon the Center of theſe Graduations is placed a moveable *Index*, upon which *Index* is erected a *Sight*, and from the upper Part of this Sight down to the Middle of the Index, is faſtened a fine Thread to caſt a Shadow upon the Middle of the ſaid Index.

This broad Braſs-Circle is croſſed at Right Angles with two Threads, and from the Ends of thoſe Threads are drawn 4 ſmall black Lines on the Inſide of the Box; alſo there are 4 right Lines drawn at right Angles one to the other on the *Card*, (which are all the Graduations generally made upon this *Card.*)

The Manner of Obſerving the Amplitude or Azimuth.

If the Obſervation be for an *Amplitude* at *Sun-riſing* or an *Azimuth* in the *Fore-noon*, you muſt turn the Inſtrument about until you bring the *Weſt Point* of the *Card* under the *Center* of the *Index*, and alſo the four Lines on the Edge of the Card, and the four Lines on the Inſide of the Box, do exactly agree or come together. But if the Obſervation be for an *Amplitude* at *Sun-ſetting*, or an *Azimuth* in the *Afternoon*, you muſt turn the *Center* of the *Index* right over the *Eaſt* Point of the *Card*, and make the Lines within the *Box*, and thoſe upon the *Card*, to concur or meet together. The Inſtrument being thus fitted for the Obſervation, you are to turn the *Index* towards the Sun, till the Shadow of the Thread falls upon the Slit of the *Sight*, and on the Middle of the *Index*, or till you

you ſee through the *Sight* the *Center* of the *Sun* cut by the ſaid *Thread*; then the *Degrees* and *Minutes* cut by the inner Edge of the *Index*, is taken for the Sun's *Magnetical Azimuth* or *Amplitude* from the *Prime Vertical*, or from the Eaſt or Weſt Points of the *Horizon*, according as the Obſervation is made before or after Noon.

When firſt I ſaw this Inſtrument uſed, I was ſurprized to find ſeveral Degrees difference in Obſervations made thereby within a few Minutes of Time of one another; notwithſtanding the *Limb* being actually divided to every 2 *Min.* But when I had conſidered the Nature thereof, I was no longer ſurprized that People ſhould differ ſo much in their Obſervations; but on the other hand, was amazed to think how People could ſo long impoſe upon their Reaſon and Judgment, in pretending to make even nice Obſervations, by ſo imperfect as well as abſurd an Inſtrument.

That the Conſtruction of the common Azimuth Compaſs is abſurd, and that the Obſervations made thereby muſt be vaſtly uncertain.

As the *Variation* of the *Needle* can be no otherwiſe found than by the Needle, it is very evident that the ſaid *Variation* cannot be found, even on *Terra Firma*, (except *Microſcopes* and other Contrivances be uſed) to greater Exactneſs than you can divide a *Circle*, whoſe *Diameter* is the Length of that *Needle* which you make uſe of; or to greater Exactneſs than you can divide the *Limb* of the *Card:* It is very abſurd then to ſuppoſe, that by the help of an *Index* and a large Circle, (fixed to the Out-ſide of the Box wherein the Card is placed) the *Variation* may be found at *Sea* to a greater Exactneſs than poſſibly can be done by ſuch an Inſtrument on *Shore.* For if an Obſervation were to be made by the common *Azimuth Compaſs* on *Terra Firma*, (where we may have Time and Opportunity ſufficient for rectifying it, which is not eaſily had at *Sea*) the ſaid Inſtrument muſt be firſt of all rectified by help of the *Card*; and this Rectification will be more or leſs accurate, according to the Goodneſs and Length of the *Needle:* It plainly appears then, that whatever Exactneſs we pretend to by the help of the *Index*, is only imaginary, ſince the Goodneſs of the Obſervation entirely depends upon the Poſition of the *Needle*; for rectifying of which, the *Index* is of no manner of help to us, and conſequently it is very abſurd to pretend to make uſe of it, even on *Terra Firma.*

But when we conſider this Inſtrument as deſigned for *Sea*-Obſervations, we ſhall find its Conſtruction to be ſtill more abſurd; and that the Obſervations made by it at Sea muſt be vaſtly uncertain: And indeed, this imaginary Improvement of having an *Index*, makes the Inſtrument leſs fit for Obſervation than even the common *Sea-Compaſs.* The only Guide for rectifying the Inſtrument, is to bring the Croſs-Lines upon the *Card*, and thoſe in the inner *Box*, ſo as apparently to agree or co-incide; but this is very difficult to be done, by reaſon of the continual *Yawing* of the *Ship* and *Motion* of the *Card*, neither can you judge of that Poſition, ſo exact as you can diſcern the Courſe pointed by the common *Card*; becauſe when you look downwards upon the *Glaſs*, the ſmalleſt Alteration in the Poſition of the Eye will make an apparent Alteration in the Poſi-

Poſition of the *Card*; whereas when you look directly before you at a perpendicular Line drawn in the Box, you are not ſo liable to err. But granting the Lines on the *Card* and thoſe in the *Box* may be exactly brought together, yet that is in reality no manner of Guide for rectifying the Inſtrument; for as the *Ship* and the *Card* are both in continual Motion, the Motion of the *Card* is rendered ambiguous, and ſo you have no manner of Certainty, whether the *Needle* at the time of Obſervation be in its own *Meridian*, or not; for it may as well be a conſiderable deal either to the *Eaſtward* or to the *Weſtward*, and which (ſo far from knowing how much) it is, you cannot judge. Again, as the Ship by her continual *yawing* from one Side to the other, continually alters the Direction of the *Compaſs-Box*, one Perſon continually moves the *Index*, and another looks through the Sight, ſaying, *Very well*, (by way of notice to the other) ſo long as he ſees the Object; and I have known this notice *very well* given, when the *Index* had been moved 15 or 20 Degrees out of its former Poſition, and conſequently the Obſervation muſt be vaſtly uncertain, ſince there is no manner of Guide to direct where or when to take it. Laſtly, when the Lines in the *Box* and thoſe on the *Card* differ very much, a third Perſon moves the *Box* at random, as much as he thinks fit, (for he has no Guide to direct him how much he ought to move it, ſince he does not know in what Poſition the *Needle* is at that time with reſpect to its own *Meridian*) then the Obſervation is begun a-new, after that perhaps another, and ſo on may be a fourth, and a fifth, *&c.* ſo long as there is an Opportunity. All theſe Obſervations are preciſely taken to a *Minute* or two, after that lumped together, then a *Mean* is taken betwixt them all; which Mean (though it may differ conſiderably from all the Obſervations) is taken to be exact beyond all Contradiction; and yet nothing more common than the Morning and next Evening *Means* (as they are called) to differ $\frac{1}{2}$ *Point*, more or leſs, and ſometimes conſiderably more. Sometimes theſe *Means* will be compared together, then a new one taken betwixt them; but this is not always done, for ſome rather than allow the Obſervations to be faulty, will inſiſt that there was a great *Load-ſtone* under or near the Ship, and ſo account for the Difference that way.

From what has been ſhewed above, it appears, that the common *Azimuth-Compaſs* is one of the abſurdeſt Inſtruments that ever was uſed; for the Object in the Heavens, which is the only Guide for rectifying it, is not properly regarded, ſince we only turn a moveable *Index* to the Object, according as the Ship by her yawing turns the *Compaſs-Box*: And perhaps among the Catalogue of vulgar Errors, there is not an Inſtance wherein People impoſe more upon themſelves, than in their common Notion concerning the Accuracy of Obſervations made by this Inſtrument.

Having thus ſhewed the Defects of the common *Azimuth Compaſs*, I ſhall next give the Deſcription of a new one, whereby the *Magnetical Variation* may be found to as great an Exactneſs as can be reaſonably expected to be done at Sea.

The Deſcription of a new AZIMUTH-COMPASS.

This Inſtrument conſiſts of a *Card* and two *Boxes*, and is hung in two Braſs *Rings*, after the ſame manner with the common *Sea-Compaſs*; the innermoſt *Box* wherein the *Card* is placed, is made to turn eaſily in the (innermoſt) *Ring* without diſturbing the outward *Box*; to the out-ſide of the ſaid innermoſt Box is fixed two upright *Sights* (diametrically oppoſite) for rectifying the Inſtrument, by keeping the ſaid Sights directly to the *Object*.

Plate 11. *Fig.* 3. Thus, A is the round Braſs *Box* wherein the *Card* is placed, at the bottom of which is fixed a large Piece of Lead to keep it perpendicular to the *Horizon*; B B is the ſquare Wooden *Box* which contains all the reſt; *a* the outward *Ring*, which is hung upon two *cylindrical Pins* fixed diametrically oppoſite in the *Box* B B; the inward *Ring* is hung upon two *cylindrical Pins* fixed in the outward *Ring* at right Angles to thoſe upon which the ſaid outward *Ring* is hung: The Box A is (as above ſaid) hung in the inward *Ring*, and the two Rings turn eaſily round their reſpective *Axes*, ſo that the *Box* A (by its own weight) keeps always nearly in the ſame Poſition, notwithſtanding the Motion or Rolling of the Ship. C, D are two upright *Sights*, ſcrewed diametrically oppoſite to the out-ſide of the inward *Box*; C is the *Sight* which is next the Eye in time of Obſervation, in which is made a Slit ſo fine, that you can bear looking through it at the *Sun*, without being hurt by the Splendor of his *Rays*: D is the *Object-Sight*, having a wide Slit or Opening, in the middle of which is fixed a fine Thread to cut the Object; this Sight is alſo made ſo long, as to take the Bearing of any Object whoſe Altitude is not above 10 or 12 Degrees. E is an *Index* of about 7 or 8 Inches long, having two *Sights a*, *b*, and a ſhort *Pedeſtal d*; the *Index* turns upon the head of its *Pedeſtal* to any Elevation not exceeding 60 Degrees, and the ſaid *Pedeſtal* is to be ſcrewed on in the room of the *Sight* C, when the Altitude of the *Object* is above the reach of the *Sight* D. In the *Sight a* is made a Slit wide enough to look through at a *Star*; and in the *Sight b* is a wide Opening having a pretty large Thread in the middle for cutting the *Object* in time of Obſervation; alſo under this Thread is a ſmall *Hole* to let the Beams of the Sun paſs through (and ſo form a little white Spot) upon the Slit in the *Sight a*.

The *Glaſs* (which covers the *Box* wherein the *Card* is placed) is fixed in a *Ring*, which is faſtened to the Box by an *Hinge*, ſo that it may be eaſily lifted up and ſhut, for the eaſier placing or diſplacing the *Card* upon or off the *Center Pin*, and alſo for cleaning this Pin, *&c.* as occaſion requires. The *Card* is graduated like the Specimen in *Plate* 3. Within the *Box* is drawn two upright black Lines, exactly under the middle of the two upright *Sights*; and at 90 Degrees diſtance from theſe, are drawn two other black Lines after the ſame manner.

The

The Manner of obſerving the Magnetical Amplitude or Azimuth of the Sun or any Fixed Star, by this Inſtrument.

If the Obſervation be for the *Sun's Amplitude*, or his *Azimuth* when he is not above 12 Degrees above the *Horizon*, fix on the two long *Sights* C, D; then taking hold of the bottom of the *Sight* C, turn the Inſtrument about until looking through this *Sight*, you ſee the Thread in the oppoſite *Sight* cut the *Center* of the *Sun*; then holding your Hand faſt, keep ſtill the ſame Sight of the Sun, (by which means the Inſtrument will be kept entirely free from being affected by the *Yawing* of the Ship,) and at the ſame time let another Perſon carefully obſerve what Degree upon the *Card* is cut by the *Line* in the *Box* which is drawn under the *Object-Sight*, and that Degree is the *Magnetical Amplitude* or *Azimuth* of the *Sun* from the *North* or *South*-part of the *Meridian*, according to which it is that is ſpecified by the *Card*. But if the *Card* has not done vibrating, let your Aſſiſtant carefully obſerve what number of Degrees are cut by the foreſaid Line at both Extremes of the *Vibration*; and the middle between theſe two Vibrations will be the *Magnetical Amplitude* or *Azimuth* required. Or, if it be thought more convenient, your Aſſiſtant may mark the Degree upon the *Card* which is cut by one of the Lines in the Box, which are a Quadrant diſtance from the former; then counting 90 Degrees from that Point towards the Sun, will give the *Magnetical Amplitude* or *Azimuth* from the *North* or *South-part* of the *Meridian*, as before.

Note, The Obſerver muſt take care that he keeps the Inſtrument from turning with the Ship during the time of Obſervation, by conſtantly keeping the middle of the *Sight* and the *Sun* in the ſame Direction; and the Aſſiſtant muſt place himſelf ſo, that the Line which he obſerves to cut the *Card*, be directly before him: And thus if due care be taken in the Obſervation, I am ſatisfied the *Magnetical Amplitude* or *Azimuth* may be found to leſs than a *Degree*, which is ſufficiently exact for all Uſes in Navigation; and I believe above five times the Exactneſs of whatever could be done hitherto, notwithſtanding the pretended Nicety of ſome People in obſerving even the odd Minutes.

Note, It is beſt to take the *Sun's Amplitude* when its lower Limb is about $\frac{1}{4}$ Degree above the *Horizon*, becauſe the *Refraction* makes the Sun appear ſo high, when his Center is really in the *Horizon:* but indeed for Practice, (if the Place of Obſervation be not too far from the *Equator*,) the Bearing of the Sun may be very well taken for an *Amplitude*, at any Height not exceeding one or two *Degrees*.

If you would take the *Sun's Magnetical Azimuth*, when he is too high to be ſeen through the *Sights* C, D; then you muſt fix on the *Index*, and turn the Inſtrument about until the *Spot* coming through the little Hole in the *Object-Sight*, falls ſomewhere on the *Slit* in the other *Sight*; then the Degree upon the *Card* which is cut by the *Line* drawn in the fore-part of the

Box, is the *Azimuth* required. After this manner the Obſervation may at any time be taken by one Perſon only; but, I think, it is much better to have two Obſervers, for then each may better attend to its reſpective Part; neither is it any manner of Objection to have two Perſons on board a Ship (where there are always ſo many Hands, that two may be eaſily ſpared) to an Obſervation, except it be ſaid that you cannot have an Aſſiſtant that is qualified, or one whom you can depend upon, which I am very loth to ſuppoſe. Alſo, if the Obſervation be of a *Fixed Star*, you muſt likewiſe uſe the *Index*, and putting your Eye to the proper *Sight*, turn the Inſtrument about, and elevate or depreſs the *Index*, until the Thread in the *Object-Sight* (*b*) cuts the *Star*; then the black Line in the fore-part of the Box will point the *Azimuth* upon the *Card*.

Note, When you obſerve the *Magnetick Azimuth* of any Object, its Altitude muſt be likewiſe taken at the ſame time, in order to find the true *Azimuth*.

This Inſtrument is very uſeful for finding the *Lee-Way*, and alſo for ſetting the *Bearings* of *Headlands*, &c. for having directed the *Index* to the Wake of the Ship, the Card will point out the true *Courſe*; the Difference of which, and the Courſe given by the *Compaſs* in the *Bittacle*, is the Lee-Way required. After the ſame manner you may ſet the Bearing of any Place. This Inſtrument may be likewiſe fitted as a good Compaſs to ſteer by; for having taken off the Sights, and two Marks being made, one on the inner Ring, and the other on the inner Box, as a Direction for placing the *Meridian* of the ſaid *Box*, (or the Direction of the *Sights*) parallel to the Sides of the ſquare wooden Box; the Inſtrument being rectified by the ſaid Direction, you may place it in the *Bittacle*, (or elſewhere as you think convenient) and the *Card* will point out the true *Courſe* ſteered. Having thus explained the Uſe of this Inſtrument, I ſhall next ſhew how to determine the Variation.

Having by the Directions above found the *Magnetical Amplitude*, and by the *Latitude* of the Place, and the *Declination* of the Object, found the true *Amplitude* by *Caſe* 1. of *Prob.* II. *Sect.* 2. of the laſt *Chapter*; or having by the *Sun* or *Star's Declination*, *Altitude*, and *Latitude* of the Place, found the true *Azimuth* by *Prob.* III. of the ſame *Section*, and alſo the *Magnetick Azimuth* by the Directions above: The *Variation* of the *Compaſs* may be found as followeth.

If the *True* and *Magnetick Amplitude*, or the *True* and *Magnetick Azimuth* be the ſame, it is evident there is no *Variation*, but the *Needle* points exactly true.

N. B. For the eaſier determining the *Variation*, let the *True* and *Magnetick Amplitude* or *Azimuth* be always counted from the *North*-part of the *Meridian*; then the *Variation* will be always found by the two following Rules:

I. *If the Obſervation be made in the Forenoon, and the* {*True* / *Magnetick*} *Amplitude or Azimuth be the greateſt, their Difference is the Variation* {*Eaſterly* / *Weſterly*.

II. *If the Obſervation be made in the Afternoon, and the* {*True* / *Magnetick*} *Amplitude or Azimuth be the greateſt, their Difference is the Variation* {*Weſterly* / *Eaſterly*.

The Circle N, E, S, W, repreſents the *Horizon*, N the true *North* Point, S. the *South*, and E, W, the *Eaſt* and *Weſt* Points; *n* the *North* Point of the *Magnetical Meridian*, *s* the *South* Point thereof, *e* and *w* the *Magnetical Eaſt* and *Weſt*, or the *Eaſt* and *Weſt* Points inſpected by the *Card*; N ⊙ is the Meaſure of the true *Amplitude* or *Azimuth* from the *North*, *n* ⊙ the *Magnetical Amplitude* or *Azimuth*, or the Amplitude or Azimuth from the *North* Point of the *Card*, and N *n* the VARIATION. Theſe Schemes being thus explained, the reaſon of the following *Examples* will be very evident. *Fig.* 4, 5, 6, 7, 8, & 9.

Examp. 1. At *Sun-riſing*.

The Sun's true Amplitude N E^{ly}	82° 30′ N ⊙	*Fig.* 4.
The Magnetical Amplitude N E^{ly}	74 *n* ⊙	
Variation E^{ly}	8 30 N *n*	

Examp. 2. At *Sun-riſing*.

The true Amplitude from the North	86° 40′ N ⊙	
The Magnetical Ampl. from the North	98 *n* ⊙	*Fig.* 5.
Variation Weſterly	11 20 N *n*	

Examp. 3. At *Sun-ſetting*.

The true Amplitude from the North	112° 20′ N ⊙	
The Magnet. Ampl. from the North	98 *n* ⊙	*Fig.* 6.
Variation Weſterly	14 20 N *n*	

Examp. 4. At *Sun-ſetting*.

The true Amplitude from the North	76° 00′ N ⊙	
The Magnet. Ampl. from the North	97 *n* ⊙	*Fig.* 7.
Variation Eaſterly	21 00 N *n*	

Examp.

Box, is the *Azimuth* required. After this manner the Obſervation may at any time be taken by one Perſon only; but, I think, it is much better to have two Obſervers, for then each may better attend to its reſpective Part; neither is it any manner of Objection to have two Perſons on board a Ship (where there are always ſo many Hands, that two may be eaſily ſpared) to an Obſervation, except it be ſaid that you cannot have an Aſſiſtant that is qualified, or one whom you can depend upon, which I am very loth to ſuppoſe. Alſo, if the Obſervation be of a *Fixed Star*, you muſt likewiſe uſe the *Index*, and putting your Eye to the proper *Sight*, turn the Inſtrument about, and elevate or depreſs the *Index*, until the Thread in the *Object-Sight* (*b*) cuts the *Star*; then the black Line in the fore-part of the Box will point the *Azimuth* upon the *Card*.

Note, When you obſerve the *Magnetick Azimuth* of any Object, its Altitude muſt be likewiſe taken at the ſame time, in order to find the true *Azimuth*.

This Inſtrument is very uſeful for finding the *Lee-Way*, and alſo for ſetting the *Bearings* of *Headlands*, &c. for having directed the *Index* to the Wake of the Ship, the Card will point out the true *Courſe*; the Difference of which, and the Courſe given by the *Compaſs* in the *Bittacle*, is the LEE-WAY required. After the ſame manner you may ſet the Bearing of any Place. This Inſtrument may be likewiſe fitted as a good Compaſs to ſteer by; for having taken off the Sights, and two Marks being made, one on the inner Ring, and the other on the inner Box, as a Direction for placing the *Meridian* of the ſaid *Box*, (or the Direction of the *Sights*) parallel to the Sides of the ſquare wooden Box; the Inſtrument being rectified by the ſaid Direction, you may place it in the *Bittacle*, (or elſewhere as you think convenient) and the *Card* will point out the true *Courſe* ſteered. Having thus explained the Uſe of this Inſtrument, I ſhall next ſhew how to determine the Variation.

Having by the Directions above found the *Magnetical Amplitude*, and by the *Latitude* of the Place, and the *Declination* of the Object, found the true *Amplitude* by *Caſe* 1. of *Prob.* II. *Sect.* 2. of the laſt *Chapter*; or having by the *Sun* or *Star's Declination*, *Altitude*, and *Latitude* of the Place, found the true *Azimuth* by *Prob.* III. of the ſame *Section*, and alſo the *Magnetick Azimuth* by the Directions above: The *Variation* of the *Compaſs* may be found as followeth.

If the *True* and *Magnetick Amplitude*, or the *True* and *Magnetick Azimuth* be the ſame, it is evident there is no *Variation*, but the *Needle* points exactly true.

N. B. For the eaſier determining the *Variation*, let the *True* and *Magnetick Amplitude* or *Azimuth* be always counted from the *North*-part of the *Meridian*; then the *Variation* will be always found by the two following Rules:

I. *If the Obſervation be made in the Forenoon, and the* {*True* / *Magnetick*} *Amplitude or Azimuth be the greateſt, their Difference is the Variation* {*Eaſterly* / *Weſterly*.

II. *If the Obſervation be made in the Afternoon, and the* {*True* / *Magnetick*} *Amplitude or Azimuth be the greateſt, their Difference is the Variation* {*Weſterly* / *Eaſterly*.

The Circle N, E, S, W, repreſents the *Horizon*, N the true *North* Point, S the *South*, and E, W, the *Eaſt* and *Weſt* Points; *n* the *North* Point of the *Magnetical Meridian*, *s* the *South* Point thereof, *e* and *w* the *Magnetical Eaſt* and *Weſt*, or the *Eaſt* and *Weſt* Points inſpected by the *Card*; N ⊙ is the Meaſure of the true *Amplitude* or *Azimuth* from the *North*, *n* ⊙ the *Magnetical Amplitude* or *Azimuth*, or the Amplitude or Azimuth from the *North* Point of the *Card*, and N *n* the VARIATION. Theſe Schemes being thus explained, the reaſon of the following *Examples* will be very evident. *Fig.* 4, 5, 6, 7, 8, & 9.

Examp. 1. At *Sun-riſing.*

The Sun's true Amplitude N E^{ly}	82° 30′ N ⊙	*Fig.* 4.
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Variation E^{ly}	8 30 N *n*	

Examp. 2. At *Sun-riſing.*

The true Amplitude from the North	86° 40′ N ⊙	
The Magnetical Ampl. from the North	98 *n* ⊙	*Fig.* 5.
Variation Weſterly	11 20 N *n*	

Examp. 3. At *Sun-ſetting.*

The true Amplitude from the North	112° 20′ N ⊙	
The Magnet. Ampl. from the North	98 *n* ⊙	*Fig.* 6.
Variation Weſterly	14 20 N *n*	

Examp. 4. At *Sun-ſetting.*

The true Amplitude from the North	76° 00′ N ⊙	
The Magnet. Ampl. from the North	97 *n* ⊙	*Fig.* 7.
Variation Eaſterly	21 00 N *n*	

Examp.

Examp. 5. In the *Forenoon.*

Fig. 8.

The true Azimuth from the North	150° 30′	N ⊙
The Magnetick Azim. from the North	132	*n* ⊙
Variation Eaſterly	18 30	N *n*

Examp. 6. In the *Afternoon.*

Fig. 9.

The true Azimuth from the North	74° 30′	N ⊙
The Magnet. Azimuth from the North	52	*n* ⊙
Variation Weſterly	22 30	N *n*

SECT. III.

The Method of keeping and correcting a Sea-Journal.

THE moſt remarkable Occurrences that happen at Sea, (which any way relate to the Motion of the Ship) are punctually ſet down upon a *Board* divided into proper Columns, called the LOG-BOARD. The Particulars that ought to be noted upon the *Log-Board*, are as follow:

1. The *Log* ought to be hove once every Hour, and then the Number of *Knots*, and *decimal Parts* of a *Knot*, (if the *Log-Line* is decimally divided) muſt be entered into their proper Columns upon the *Log-board*; and this Experiment being repeated hourly during the whole 24 Hours, the Sum of the *Knots* and *Parts* upon the *Log-board* do ſhew the number of Miles ſailed in that time. *N. B.* If the *Gale* has not been the ſame during the preceding Hour, as at the time of the Experiment, or if any *Sail* has been *ſet* or *handed* (ſo as to cauſe an Alteration in the Velocity of the Ship) the Artiſt muſt make a diſcretional Allowance for the *Drift* of the Ship, and accordingly leſſen or augment the Diſtance given by the *Log*, as the Caſe requires.

2. The *Courſes ſteered* according to the *Compaſs*, muſt be carefully noted, and ſet down in a proper Column, each in its reſpective Place, ſo often as the Courſe is altered. Alſo the Alteration of *Winds*, *Weather*, &c. is uſually ſet down in another Column: likewiſe, what Sails have been ſet or handed, *&c.*

3. The *Lee-Way*, if there be any, ought to be carefully obſerved, as often as may be judged neceſſary; and theſe Obſervations muſt be punctually ſet down by the Officer of the reſpective Watch; at leaſt if no Obſervation be made, he ought to ſet down the *Lee-Way* according

to

to his Judgment, once or twice in the *Watch*. And by this means, the Courſe made good may be found to a much greater Certainty and Exactneſs, than by the common Method of allowing for *Lee-Way*, when the Day's Account comes to be worked, (which is generally once in 24 Hours.) For an Obſervation muſt certainly be better than any Gueſs; but if no Obſervation be made, the Perſon who is upon *Deck*, and has the Care of the *Watch*, is better able to make proper Allowances while things are freſh in his Memory, and while he is an Eye-witneſs of the ſeveral Accidents that happen; and certainly much more capable than another, who might not be at all upon *Deck* during the whole *Watch*.

I often admired to ſee how particularly every thing is ſtated upon the *Log-board*, excepting the *Lee-Way*; and yet that (which is one of the moſt material Articles, ſince the *Courſe* according to the *Compaſs* muſt be corrected by it) only allowed for next day, according to every one's Fancy; thereby, as it were, keeping as many different *Journals* as there are *Artiſts* (ſo called) on board the Ship, and yet not one regular *Journal* properly amongſt them all, ſince one of the moſt material Articles is only gueſſed at. I have reaſoned this Caſe with ſome People, and their Argument was, that ſince the ſeveral handing and ſetting of Sails is put down upon the *Log-board*, they think that to be ſufficient for determining the *Lee-Way*; and beſides, that they can't ſo well truſt to another's Judgment or Care in entering it upon the *Board*. The former of theſe Arguments I have already ſhewed to be unreaſonable in the laſt *Section*; and indeed it is no more reaſonable than if a Perſon would pretend to trace the Way of a Ship during a whole *Voyage*, by the help of an old *Log-Book* (having only the ſeveral Obſervations of the *Latitude* added to it) kept on board of her a Twelve-month before: As to the later, I am very loth to ſuppoſe, that a Perſon who has ſo great a Charge upon him as the Management of a Ship, would be ſo negligent, or could be ſo ignorant, as not to make proper Entries of the *Lee-Way* upon the *Log-board*, were it but the Cuſtom ſo to do; and indeed to queſtion a Perſon's Abilities in this Point, would be as great an Affront done to him, as if he were ſuſpected of not being qualified to heave the *Log*, or to note down the proper *Courſe* according to the *Compaſs*.

4. If you ſuſpect any *Currents*, they ought to be tried, and the *Setting* and *Drift* put down on the *Log-board*: Alſo the *Variation* of the *Compaſs* ought to be obſerved once in two or three Days, if Opportunity offers; however the Allowance made for it, ought to be daily expreſſed.

5. When you are within ſight of *Land*, you may ſet the *Bearings* of ſome remarkable Parts of it, from different Stations, and thence you'll be able to correct any Error that may be in the Deſcription of that Part of the *Coaſt*, &c. Alſo when you are within *Soundings*, the *Quality* of the *Ground* and the *Depth* of the *Water* may be properly entered upon the Log-board.

Every Day at *Noon* this Account is taken off the *Log-board*, and entered into a *Log-Book*, and from thence every body that keep a Reckoning tranſcribe it into their own *Log* or *Journal-Books*.

The Method (I would recommend) for dividing a Journal-Book, is as followeth:

Plate 12. The firſt ſix Columns reaching to the bottom repreſent the *Log-Book*, wherein muſt be inſerted the ſeveral Particulars noted upon the *Log-board*; alſo the *Log-board* ought to be divided after this manner. In the firſt *Column* marked H are inſerted the *Hours* of the *Day*, beginning at *Noon*, and ſo numbered to twice 12, until they end at *Noon* next Day; In the ſecond and third *Columns* muſt be inſerted the *Knots* and *Parts* run each Hour, the *Column* K ſignifying *Knots*, and that maked P^ts^ the *Decimal Parts* of a *Knot:* In the fourth *Column* muſt be noted the ſeveral *Courſes*, in the fifth the *Winds*, and in the ſixth mark'd L. W the *Lee-Way*. And in the 5th Column may be noted the ſetting or handing of Sails, (if any body has the Curioſity;) alſo the Quality of the *Weather*, and if a *Current* was tried, its *Setting* and *Drift* muſt be put down in the *Log-Book*, as alſo any other Particular which may be thought convenient.

This *Table* is here divided into Parts correſponding to the ſeveral *Watches* kept on board a Ship, but it is beſt to have the Diviſions upon the *Log-board* for every Hour: The Dots in the Lines are made as a Direction for placing the ſeveral Particulars right againſt the reſpective Hours.

The ſmall Table on the right-hand is a *Traverſe-Table*, made ſo long as to contain all the *Traverſes* requiſite to be made in any *Day's Work*. In the firſt of theſe Columns muſt be placed the ſeveral *Courſes* corrected, by making due allowance for *Variation* and *Lee-Way*, (if there be any.) In the ſecond Column muſt be placed the *Diſtance* correſponding to each ſingle Courſe, and in the other four Columns muſt be placed the Alterations of *Latitude* and *Departure* anſwering to each Courſe, as has been ſhewed in *Sect.* 3. of *Chap.* II. The two *Compaſſes* are drawn in the blank Space under the firſt Column, as a Guide for correcting the ſeveral Courſes (by allowing for *Variation* and *Lee-Way*,) with the more Eaſe and Certainty; the one having the North Point upwards, and the other the South, to be uſed according as the Courſe ſteered is Northerly or Southerly.

In the blank Column at the head of the Table may be properly put the *Running Title*, viz. the Name of the Ship, the Voyage, and the Date of the Year: And in the blank Column at the head of the *Traverſe Table*, may be noted the Day of the Week, and the Date of the Month.

Underneath the Traverſe Table, againſt *Latitude* by { *Account* — / *Obſervation* — } muſt be ſet down the { computed Latitude according to dead Reckoning / the Lat. by Obſervation (if any had been made.) } If theſe two agree, the Reckoning is ſuppoſed to be exact; but if they differ above 10 or 15 Min. the *Departure* muſt be corrected by the Rules here-

hereafter given; and the *Departure* ſo corrected muſt be ſet down againſt *Cor. Departure*, and in the ſame Line againſt *Rule*, muſt be ſet down the N° of that Rule whereby you corrected the *Departure*; by which means you'll know at any time afterwards what allowance you then made, which may be of good help to you when you come to correct another Day's Work.

Againſt *Lat.* { *Laſt* ⊙ — / *This Day* ⊙ — } *M. Parts*, muſt be noted the *Latitude* by { the laſt preceding / this Day's } Obſervation; and againſt theſe the reſpective *Meridional Parts*, and underneath the *Proper* and *Meridional* Difference of Latitude. Underneath is expreſſed the Proportion for finding the *Difference* of *Longitude*, which muſt be accordingly wrought as has been exemplified in *Sect.* 1. of *Chap.* II.

At the bottom may be ſet down the *Latitude* the Ship was in at *Noon*, and the *Longitude* from the laſt known *Land*; and alſo the *Bearing* and *Diſtance* of ſome remarkable Place, if it be thought convenient.

N. B. The Column marked *Winds*, &c. is made ſufficiently wide, for inſerting any remarkable Occurrences, (which ought to be ſet down againſt the reſpective Hours wherein they happen) as the Quality of *Soundings*, *viz.* the Depth of Water, and the Quality of the Ground; the Alteration of Weather; any remarkable *Fiſh* that had been ſeen; the Diſcovery of *Sands* or *Shoals*, &c. alſo the Handing of *Sails*, &c. if it be thought convenient. And when the Ship is cloſe upon a Wind, there is no neceſſity of ſetting down the ſeveral Alterations of it, (for the *Courſes* expreſſed in the next *Col.* will determine that) if you want room for any thing elſe.

This Method of Journalizing a Day's Work, and noting in the *Journal-Book*, all the Occurrences that happened, and alſo working the *Traverſes*, and the proportion for finding the *Difference* of *Longitude*, &c. in the ſame Book, is undoubtedly far preferable to any other; and indeed is very neceſſary, for by this means a Perſon is capable (if need be) of over-looking a whole *Voyage*, and correcting any Miſtake (of his) which might have ſlipt in it during the whole time; which otherwiſe could not be done. And if Mariners would have their Books printed or ruled like this Specimen, they would ſave themſelves a great deal of trouble, beſides the advantage of having a ready and true Direction to proceed in every Particular.

Having thus explained the Nature and Method of a *Journal-Book*, (which in my humble Opinion is far preferable to any other Form that has hitherto been uſed;) I ſhall next give Directions for working a Day's Reckoning, by making proper allowance for *Variation*, *Lee-Way*, &c.

Of Working a Day's Account.

1. If there be but one *Courſe* upon the *Log-board*, then you muſt add all the *Knots* and *Parts*, and take the Total for the *Diſtance ſailed*: But if the

Ship has ſailed on ſeveral *Courſes* during the 24 Hours, draw a Daſh at the end of each *Courſe*, and add up the *Knots* and *Parts* correſponding to theſe ſeveral *Courſes*, and ſet down the ſeveral *Sums*; thus, in the firſt *Day's Work*, at end of this *Section*, the *Diſtance ſailed* on the firſt *Courſe* is 22 *Miles*, on the ſecond 32.5 *m.* and on the laſt Courſe 65 *m.*

2. The ſeveral *Courſes* muſt be corrected by allowing for *Variation* and *Lee-Way*, (if there be any;) which will admit of three *Caſes*.

1. When there is only *Variation*, but no Lee-Way.
2. When there is only *Lee-Way*, but no Variation.
3. When there is both *Variation* and *Lee-Way*.

1. *The Courſe ſteered by the Compaſs and the Variation, being given; to find the true Courſe.*

If the *Variation* (for inſtance) be 1 *Point* {Eaſterly / Weſterly} then the *North* Point of the *Card*, points the true {N *b* E, / N *b* W,} and conſequently all the other *Points* of the *Card* are ſo much more {Eaſterly / Weſterly} than they ought to be: Or which is all one, the true *Points* of the *Horizon* are more to the {Left / Right} hand from the *North*, than the *Points* upon the *Card*; whence we draw this general Rule for finding the true *Courſe*.

If you ſuppoſe yourſelf to ſtand in the Center of the *Compaſs*, with your Face turned towards that *Point* upon which the Ship is *ſteered*, and if the *Variation* be {Eaſterly / Weſterly} count the Quantity thereof from the Courſe ſteered, towards the {Right / Left} hand, and the *Point* of the *Compaſs* thus found will be the true *Courſe ſteered*: Thus, if the Courſe ſteered be N E, and the *Variation* be 1 *Point* Weſterly, the true *Courſe* will be N E *b* N, 1 *Point* more towards the left-hand, (or nearer to the North;) but if the *Variation* had been 1 *Point* Eaſterly, then the true *Courſe* would have been N E *b* E, 1 *Point* more towards the right-hand. After this manner were corrected the ſeveral Courſes in the firſt *Day's Work*, at the end of this *Section*, (where is no *Lee-Way*;) thus the firſt *Courſe* according to the *Compaſs* being S W *b* S, and the *Variation* reckoned 1 $\frac{1}{2}$ *Point* Weſterly, the true Courſe is S *b* W $\frac{1}{2}$ W, which is accordingly ſet down in the *Traverſe Table*; and the ſame Method muſt be obſerved in all other Caſes whatſoever.

2. *When*

2. *When there is no Variation, the Course steered by the Compass, and the Lee-Way, being given; to find the true Course.*

If you suppose yourself to stand in the *Center* of the *Compass*, with your Face turned directly to *Windward*, and if the {*Star-board* / *Lar-board*} *Tacks* be on *board*, then the *Lee-Way* counted from the Course steered by the Compass towards the {*Left* / *Right*} *hand*, is the true *Course* required. Thus, for Example, if the *Wind* be at S E, and the Ship *steers* E N E, making 1 *Point Lee-Way*, the true *Course* (no allowance being made for *Variation*) will be N E *b* E; but if the *Wind* had been at *North*, and the *Course steered* and *Lee-Way*, the same as before, the true *Course* would have been E *b* N, 1 *Point* more towards the right-hand; and the same Rule will hold good in all other Cases.

3. *The Course steered by the Compass, the Variation and Lee-Way, being given; to find the true Course.*

1. Find the Course steered, by allowing for *Variation* only; then from that Course allow for *Lee-Way*, and the *Course* last found will be the true Course required. Thus, for Example, let the *Course steered* be N N E, the *Wind* at East, the *Lee-Way* 1 *Point*, and the *Variation* 1 $\frac{1}{2}$ *Point* Westerly; the Course corrected by the *Variation* is N $\frac{1}{2}$ E, and the *Lee-Way* being counted from that Point towards the left-hand, (because the *Star-board Tack* is on *board*) gives the true *Course* N $\frac{1}{2}$ W. Again, let the Course *steered* be N E, the *Variation* 1 *Point* Westerly, the *Lar-board Tacks* on *board*, and the *Lee-Way* 1 $\frac{1}{2}$ *Point*, the true *Course* will be N E $\frac{1}{2}$ E; because the *Lee-Way* and the *Variation* in this Case destroy each other, and because the *Lee-Way* is greatest, the Excess = $\frac{1}{2}$ *Point* is to be allowed for *Lee-Way* from the Course steered: but if the *Variation* had been greatest, the Difference must have been allowed for *Variation*, and thence the Course corrected, without any allowance for *Lee-Way*.

The easiest way for Beginners to correct the Ship's Course, is to have the *Compass* drawn in some convenient place in the *Journal-Book*; then noting down the Place of the *Wind*, the *Variation*, and the *Course* steered, the true *Course* may be easily found: thus, let the *Data* be the same as in the last *Example*, having drawn the *Compass*, and laid the *Variation* = 1 *Point* Westerly from N to *n*, then will *n* be the North Point of the *Magnetick Meridian*, or the North Point of the *Card*; from *n* lay 4 *Points* (on the right-hand) to *a*, then will N *a* be 3 *Points*, and consequently *a* is the true N E *b* N, the Course corrected by the *Variation*. Again, because the *Lar-board Tacks* are on board, the *Wind* is to the left-hand, (some- *Plate* 11. *Fig.* 10.

where towards *d*,) and becauſe the Ship endeavours to ſail from C towards *a*, the *Lee-Way* is to the right-hand, therefore lay the Quantity thereof = 1 ½ *Point* from *a* to *b*; then will *b* be the true N E ½ E *Point*, the *Courſe* upon which the Ship has made her Way good.

The *Courſes* in the *Log-book* being corrected, they muſt be entered in their proper Column in the *Traverſe Table*, and againſt them their reſpective *Diſtances*; then the ſeveral Alterations of *Latitude* and *Departure* muſt be found, and the whole Difference of *Latitude* and *Departure* made good muſt be ſet down in their proper Columns at the bottom of the Table: then by *both Latitudes* and the *Departure*, find the *Difference* of *Longitude*, to which add the whole Difference of Longitude reckoned the Day before, and the Sum will be the whole Difference of Longitude from the laſt known *Land*. But if the Ship ſailed on the ſame Courſe during the whole 24 Hours, you need not find the *Departure*, but inſtead thereof find the *Difference of Longitude* directly by both the *Latitudes*, the *Courſe*, and *Diſtance*, which add to the Difference of Longitude reckoned the preceding Day, as before.

Every *Noon*, if poſſible, the *Latitude* ought to be obſerved; but if an Obſervation cannot be had by the *Sun*, you muſt endeavour to find the *Latitude* at Night, by taking the Meridian Altitude of ſome known *Star*, (though it is much better if you can obſerve at Noon) and if the *Latitude* by dead Reckoning agrees with the *Latitude* by Obſervation, the Reckoning may be ſuppoſed to be good. But by reaſon of the ſeveral Accidents that may attend the Ship in one Day's Running, ſuch as *ſwelling Seas*, different Rates of ſailing between the times of heaving the *Log*, want of Care at the Helm in letting the Ship *yaw* or fall off, accidental *Currents*, ſudden *Storms*, when no Account can be kept, *&c.* the *Latitude* by dead Reckoning, and the *Latitude* by Obſervation may very often differ: then it is neceſſary that proper Corrections be made in the *Difference of Longitude*. But if the Latitude by dead Reckoning, and the Latitude by Obſervation agree within 10 or 12 *Minutes*, you need not make any Correction at that time, till you ſee how they agree at the next Obſervation; becauſe in reality the *Latitude* cannot be found at Sea to leſs than 10 or 15 *min.* by the Inſtruments now in uſe, notwithſtanding the pretence of ſome People to obſerve to 5 *min.* And nothing is more common than for two good Obſervers to differ 20 *Minutes* from each other in two Day's Obſervations; one perhaps in one Day making the *Altitude* 10 *Min.* more, and in the next Day as much leſs than the other; and I found myſelf that I could hardly diſcern any Difference in my Obſervation, in moving the *Sight-Vane* 10 *Minutes* upon the *Quadrant*: And whoever conſiders the Nature of that Inſtrument, may be rather ſurprized, that Sailors can obſerve ſo exact as they really do, than that they ſhould differ ſo much from one another. But when the Difference between the *Latitude* by dead Reckoning and the *Lat.* by Obſervation, is greater than ¼ Degree, you muſt endeavour to rectify your Account by the following Directions.

Of

Of Correcting a Day's Work.

This indeed is a very nice and difficult Article, and ought to be managed with a great deal of Diſcretion, elſe it is much better to let the Reckoning ſtand without any Alteration; and indeed after proper Corrections have been made in the *Courſes* entered in the *Traverſe Table*, there can be but little more done till ſuch time as the *Longitude* be found, or ſome other Improvement made in Navigation. However, I ſhall here give ſuch Directions as will diſcover where it is moſt probable that the Error lies, and afterwards ſhew how to correct it.

Firſt, you ought to try if there be any *Current*, and if you find there is, you muſt ſet down its *Setting*, and againſt that its *Drift* in 24 Hours, in the *Traverſe Table*; or by the Setting and Drift find a new *Difference* of *Latitude* and *Departure*, and theſe muſt be either added or ſubſtracted from the computed *Diff.* of *Lat.* and *Depar.* as the Caſe requires: Then if the new Latitude agrees with the Latitude by Obſervation, you need not proceed any farther in your Corrections; but if they differ, there is ſtill an Error either in the *Courſe* or the *Diſtance*, and to find wherein it is moſt probable the Error lies, obſerve,

1. If the *Courſes* be for the moſt part near the *Meridian*, as within 2 or 2 ½ *Points* from it, it is moſt probable the Error lies in the *Diſtance*, becauſe it would require a very great Error in the *Courſe*, to make any conſiderable Error in the *Latitude*; but it being here, that the *Diſtance* and the *Difference* of *Latitude* approach neareſt to Equality, a ſmall Error in one will therefore be more viſible in the other.

2. If the *Courſes* for the moſt part be near or within 2 *Points* of the *Eaſt* or *Weſt*, it will require a very great Error in the *Diſtance* to make any ſenſible Error in the *Latitude*, and therefore it is moſt probable, that in ſuch Caſes the *Error* lies in the *Courſe*.

3. But if the *Courſes* be for the moſt part near the middle of the *Quadrant*, or within 2 and 6 *Points* from the *Meridian*, the Error may be partly in the *Courſe* and partly in the *Diſtance*, ſince both Errors are capable of producing nearly the ſame Error in the *Latitude*; or the Error may be either in the Courſe or in the Diſtance, and yet the other true.

Having by the Directions above found where it is moſt probable the Error lies, the Reckoning may be corrected by the following Rules.

Rule 1. When the Error is ſuppoſed to be in the *Diſtance*, to find the true *Departure*, it will be, (if the ſaid Error be ſimilar, or the ſame throughout the whole Day's Work,)

As the Difference of Latitude by dead Reckoning,
To the true Difference of Latitude:
So the Departure by dead Reckoning,
To the true Departure. Alſo,

So

So is the Diſtance by dead Reckoning, To the true direct Diſtance; or,
So is any particular Diſtance run, To the true correſpondent Diſtance.

Admit a Ship from the *Latitude* of 40° 00′ N, after having ſailed upon ſeveral Courſes in the ſpace of 24 Hours, her *Difference of Latitude* is computed to be 80 Miles N^ly, and *Departure* 52 Miles E^ly; but by a good Obſervation ſhe is found to be in the *Latitude* of 41° 50′ N; and ſuppoſing the Error to proceed from a Fault in the ſeveral *Diſtances* run; I would know the true *Departure.*

Having the true Difference of Latitude = 110 Miles, (betwixt the Latitude departed from, and the Latitude obtained by Obſervation) it will be,

Plate 11. Fig. 11.

As the computed Difference of Latitude B A = 80 *m.*	8.09691
To the true Difference of Latitude B D = 110 *m.*	2.04139
So is the computed Departure A C = 52 *m.*	1.71600
To the true Departure D E = 71.5	1.85430

That this Method of correcting the *Departure* is true when there is no Error in the *Courſe*, may be eaſily demonſtrated: If the Ship ſailed but upon one Courſe, it is evident from the Property of ſimilar Triangles, that it will be B A : B D :: A C : D E. And again, becauſe while the Ship continues upon the ſame *Rhumb*, the Error in the *Departure* will be proportional to the Error in the *Difference of Latitude*; it follows, *As the Sum of the whole Difference of Latitude by Account, To the Sum of the whole Departures by Account: So is the true Difference of Latitude, To the true Departure.*

Rule 2. When the Error is in the *Courſe*, if the ſame be regular, (*i. e.* if it runs equally the ſame, and the ſame way throughout the whole Day's Work,) the direct Diſtance found by Computation will notwithſtanding be true; wherefore to find the true *Departure*, it will be,

1. *As the direct Diſtance, To the true Difference of Latitude:*
   So is the Radius, To the Co-Sine of the Courſe. Then,
2. *As the Radius, To the Tangent of the Courſe:*
   So is the true Difference of Latitude, To the true Departure.

That in this *Caſe*, the direct Diſtance found by Computation is true, may be thus demonſtrated.

Fig. 12. Let A B, B C, C D be a *Traverſe* or a compound Courſe, and *a b*, *b c*, *c d* another *Traverſe*; in which *Traverſes* let the Diſtances A B *ab*, B C *bc*, C D *cd* be equal, and let *b m* be parallel to B C, and *cn* parallel to C D: Now if the ∠ˢ B *a b*, *m b c*, *n c d*, be all equal one to the other, (*i. e.* if the Error in each Courſe is the ſame, and the ſame way;) I ſay the direct Diſtance made good A D, is equal to the direct Diſtance by Computation A *d*. Produce C B to *a*.

1. In the △ˢ A B C, A *a* C, having the ſame common ∠ at C, it is evident, that the ∠ A *a* C is leſs than the ∠ A B C, by the Quantity B A *a*; wherefore in the △ A *b c* becauſe the ∠ *m b c* = B A *b*, the ∠ A *b c* = A B C, and becauſe the containing Sides A *b*, *b c* and A B, B C are equal, the ſubtending Sides A C, A *c* are alſo equal. After the ſame manner will be proved,

Fig. I.

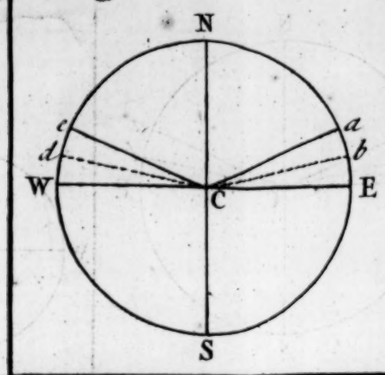


Fig. II

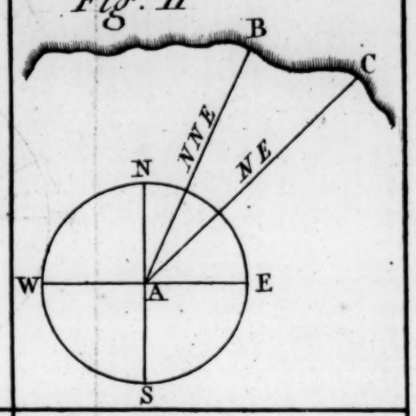


Fig. III

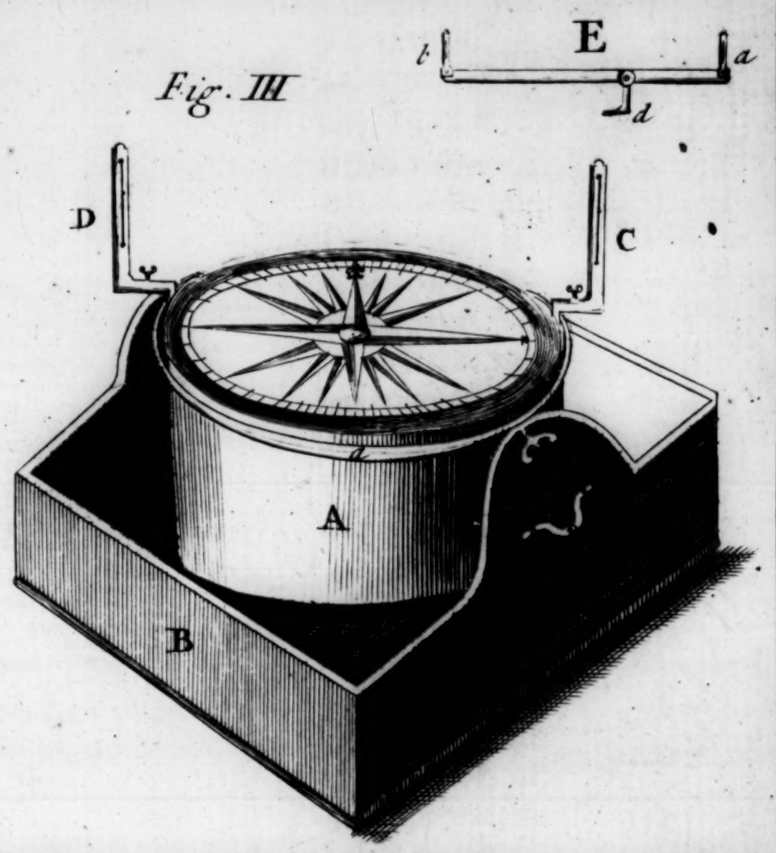


Fig. IV

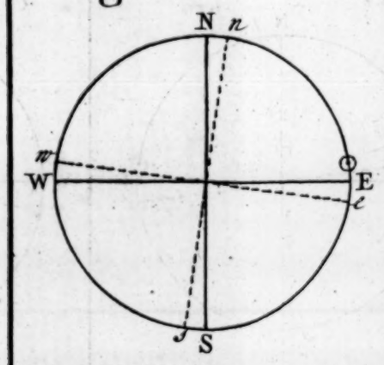


Fig. V

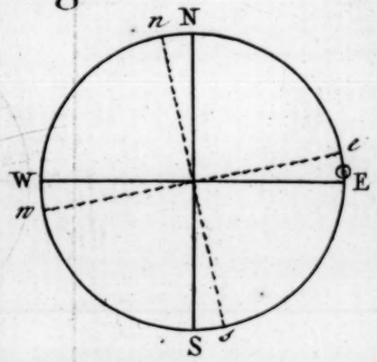


Fig. VI

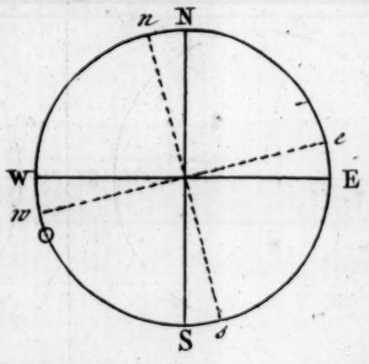


Fig. VII

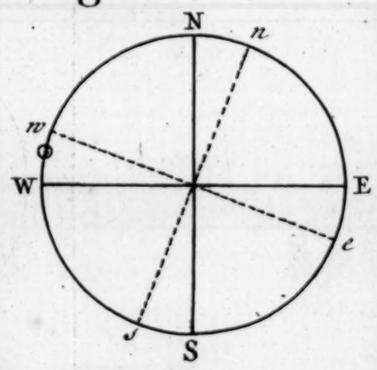


Fig. VIII

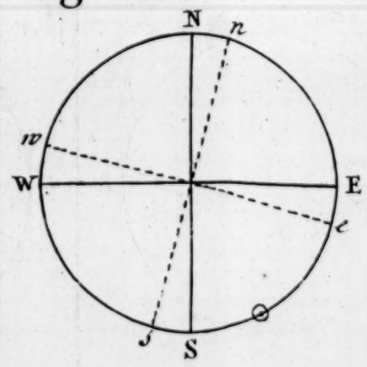


Fig. IX

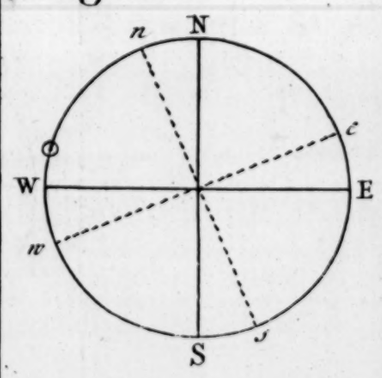
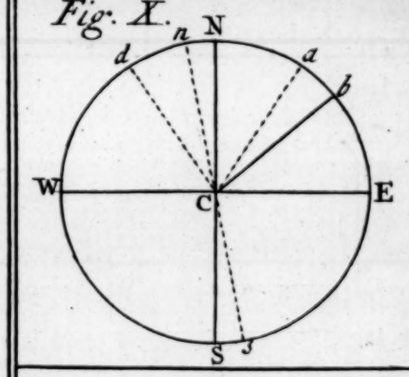


Fig. X.



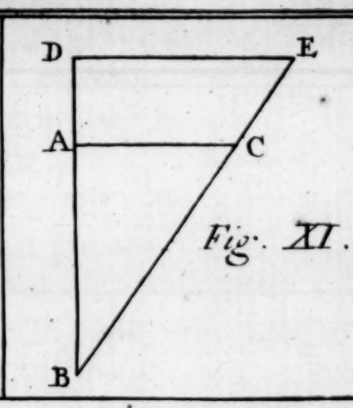


Fig. XI.

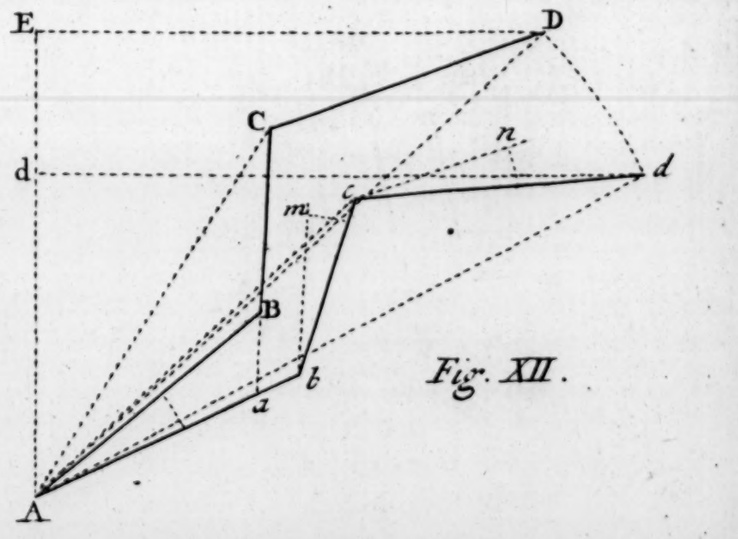


Fig. XII.

proved, that the ∠ *b c d* is equal to B C D, and conſequently A *c d* = A C D, and therefore the direct Diſtance A *d* (by Computation) is equal to the true direct Diſtance A D; and ſo will it be, let the number of Courſes be what it will, W.W.D. It is alſo evident, that the ∠ D A *d* the whole Error in the Courſe, is equal to the ∠ B A *b* the Error in each ſingle Courſe; whence the Error in the Courſes may be found.

3. When the Courſes are for the moſt part near the middle of the Quadrant, it is very difficult to determine, whether the *Error* be in the *Courſe* or in the *Diſtance*, or it may be in both; for either of theſe Errors are capable of producing nearly the ſame Alteration in the *Latitude*, and yet a contrary Error in the *Departure*. For inſtance, let the Courſes be nearer the Meridian than what they were ſuppoſed to be, this Error will make the true *Difference of Latitude* greater than the computed *Difference of Latitude*; but the true *Departure* will be leſs than the computed *Departure*: And if the Courſes be true, and the Diſtance ſailed greater than what was given by the *Log*, this may create the ſame *Error* in the *Latitude*, as before; but the Error in the *Departure* will be the contrary way, *i. e.* the true Departure will be greater than the Departure by dead Reckoning. Wherefore, when a Ship ſails near the middle of the Quadrant, or betwixt 3 and 5 *Points* from the *Meridian*, it is beſt not to make any alteration in the *Difference* of *Latitude* by dead Reckoning, tho' the Difference of Latitude is found to be erroneous: For if the Error be both in the Courſe and Diſtance, the computed Difference of Longitude may be true, notwithſtanding the Diff. of Latitude being erroneous; and if the Diff. of Longitude ſhould be erroneous, we are ſo far from knowing how much the Error is, that we can hardly pronounce which way it lies, as whether it makes the Longitude greater or leſs than what has been found by Computation.

The *Day's Work* being wrought, (and corrected if need be) according to the above Directions, the *Latitude* the Ship is in, and the *Longitude* from the laſt known *Land*, muſt be daily expreſſed in the *Journal-Book*; by which the Ship's Place upon the *Chart* may be readily found, and thence the Bearing and Diſtance of any known Place, (by the Directions given in *Sect.* 2. of *Chap.* II.) ſufficiently exact for all Uſes in Navigation; whence it appears, that to find daily the Bearing and Diſtance of the Place departed from in Numbers, is entirely uſeleſs, and is ſo much time and trouble loſt.

For the further illuſtrating the Rules delivered in this Section, and the Form of keeping a Journal, I have ſubjoined the following Specimen of three Days Work.

The

The NEPTUNE *from the* Lizard *towards* Barbadoes. 1729.

H.	K.	Pts	Course.	Winds & Weather, &c.	L.W.
1					
2					
3			*The* Lizard *in Lat.* 50° 00′ N. NE*b*N 5 Leag.		
4	3	2	SW*b*W	North	
5	4	3			
6	4	6			
7	4	8			
8	5		21.9		
9	5		S S W	N E	
10	5	2			
11	5	2			
12	5	4			
1	5	7			
2	6		32.5		
3	6	2	SW ½ W	N N E	
4	6	4			
5	6	2			
6	6	3			
7	6	5			
8	6	8			
9	6	4			
10	6	6			
11	6	7			
12	7		65.1		
			119.5 *Miles.*	*Variation I reckon* 1 ½ *Point* [W^ly.	

Monday, November 10th.

Courses corrected.	*Dist.*	*Diff. Lat.* N.	*Diff. Lat.* S.	*Depar.* E.	*Depar.* W.
S*b*W ½ W	15		14.4		4.3
SW ½ S	22		17		14
S ½ W	32.5		32.4		3.2
SW*b*S	65		54		36.1
			117.8		57.6

Lat. by { *Account*—48° 02′ N. / *Observ.*—48 08

Cor. *Dep.* 57.6 *m. Westerly.*

L. { *Lizard* —50° 00′ N—3475 / *To-day* ⊙—48 08—3304 } *M.P*

1 52 — 171

60

112

Prop. Diff. Lat. 112	7.9508
M. Diff. Lat. 171	2.2330
Cor. *Depart.* 57.6	1.7604
Diff. Long. 88	1.9442

At Noon Lat. 48° 08′ N, *Longitude from the* Lizard 1° 28′ W^ly.

For the moſt part of theſe 24 Hours moderate Gales and Fair-Weather.

Note, Becauſe the *Lizard* bore N E *b* N 5 Leagues, the firſt Courſe is S W *b* S 15 Miles, which being corrected by allowing 1 ½ Point W^ly Variation, is S *b* W ½ W, which is accordingly entered in the Traverſe-Table: After the ſame manner the three Courſes upon the Log-board are corrected by allowing 1 ½ Point Variation, (there being no Lee-Way made) againſt each of which is ſet the reſpective Diſtance.

The

The NEPTUNE *from the* Lizard *towards* Barbadoes. 1729.

H.	K.	Pts	*Courses.*	*Winds & Weather, &c.*	L. W.
1	7		SW*b*W	N*b*W	
2	6	4			
3	6	2			
4	6		25.6		
5	6		SSW	West	½
6	5	4			
7	5				
8	5	2			
9	4	3	26	*Took a Reef in each T. S.*	1
10	4	5	WSW	NW	
11	4	7	SW		
12	4	6	SW*b*W		
1	4	4	18.2	*Close reefed the T. S.*	1¼
2	3	8	South.	WSW	
3	3	2			
4	2	7	9.7	*Took in the F. T. S.*	3
5	2	5			
6	2	3			
7	2	6			
8	3		S *b* W	*Handed M. T. S.*	4
9	2	5			
10	2	3			
11	2	2			
12	2		19.4		
	98.8 *m.*		*Variat.* { allowed 1¼ *P.* }	① 13° ½	W^ly.

Tuesday, November 11th.

Courses corrected.	*Dist.*	*Diff. Lat.* N.	*Diff. Lat.* S.	*Depar.* E.	*Depar.* W.
SW ¼ S	25.6		18.9		17.1
S ¼ W	26		26		1.3
SSW ¾ W	17		14.6		8.7
SE *b* S	10		8.3	5.5	
SE ¼ E	19.4		13	14.4	
			80.8	19.9	27.1
					19.9
			80.8		7.2

Lat. by { *Account* — 46° 47′ N. / *Observ.* — 47 10 }

Rule 1. Cor. *Dep.* 5 *m*, *Westerly.*

L. { *Last* ⊙ — 48° 08′ — 3304 / *To-day* ⊙ — 47 10 — 3217 } *M. P.*

58 87

Prop. Diff. Lat. 58	8̄.2366
Merid. Diff. Lat. 87	1.9395
Cor. *Depart.* 5	0.6990
Diff. Long. 7.5	0.8751
1° 28′	
1 35	

At Noon Lat. 47° 10′ N. *Longitude from the* Lizard 1° 35′ W^ly.

The first Part moderate Gales and fair, the middle and later Part squally with hard Showers of Rain.

Note, 1. I contracted the three Courses WSW, SW, SW*b*W, because they are close together, into one, taking the middlemost which is SW*b*W, and also because the Distance run upon this Course is the greatest, and the Distance I lessened one Mile: And the like may be done in all such Cases, so that you need never have above 6 or 8 *Courses* in the *Traverse-Table*.

Note, 2. Because when the *Fore-Top Sail* was taken in the *Lee-Way* was altered, the Ship makes a new Course from 4 A. M. tho' her Head is still kept directly *South*.

Note, 3. In correcting the Courses, it is sufficient, that the *Variation* be counted to a ¼ *Point*: thus, tho' the *Variation* has been observed above to be 13½ Degrees, in correcting the several Courses I allowed just 1¼ *Point*, which is somewhat more than is given by the Observation.

Note, 4. The *Rule* 1. against *Cor. Dep.* denotes, that the *Departure* was corrected by the first *Rule* aforegoing, or that the *Error* in the *Latitude* was attributed to an Error in the several *Distances*, because the *Courses* are partly on each side the *South*-part of the *Meridian*, and not very far from it.

The NEPTUNE *from the* Lizard *towards* Barbadoes. 1729.

H.	K.	Pts	Course.	Winds & Weather, &c.	L.W.
1	2	5	S S W	WSW	4½
2	3			*Reefed Courses.*	5
3	3	8		*Violent Squalls.*	
4	3	5	12.8	*Handed Fore-Sail.*	5½
5	3	4	S W	*Hard Showers of Rain.*	
6	3	8		*Set the Fore-Sail.*	5
7	3	9			
8	4	2	15.3		
9	4	8			
10	5		9.8	*Let the Reefs out of the*	3
11	5	4		*Courses, and set the T.S.*	
12	5	8		*Unreefed the Top-S.*	¾
1	6	3	17.5	*A great Sea setting* NE*b*N.	
2	6	6	WSW	N *b* W	0
3	6	8			
4	7	2			
5	7	4		*A brisk & steady Gale.*	
6	7	7			
7	8				
8	8	2			
9	8	1			
10	7	9			
11	8	2			
12	7	8	83.9		
	139.3 *m.*		*Variat.*	{⊙ allowed 1¼ *P.* Degrees} W^ly.	

Wednesday, November 12th.

Courses corrected.	Dist.	Diff. Lat. N.	Diff. Lat. S.	Depar. E.	Depar. W.
SE ¼E	12		8	8.9	
SSE ¼E	15		13.5	6.4	
South.	10		10		
S S W	17		15.7		6.5
SW ¾W	84		50		67.5
			97.2	15.3	74
					15.3
			97.2		58.7

Lat. by { *Account* —45° 33′ N. / *Observ.* —45 54 } 19

Cor. *Dep.* 45 *m. Westerly.*

L. { *Last* ⊙ —47° 10′ N —3217 / *To-day* ⊙ —45 54 —3107 } *M.P*

1 16 — 110

60

76

Prop. Diff. Lat. 76		8.1192
M. Diff. Lat. 110		2.0414
Depart. 45		1.6532
Diff. Long. 65		1.8138
	1° 05′	
	1 35	
	2 40	

At Noon Lat. 45° 54′ N. *Longitude from the* Lizard 2° 40′ W^ly.

The first Part squally with hard Showers, the middle and later a fresh Gale and Fair Weather.

Note, 1. In correcting the first Course, because the three different *Lee-Ways* are near to one another, I took a Mean betwixt them, and so contracted them into one Course, and lessened the Distance: and the like may be done in all such Cases.

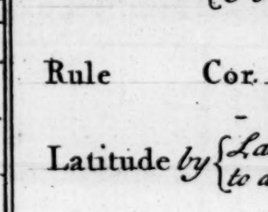
Note, 2. The *Error* in the *Latitude* in this Day's Work I attribute to the great Swell, setting NE*b*N, and the Departure I corrected accordingly: Thus, with 19 Miles (the Difference betwixt the Latitude by dead Reckoning, and the Latitude by Observation) and the *Setting* of the *Sea* NE*b*N, I found a new *Departure* = 13 *m.* then because the Setting of the Swell is Easterly, and the Departure by dead Reckoning is Westerly, I take the Difference = 45 *m.* Westerly for the *Cor. Departure.*

Note, When you would find the Bearing and Distance of the Place you are bound for, (or of any other) substract the Longitude made good, from the Longitude of the said Place, and with that Difference, and the Difference betwixt the Lat. you are in, and the Lat. of the said Place, you may find the Bearing and Distance required. *Vide* Page 108.

The

H.	K.	Pts	Courses	Wind & Weather &c.	L.W.
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
			Miles	Variation { ⊙ / *Allowd*	

Courses *Corrected*	*Dist.*	*Diff of Lat.* N.	*Diff of Lat.* S.	*Departure* E.	*Departure* W.



Latitude *by* { *Account* / *Observat.*

Rule Cor. *Depart.*

Latitude *by* { *Last* ⊙ / *to day* ⊙ } *M.P.*

Prop. Diff. Lat.

M. Diff. Lat.

Cor. *Departure*

Diff. Longitude

At Noon { *Lat* / *Long.de* *from*

The Method of working and correcting a *Day's Work*, being already ſhewed at large, and the ſeveral Steps in the foregoing Specimen of a *Journal* being fully explained by the *Notes* underneath, I need enlarge no farther upon that Head; and I think there is hardly any Occurrence which may happen in *Navigation*, but has been ſomewhere or other fully explained. But before I take my leave of the ingenious Mariner, I muſt again caution him againſt that common Method of keeping Reckonings by *Meridional Diſtance*; the Error and Abſurdity of which Method has been largely ſhewed in *Sect.* 3. of *Chap.* I. And indeed, according to *Geometrical* Strictneſs, the *Difference of Longitude* ought to be found for every ſingle Courſe; but in Practice it is ſufficiently exact if it be found once in every 24 Hours, according to the Examples in the preceding *Journal*.

And thus have I brought this Work to a Concluſion; in which I have endeavoured to render the whole Scheme of Navigation more correct and perfect than has been done hitherto, and alſo every where plain and eaſy to the meaneſt Capacity: I hope what I have herein advanced may prove uſeful and acceptable to the induſtrious Sea-Man, to whom I heartily wiſh all deſirable Succeſs.

TABLES

OF THE

SUN'S DECLINATION

For the Years 1729, 1730, 1731, 1732,

Every Day at Noon, under the *Meridian* of LONDON:

With a TABLE of *Proportional Parts* to make them ſerve under any other *Meridian*.

TOGETHER

With a TABLE of the SUN'S PLACE and RIGHT ASCENSION for the Year 1730.

ALSO

A TABLE of the *Right Aſcenſion* and *Declination* of the principal FIXED STARS, together with an Eſtimate of their *Meridian Altitudes* in every 10 Degrees of *Latitude*, from 60° *North* to 60° *South*.

To which is added,

A TABLE, ſhewing the *Right Aſcenſion* of that *Point* of the *Heavens* which comes to the *Meridian* on the 1ſt and 16th Day of each *Month*, at every two Hours of the Night, from 8 o'-Clock in the Evening, till 4 o'-Clock in the Morning.

By the two laſt *Tables* may be readily found which *Star* is propereſt for obſerving the *Latitude* by, in all Places, and at all Times of the Year.

A TABLE *of the* Sun's *Declination for the*

Days.	Jan. S. °	′	Diff. ′	Feb. S. °	′	Diff. ′	March. S. * °	′	Diff. ′	April. N. °	′	Diff. ′	May. N. °	′	Diff. ′	June. N. °	′	Diff. ′	Days.
1	21	37		13	36		03	12		08	47		18	12		23	12		1
2	21	27	10	13	16	20	02	49	23	09	08	21	18	27	15	23	15	3	2
3	21	16	11	12	56	20	02	25	24	09	30	22	18	42	15	23	18	3	3
4	21	05	11	12	35	21	02	01	24	09	51	21	18	56	14	23	21	3	4
5	20	54	11	12	14	21	01	37	24	10	13	22	19	10	14	23	23	2	5
6	20	42	12	11	53	21	01	14	23	10	34	21	19	23	13	23	25	2	6
7	20	30	12	11	32	21	00	50	24	10	55	21	19	37	14	23	27	2	7
8	20	17	13	11	11	21	00	26	24	11	16	21	19	50	13	23	28	1	8
9	20	04	13	10	49	22	00	03	23	11	36	20	20	03	13	23	29	1	9
10	19	51	13	10	27	22	N	21	24	11	57	21	20	15	12	23	29	0	10
11	19	37	14	10	05	22	00	44	23	12	17	20	20	27	12	23	29	0	11
12	19	23	14	09	43	22	01	08	24	12	37	20	20	38	11	23	28	1	12
13	19	09	14	09	21	22	01	32	24	12	57	20	20	49	11	23	27	1	13
14	18	54	15	08	59	22	01	55	23	13	16	19	21	00	11	23	26	1	14
15	18	39	15	08	37	22	02	19	24	13	36	20	21	11	11	23	24	2	15
16	18	24	15	08	14	23	02	42	23	13	55	19	21	21	10	23	22	2	16
17	18	08	16	07	51	23	03	06	24	14	14	19	21	31	10	23	19	3	17
18	17	52	16	07	28	23	03	29	23	14	33	19	21	40	9	23	16	3	18
19	17	35	17	07	05	23	03	52	23	14	51	18	21	49	9	23	13	3	19
20	17	18	17	06	42	23	04	16	24	15	09	18	21	58	9	23	09	4	20
21	17	01	17	06	19	23	04	39	23	15	27	18	22	06	8	23	05	4	21
22	16	44	17	05	56	23	05	02	23	15	45	18	22	14	8	23	00	5	22
23	16	26	18	05	33	23	05	25	23	16	03	18	22	22	8	22	55	5	23
24	16	08	18	05	10	23	05	48	23	16	20	17	22	29	7	22	50	5	24
25	15	50	18	04	46	24	06	11	23	16	37	17	22	36	7	22	44	6	25
26	15	32	18	04	23	23	06	33	22	16	53	16	22	42	6	22	37	7	26
27	15	13	19	03	59	24	06	56	23	17	10	17	22	48	6	22	30	7	27
28	14	54	19	03	36	23	07	18	22	17	26	16	22	54	6	22	23	7	28
29	14	35	19				07	41	23	17	41	15	22	59	5	22	16	7	29
30	14	16	19				08	03	22	17	57	16	23	04	5	22	08	8	30
31	13	56	20				08	25	22				23	08	4				31

Years 1729, 1733, 1737, (*each being the first after Leap-Year.*)

Days.	July. N. °	′	Diff. ′	Aug. N. °	′	Diff. ′	Sept. N.* °	′	Diff. ′	Octob. S. °	′	Diff. ′	Nov. S. °	′	Diff. ′	Decem. S. °	′	Diff. ′	Days.
1	22	00		14	58		04	08		07	30		17	48		23	09		1
2	21	51	9	14	40	18	03	45	23	07	52	22	18	04	16	23	13	4	2
3	21	42	9	14	22	18	03	22	23	08	15	23	18	20	16	23	17	4	3
4	21	33	9	14	03	19	02	58	24	08	37	22	18	36	16	23	20	3	4
5	21	23	10	13	44	19	02	35	23	08	59	22	18	51	15	23	23	3	5
6	21	13	10	13	25	19	02	12	23	09	21	22	19	06	15	23	25	2	6
7	21	03	10	13	05	20	01	49	23	09	43	22	19	20	14	23	27	2	7
8	20	52	11	12	46	19	01	25	24	10	05	22	19	34	14	23	28	1	8
9	20	41	11	12	26	20	01	02	23	10	27	22	19	48	14	23	29	1	9
10	20	29	12	12	06	20	00	39	23	10	48	21	20	01	13	23	29	0	10
11	20	17	12	11	46	20	00	15	24	11	10	22	20	14	13	23	29	0	11
12	20	05	12	11	26	20	S	08	23	11	31	21	20	27	13	23	28	1	12
13	19	53	12	11	05	21	00	32	24	11	52	21	20	39	12	23	27	1	13
14	19	40	13	10	44	21	00	55	23	12	13	21	20	51	12	23	26	1	14
15	19	27	13	10	24	20	01	19	24	12	33	20	21	02	11	23	24	2	15
16	19	14	13	10	03	21	01	42	23	12	54	21	21	13	11	23	21	3	16
17	19	00	14	09	41	22	02	06	24	13	14	20	21	24	11	23	18	3	17
18	18	46	14	09	20	21	02	29	23	13	34	20	21	34	10	23	15	3	18
19	18	31	15	08	58	22	02	53	24	13	54	20	21	44	10	23	11	4	19
20	18	17	14	08	37	21	03	16	23	14	13	19	21	54	10	23	07	4	20
21	18	02	15	08	15	22	03	39	23	14	33	20	22	03	9	23	02	5	21
22	17	46	16	07	53	22	04	03	24	14	52	19	22	11	8	22	57	5	22
23	17	31	15	07	31	22	04	26	23	15	11	19	22	20	9	22	51	6	23
24	17	15	16	07	09	22	04	49	23	15	29	18	22	27	7	22	45	6	24
25	16	59	16	06	47	22	05	12	23	15	48	19	22	34	7	22	38	7	25
26	16	42	17	06	24	23	05	35	23	16	06	18	22	41	7	22	31	7	26
27	16	26	16	06	02	22	05	58	23	16	24	18	22	48	7	22	24	7	27
28	16	09	17	05	39	23	06	21	23	16	41	17	22	54	6	22	16	8	28
29	15	51	18	05	16	23	06	44	23	16	58	17	22	59	5	22	07	9	29
30	15	34	17	04	54	22	07	07	23	17	15	17	23	04	5	21	58	9	30
31	15	16	18	04	31	23				17	32	17				21	49	9	31

A Table for adjusting the Sun's Declination to any Meridian.

Daily Diff. in Declin. M.	Degrees of Longitude either East or West. 15	30	45	60	75	90	105	120	135	150	165	180
3	0	0	0	0	0	0	1	1	1	1	1	1
6	0	0	0	1	1	1	1	2	2	2	2	2
9	0	0	1	1	2	2	2	3	3	3	4	4
12	0	1	1	2	2	3	3	4	4	5	5	6
15	0	1	2	2	3	4	4	5	5	6	6	7
18	1	1	2	3	3	4	5	6	7	8	8	9
21	1	2	3	4	4	5	6	7	8	9	10	11
24		2	3	4	5	6	7	8	9	10	11	12

A TABLE *of the* Sun's *Declination for the*

Days.	Jan. S. °	′	Diff. ′	Feb. S. °	′	Diff. ′	March. S. * °	′	Diff. ′	April. N. °	′	Diff. ′	May. N. °	′	Diff. ′	June. N. °	′	Diff. ′	Days.
1	21	39		13	40		03	18		08	41		18	08		23	11		1
2	21	29	10	13	20	20	02	54	24	09	03	22	18	23	15	23	15	4	2
3	21	19	10	13	00	20	02	31	23	09	25	22	18	38	15	23	18	3	3
4	21	08	11	12	39	21	02	07	24	09	46	21	18	52	14	23	21	3	4
5	20	57	11	12	19	20	01	43	24	10	08	22	19	06	14	23	23	2	5
6	20	45	12	11	58	21	01	20	23	10	29	21	19	20	14	23	25	2	6
7	20	33	12	11	37	21	00	56	24	10	50	21	19	34	14	23	27	2	7
8	20	20	13	11	15	22	00	32	24	11	11	21	19	47	13	23	28	1	8
9	20	07	13	10	54	21	00	09	23	11	31	20	19	59	12	23	29	1	9
10	19	54	13	10	32	22	N	14	23	11	52	21	20	12	13	23	29	0	10
11	19	41	13	10	10	22	00	38	24	12	12	20	20	24	12	23	29	0	11
12	19	27	14	09	48	22	01	02	24	12	32	20	20	36	12	23	28	1	12
13	19	12	15	09	26	22	01	26	24	12	52	20	20	47	11	23	27	1	13
14	18	58	14	09	04	22	01	49	23	13	12	20	20	58	11	23	26	1	14
15	18	43	15	08	42	22	02	13	24	13	31	19	21	08	10	23	24	2	15
16	18	27	16	08	19	23	02	36	23	13	50	19	21	19	11	23	22	2	16
17	18	11	16	07	57	22	03	00	24	14	09	19	21	29	10	23	20	2	17
18	17	55	16	07	34	23	03	23	23	14	28	19	21	38	9	23	17	3	18
19	17	39	16	07	11	23	03	47	24	14	47	19	21	47	9	23	13	4	19
20	17	22	17	06	48	23	04	10	23	15	05	18	21	56	9	23	10	3	20
21	17	05	17	06	25	23	04	33	23	15	23	18	22	05	9	23	06	4	21
22	16	48	17	06	02	23	04	56	23	15	41	18	22	13	8	23	01	5	22
23	16	31	17	05	39	23	05	19	23	15	58	17	22	20	7	22	56	5	23
24	16	13	18	05	15	24	05	42	23	16	15	17	22	27	7	22	51	5	24
25	15	55	18	04	52	23	06	05	23	16	32	17	22	34	7	22	45	6	25
26	15	36	19	04	29	23	06	28	23	16	49	17	22	41	7	22	39	6	26
27	15	18	18	04	05	24	06	50	22	17	05	16	22	47	6	22	32	7	27
28	14	58	20	03	42	23	07	13	23	17	22	17	22	53	6	22	25	7	28
29	14	39	19				07	35	22	17	38	16	22	58	5	22	18	7	29
30	14	20	19				07	57	22	17	53	15	23	03	5	22	10	8	30
31	14	00	20				08	19	22				23	07	4				31

Years 1730, 1734, 1738, (*each being the second after Leap-Year.*)

Days.	July. N. °	′	Diff. ′	Aug. N. °	′	Diff. ′	Sept. N. * °	′	Diff. ′	Octob. S. °	′	Diff. ′	Nov. S. °	′	Diff. ′	Decem. S. °	′	Diff. ′	Days.
1	22	02		15	02		04	13		07	24		17	44		23	08		1
2	21	53	9	14	44	18	03	50	23	07	47	23	18	00	16	23	12	4	2
3	21	44	9	14	26	18	03	27	23	08	09	22	18	16	16	23	16	4	3
4	21	35	9	14	07	19	03	04	23	08	32	23	18	32	16	23	19	3	4
5	21	26	9	13	48	19	02	41	23	08	54	22	18	47	15	23	22	3	5
6	21	16	10	13	29	19	02	17	24	09	16	22	19	02	15	23	24	2	6
7	21	06	10	13	10	19	01	54	23	09	38	22	19	17	15	23	26	2	7
8	20	55	11	12	51	19	01	31	23	10	00	22	19	31	14	23	27	1	8
9	20	44	11	12	31	20	01	07	24	10	21	21	19	45	14	23	28	1	9
10	20	33	11	12	11	20	00	44	23	10	43	22	19	58	13	23	29	1	10
11	20	21	12	11	51	20	00	21	23	11	04	21	20	11	13	23	29	0	11
12	20	09	12	11	31	20	S	03	24	11	26	22	20	24	13	23	28	1	12
13	19	57	12	11	10	21	00	26	23	11	47	21	20	36	12	23	27	1	13
14	19	44	13	10	49	21	00	50	24	12	08	21	20	48	12	23	26	1	14
15	19	31	13	10	28	21	01	13	23	12	28	20	21	00	12	23	24	2	15
16	19	18	13	10	07	21	01	37	24	12	49	21	21	11	11	23	22	2	16
17	19	04	14	09	46	21	02	00	23	13	09	20	21	22	11	23	19	3	17
18	18	50	14	09	25	21	02	23	23	13	29	20	21	32	10	23	16	3	18
19	18	35	15	09	04	21	02	47	24	13	49	20	21	42	10	23	12	4	19
20	18	20	15	08	42	22	03	10	23	14	09	20	21	51	9	23	08	4	20
21	18	05	15	08	20	22	03	34	24	14	28	19	22	00	9	23	03	5	21
22	17	51	15	07	58	22	03	57	23	14	47	19	22	09	9	22	58	5	22
23	17	35	16	07	36	22	04	20	23	15	06	19	22	17	8	22	52	6	23
24	17	19	16	07	14	22	04	44	24	15	25	19	22	25	8	22	46	6	24
25	17	03	16	06	52	22	05	07	23	15	43	18	22	33	8	22	40	6	25
26	16	46	17	06	30	22	05	30	23	16	01	18	22	40	7	22	33	7	26
27	16	30	16	06	07	23	05	53	23	16	19	18	22	46	6	22	25	8	27
28	16	13	17	05	44	23	06	16	23	16	37	18	22	52	6	22	17	8	28
29	15	56	17	05	22	22	06	39	23	16	54	17	22	58	6	22	09	8	29
30	15	39	17	04	59	23	07	02	23	17	11	17	23	03	5	22	00	9	30
31	15	21	18	04	36	23				17	28	17				21	51	9	31

A Table for adjusting the Sun's Declination to any Meridian.

Daily Diff. in Declin.	Degrees of Longitude either East or West.											
M.	15	30	45	60	75	90	105	120	135	150	165	180
3	0	0	0	0	0	0	1	1	1	1	1	1
6	0	0	0	1	1	1	1	2	2	2	2	3
9	0	0	1	1	2	2	2	3	3	3	4	4
12	0	1	1	2	2	3	3	4	4	5	5	6
15	0	1	2	2	3	4	4	5	5	6	6	7
18	1	1	2	3	3	4	5	6	7	8	8	9
21	1	2	3	4	4	5	6	7	8	9	10	11
24	1	2	3	4	5	6	7	8	9	10	11	12

A TABLE *of the* Sun's *Declination for the*

Days.	Jan. S. °	′	Diff. ′	Feb. S. °	′	Diff. ′	March. S. * °	′	Diff. ′	April. N. °	′	Diff. ′	May. N. °	′	Diff. ′	June. N. °	′	Diff. ′	Days.
1	21	42		13	45		03	24		08	36		18	05		23	10		1
2	21	32	10	13	25	20	03	00	24	08	58	22	18	20	15	23	14	4	2
3	21	21	11	13	05	20	02	36	24	09	20	22	18	35	15	23	17	3	3
4	21	10	11	12	44	21	02	13	23	09	41	21	18	49	14	23	20	3	4
5	20	59	11	12	24	20	01	49	24	10	03	22	19	03	14	23	23	3	5
6	20	48	11	12	03	21	01	25	24	10	24	21	19	17	14	23	25	2	6
7	20	36	12	11	42	21	01	02	23	10	45	21	19	30	13	23	26	1	7
8	20	23	13	11	21	21	00	38	24	11	06	21	19	43	13	23	27	1	8
9	20	10	13	10	59	22	00	14	24	11	27	21	19	56	13	23	28	1	9
10	19	57	13	10	38	21	N	09	23	11	47	20	20	09	13	23	29	1	10
11	19	44	13	10	16	22	00	33	24	12	07	20	20	21	12	23	29	0	11
12	19	30	14	09	54	22	00	56	23	12	27	20	20	33	12	23	29	0	12
13	19	16	14	09	32	22	01	20	24	12	47	20	20	44	11	23	28	1	13
14	19	01	15	09	10	22	01	44	24	13	07	20	20	55	11	23	27	1	14
15	18	46	15	08	47	23	02	07	23	13	26	19	21	06	11	23	25	2	15
16	18	31	15	08	25	22	02	31	24	13	46	20	21	16	10	23	23	2	16
17	18	15	16	08	02	23	02	54	23	14	05	19	21	26	10	23	21	2	17
18	17	59	16	07	39	23	03	18	24	14	24	19	21	36	10	23	18	3	18
19	17	43	16	07	17	22	03	41	23	14	42	18	21	45	9	23	14	4	19
20	17	26	17	06	54	23	04	04	23	15	01	19	21	54	9	23	11	3	20
21	17	09	17	06	31	23	04	27	23	15	19	18	22	03	9	23	07	4	21
22	16	52	17	06	08	23	04	51	24	15	37	18	22	11	8	23	02	5	22
23	16	35	17	05	44	24	05	14	23	15	54	17	22	18	7	22	57	5	23
24	16	17	18	05	21	23	05	37	23	16	11	17	22	26	8	22	52	5	24
25	15	59	18	04	58	23	05	59	22	16	28	17	22	33	7	22	46	6	25
26	15	41	18	04	34	24	06	22	23	16	45	17	22	39	6	22	40	6	26
27	15	22	19	04	11	23	06	45	23	17	02	17	22	45	6	22	34	6	27
28	15	03	19	03	47	24	07	07	22	17	18	16	22	51	6	22	27	7	28
29	14	44	19				07	30	23	17	34	16	22	56	5	22	20	7	29
30	14	25	19				07	52	22	17	50	16	23	01	5	22	12	8	30
31	14	05	20				08	14	22				23	06	5				31

Years 1731, 1735, 1739, (*each being the third after Leap-Year.*)

Days.	July. N. °	′	Diff. ′	Aug. N. °	′	Diff. ′	Sept. N. * °	′	Diff. ′	Octob. S. °	′	Diff. ′	Nov. S. °	′	Diff. ′	Decem. S. °	′	Diff. ′	Days.
1	22	04		15	07		04	19		07	19		17	41		23	07		1
2	21	55	9	14	49	18	03	56	23	07	41	22	17	57	16	23	11	4	2
3	21	47	8	14	30	19	03	33	23	08	04	23	18	13	16	23	15	4	3
4	21	38	9	14	12	18	03	10	23	08	26	22	18	28	15	23	18	3	4
5	21	28	10	13	53	19	02	46	24	08	48	22	18	43	15	23	21	3	5
6	21	18	10	13	34	19	02	23	23	09	11	23	18	58	15	23	24	3	6
7	21	08	10	13	15	19	02	00	23	09	33	22	19	13	15	23	26	2	7
8	20	57	11	12	55	20	01	36	24	09	54	21	19	27	14	23	27	1	8
9	20	46	11	12	36	19	01	13	23	10	16	22	19	41	14	23	28	1	9
10	20	35	11	12	16	20	00	50	23	10	38	22	19	55	14	23	29	1	10
11	20	23	12	11	56	20	00	26	24	10	59	21	20	08	13	23	29	0	11
12	20	11	12	11	35	21	00	03	23	11	20	21	20	21	13	23	29	0	12
13	19	59	12	11	15	20	S	20	23	11	42	22	20	33	12	23	28	1	13
14	19	47	12	10	54	21	00	44	24	12	03	21	20	45	12	23	27	1	14
15	19	34	13	10	34	20	01	07	23	12	23	20	20	57	12	23	25	2	15
16	19	20	14	10	13	21	01	31	24	12	44	21	21	08	11	23	23	2	16
17	19	07	13	09	51	22	01	54	23	13	04	20	21	19	11	23	20	3	17
18	18	53	14	09	30	21	02	18	24	13	24	20	21	30	11	23	17	3	18
19	18	38	15	09	09	21	02	41	23	13	44	20	21	40	10	23	13	4	19
20	18	24	14	08	47	22	03	05	24	14	04	20	21	49	9	23	09	4	20
21	18	09	15	08	25	22	03	28	23	14	23	19	21	58	9	23	04	5	21
22	17	54	15	08	04	21	03	51	23	14	42	19	22	07	9	22	59	5	22
23	17	38	16	07	42	22	04	15	24	15	01	19	22	16	9	22	54	5	23
24	17	23	15	07	20	22	04	38	23	15	20	19	22	24	8	22	48	6	24
25	17	07	16	06	57	23	05	01	23	15	39	19	22	31	7	22	42	6	25
26	16	50	17	06	35	22	05	24	23	15	57	18	22	38	7	22	35	7	26
27	16	34	16	06	12	23	05	47	23	16	15	18	22	45	7	22	27	8	27
28	16	17	17	05	50	22	06	10	23	16	33	18	22	51	6	22	20	7	28
29	16	00	17	05	27	23	06	33	23	16	50	17	22	57	6	22	12	8	29
30	15	42	18	05	05	22	06	56	23	17	07	17	23	02	5	22	03	9	30
31	15	25	17	04	42	23				17	24	17				21	54	9	31

A Table for adjusting the Sun's Declination to any Meridian.

Daily Diff. in Declin. M.	Degrees of Longitude either East or West. 15	30	45	60	75	90	105	120	135	150	165	180
3	0	0	0	0	0	0	1	1	1	1	1	1
6	0	0	0	1	1	1	1	2	2	2	2	3
9	0	0	1	1	2	2	2	3	3	3	4	4
12	0	1	1	2	2	3	3	4	4	5	5	6
15	0	1	2	2	3	4	4	5	5	6	6	7
18	1	1	2	3	3	4	5	6	7	8	8	9
21	1	2	3	4	4	5	6	7	8	9	10	11
24		2	3	4	5	6	7	8	9	10	11	12

A TABLE *of the* Sun's *Declination for the*

Days.	Jan. S. °	′	Diff. ′	Feb. S. °	′	Diff. ′	March. S. * °	′	Diff. ′	April. N. °	′	Diff. ′	May. N. °	′	Diff. ′	June. N. °	′	Diff. ′	Days.
1	21	44		13	50		03	06		08	53		18	16		23	13		1
2	21	34	10	13	30	20	02	42	24	09	14	21	18	31	15	23	16	3	2
3	21	24	10	13	10	20	02	19	23	09	36	22	18	46	15	23	19	3	3
4	21	13	11	12	49	21	01	55	24	09	57	21	19	00	14	23	22	3	4
5	21	02	11	12	29	20	01	31	24	10	19	22	19	14	14	23	24	2	5
6	20	51	11	12	08	21	01	08	23	10	40	21	19	27	13	23	26	2	6
7	20	39	12	11	47	21	00	44	24	11	01	21	19	40	13	23	27	1	7
8	20	27	12	11	26	21	00	20	24	11	21	20	19	53	13	23	28	1	8
9	20	14	13	11	04	22	N	03	23	11	42	21	20	06	13	23	29	1	9
10	20	01	13	10	43	21	00	27	24	12	02	20	20	18	12	23	29	0	10
11	19	47	14	10	21	22	00	51	24	12	23	21	20	30	12	23	29	0	11
12	19	33	14	09	59	22	01	14	23	12	43	20	20	41	11	23	28	1	12
13	19	19	14	09	37	22	01	38	24	13	02	19	20	52	11	23	27	1	13
14	19	05	14	09	15	22	02	02	24	13	22	20	21	03	11	23	25	2	14
15	18	50	15	08	53	22	02	25	23	13	41	19	21	14	11	23	23	2	15
16	18	35	15	08	30	23	02	49	24	14	00	19	21	24	10	23	21	2	16
17	18	19	16	08	08	22	03	12	23	14	19	19	21	34	10	23	18	3	17
18	18	03	16	07	45	23	03	35	23	14	38	19	21	43	9	23	15	3	18
19	17	47	16	07	22	23	03	59	24	14	56	18	21	52	9	23	12	3	19
20	17	30	17	06	59	23	04	22	23	15	14	18	22	00	8	23	08	4	20
21	17	13	17	06	36	23	04	45	23	15	32	18	22	09	9	23	04	4	21
22	16	56	17	06	13	23	05	08	23	15	50	18	22	17	8	22	59	5	22
23	16	39	17	05	50	23	05	31	23	16	07	17	22	24	7	22	54	5	23
24	16	21	18	05	27	23	05	54	23	16	24	17	22	31	7	22	48	6	24
25	16	03	18	05	03	24	06	17	23	16	41	17	22	38	7	22	42	6	25
26	15	45	18	04	40	23	06	39	22	16	58	17	22	44	6	22	35	7	26
27	15	27	18	04	17	23	07	02	23	17	14	16	22	50	6	22	29	6	27
28	15	08	19	03	53	24	07	24	22	17	30	16	22	55	5	22	22	7	28
29	14	49	19	03	29	24	07	47	23	17	46	16	23	00	5	22	14	8	29
30	14	29	20				08	09	22	18	01	15	23	05	5	22	06	8	30
31	14	10	19				08	31	22				23	09	4				31

Years 1732, 1736, 1740, (*being Leap-Years.*)

Days.	July. N. °	′	Diff. ′	Aug. N. °	′	Diff. ′	Sept. N. * °	′	Diff. ′	Octob. S. °	′	Diff. ′	Nov. S. °	′	Diff. ′	Decem. S. °	′	Diff. ′	Days.
1	21	58		14	53		04	01		07	36		17	53		23	10		1
2	21	49	9	14	35	18	03	38	23	07	58	22	18	09	16	23	14	4	2
3	21	40	9	14	16	19	03	15	23	08	21	23	18	25	16	23	17	3	3
4	21	31	9	13	58	18	02	52	23	08	43	22	18	40	15	23	20	3	4
5	21	21	10	13	39	19	02	29	23	09	05	22	18	55	15	23	23	3	5
6	21	11	10	13	19	20	02	05	24	09	27	22	19	10	15	23	25	2	6
7	21	00	11	13	00	19	01	42	23	09	49	22	19	24	14	23	27	2	7
8	20	49	11	12	40	20	01	19	23	10	11	22	19	38	14	23	28	1	8
9	20	38	11	12	21	19	00	55	24	10	33	22	19	52	14	23	29	1	9
10	20	27	11	12	01	20	00	32	23	10	54	21	20	05	13	23	29	0	10
11	20	15	12	11	40	21	00	08	24	11	15	21	20	18	13	23	29	0	11
12	20	02	13	11	20	20	S	15	23	11	37	22	20	30	12	23	28	1	12
13	19	50	12	10	59	21	00	38	23	11	58	21	20	42	12	23	27	1	13
14	19	37	13	10	39	20	01	02	24	12	18	20	20	54	12	23	25	2	14
15	19	24	13	10	18	21	01	25	23	12	39	21	21	05	11	23	23	2	15
16	19	10	14	09	57	21	01	49	24	12	59	20	21	16	11	23	20	3	16
17	18	56	14	09	35	22	02	12	23	13	19	20	21	27	11	23	17	3	17
18	18	42	14	09	14	21	02	36	24	13	39	20	21	37	10	23	14	3	18
19	18	27	15	08	52	22	02	59	23	13	59	20	21	47	10	23	10	4	19
20	18	12	15	08	31	21	03	22	23	14	19	20	21	56	9	23	05	5	20
21	17	57	15	08	09	22	03	46	24	14	38	19	22	05	9	23	00	5	21
22	17	42	15	07	47	22	04	09	23	14	57	19	22	14	9	22	55	5	22
23	17	26	16	07	25	22	04	32	23	15	16	19	22	22	8	22	49	6	23
24	17	10	16	07	03	22	04	56	24	15	34	18	22	29	7	22	43	6	24
25	16	54	16	06	40	23	05	19	23	15	53	19	22	36	7	22	36	7	25
26	16	38	16	06	18	22	05	42	23	16	11	18	22	43	7	22	29	7	26
27	16	21	17	05	55	23	06	05	23	16	29	18	22	49	6	22	21	8	27
28	16	04	17	05	33	22	06	28	23	16	46	17	22	55	6	22	13	8	28
29	15	47	17	05	10	23	06	51	23	17	03	17	23	01	6	22	05	8	29
30	15	29	18	04	47	23	07	13	22	17	20	17	23	06	5	21	56	9	30
31	15	11	18	04	24	23				17	37	17				21	46	10	31

A Table for adjusting the Sun's Declination to any Meridian.

Daily Diff. in Declin. M.	Degrees of Longitude either East or West. 15	30	45	60	75	90	105	120	135	150	165	180
3	0	0	0	0	0	0	1	1	1	1	1	1
6	0	0	0	1	1	1	1	2	2	2	2	3
9	0	0	1	1	2	2	2	3	3	3	4	4
12	0	1	1	2	2	3	3	4	4	5	5	6
15	0	1	2	2	3	4	4	5	5	6	6	7
18	1	1	2	3	3	4	5	6	7	8	8	9
21	1	2	3	4	4	5	6	7	8	9	10	11
24	1	2	3	4	5	6	7	8	9	10	11	12

A TABLE *of the* Sun's *Place*

Days.	January. ☉'s P. ♑.		January. ☉'s R. Asc.		February. ☉'s P. ♒.		February. ☉'s R. Asc.		March. ☉'s P ♓.		March. ☉'s R. Asc.		April. ☉'s P ♈.		April. ☉'s R. Asc		May. ☉'s P. ♉.		May. ☉'s R. Asc		June. ☉'s P. ♊.		June. ☉'s R. Asc.	
	°	′	H.	M.	°	′	H.	M.	°	′	H.	M.	°	′	H.	M.	°	′	H.	M	°	′	H.	M
1	22	09	19	36	23	37	21	44	21	41	23	30	22	17	1	22	21	23	3	16	21	06	5	21
2	23	10	19	40	24	37	21	48	22	41	23	33	23	16	1	26	22	21	3	20	22	03	5	25
3	24	11	19	44	25	38	21	52	23	41	23	37	24	14	1	30	23	19	3	24	23	00	5	29
4	25	12	19	49	26	38	21	55	24	41	23	40	25	13	1	33	24	17	3	28	23	58	5	34
5	26	13	19	53	27	39	21	59	25	40	23	44	26	11	1	37	25	14	3	31	24	55	5	38
6	27	14	19	57	28	39	22	03	26	40	23	48	27	10	1	41	26	12	3	35	25	52	5	42
7	28	15	20	01	29	40	22	07	27	39	23	52	28	08	1	44	27	10	3	39	26	49	5	46
8	29	16	20	06	♓	40	22	11	28	39	23	55	29	07	1	48	28	07	3	43	27	46	5	50
9	♒	17	20	10	1	40	22	15	29	38	23	59	♉	05	1	52	29	05	3	47	28	44	5	54
10	01	18	20	14	2	41	22	19	♈	38	0	02	1	04	1	56	♊	03	3	51	29	41	5	59
11	02	19	20	18	3	41	22	22	1	37	0	06	2	02	1	59	1	00	3	55	♋	38	6	03
12	03	20	20	22	4	41	22	26	2	37	0	10	3	00	2	03	1	58	3	59	1	35	6	07
13	04	21	20	27	5	42	22	30	3	36	0	13	3	59	2	07	2	55	4	03	2	32	6	11
14	05	22	20	31	6	42	22	34	4	35	0	17	4	57	2	11	3	53	4	07	3	30	6	15
15	06	23	20	35	7	42	22	37	5	34	0	20	5	55	2	14	4	50	4	11	4	27	6	19
16	07	24	20	39	8	42	22	41	6	34	0	24	6	53	2	18	5	48	4	15	5	24	6	23
17	08	25	20	43	9	42	22	45	7	33	0	28	7	52	2	22	6	45	4	19	6	21	6	28
18	09	26	20	47	10	43	22	49	8	32	0	31	8	50	2	26	7	43	4	24	7	18	6	32
19	10	27	20	51	11	43	22	52	9	31	0	35	9	48	2	29	8	40	4	28	8	16	6	36
20	11	28	20	56	12	43	22	56	10	30	0	39	10	46	2	33	9	38	4	32	9	13	6	40
21	12	29	21	00	13	43	23	00	11	29	0	42	11	44	2	37	10	35	4	36	10	10	6	44
22	13	30	21	04	14	43	23	04	12	28	0	46	12	42	2	41	11	52	4	40	11	07	6	48
23	14	31	21	08	15	43	23	07	13	27	0	49	13	40	2	45	12	30	4	44	12	04	6	52
24	15	31	21	12	16	42	23	11	14	26	0	53	14	38	2	49	13	27	4	48	13	01	6	56
25	16	32	21	16	17	42	23	15	15	25	0	57	15	36	2	52	14	25	4	52	13	59	7	01
26	17	33	21	20	18	42	23	18	16	24	1	00	16	34	2	56	15	22	4	56	14	56	7	05
27	18	33	21	24	19	42	23	22	17	23	1	04	17	32	3	00	16	19	5	00	15	53	7	09
28	19	34	21	28	20	42	23	26	18	22	1	08	18	30	3	04	17	17	5	04	16	50	7	13
29	20	35	21	32					19	21	1	11	19	28	3	08	18	14	5	09	17	47	7	17
30	21	35	21	36					20	20	1	15	20	26	3	12	19	11	5	13	18	45	7	21
31	22	36	21	40					21	18	1	19					20	09	5	17				

and Right Ascension, for the Year 1730.

Days.	July. ☉'s P. ♋.		July. ☉'s R. Asc.		August. ☉'s P. ♌.		August. ☉'s R. Asc.		September. ☉'s P. ♍R.		September. ☉'s R. Asc.		October. ☉'s P. ♎.		October. ☉'s R. Asc.		November. ☉'s P. ♏.		November. ☉'s R. Asc		December. ☉'s P. ♐.		December. ☉'s R. Asc.	
	°	′	H.	M	°	′	H.	M	°	′	H.	M	°	′	H.	M	°	′	H.	M	°	′	H.	M
1	19	42	7	25	19	21	9	27	19	21	11	21	18	52	13	10	19	53	15	10	20	18	17	18
2	20	39	7	29	20	19	9	31	20	20	11	24	19	52	13	13	20	54	15	14	21	19	17	22
3	21	36	7	33	21	17	9	35	21	18	11	28	20	51	13	17	21	54	15	18	22	20	17	27
4	22	34	7	37	22	14	9	38	22	17	11	32	21	51	13	21	22	55	15	22	23	22	17	31
5	23	31	7	41	23	12	9	42	23	16	11	35	22	50	13	24	23	55	15	26	24	23	17	35
6	24	28	7	45	24	10	9	46	24	14	11	39	23	50	13	28	24	56	15	30	25	24	17	40
7	25	25	7	49	25	08	9	50	25	13	11	42	24	50	13	32	25	57	15	34	26	25	17	44
8	26	23	7	53	26	06	9	53	26	12	11	46	25	50	13	36	26	57	15	38	27	26	17	49
9	27	20	7	57	27	03	9	57	27	11	11	50	26	49	13	40	27	58	15	43	28	27	17	53
10	28	17	8	01	28	01	10	01	28	09	11	53	27	49	13	43	28	59	15	47	29	28	17	58
11	29	14	8	05	28	59	10	04	29	08	11	57	28	49	13	47	29	59	15	51	♑	29	18	02
12	♌	12	8	09	29	57	10	08	♎	07	12	00	29	49	13	51	♐	1 00	15	55	1	31	18	06
13	1	09	8	13	♍R	55	10	12	1	06	12	04	♏	49	13	55	2	01	16	00	2	32	18	11
14	2	06	8	17	1	53	10	16	2	05	12	08	1	49	13	58	3	02	16	04	3	33	18	15
15	3	04	8	21	2	51	10	19	3	04	12	11	2	49	14	02	4	03	16	08	4	34	18	20
16	4	01	8	25	3	49	10	23	4	03	12	15	3	49	14	06	5	03	16	12	5	35	18	24
17	4	59	8	29	4	47	10	27	5	02	12	18	4	49	14	10	6	04	16	17	6	37	18	29
18	5	56	8	33	5	45	10	30	6	01	12	22	5	49	14	14	7	05	16	21	7	38	18	33
19	6	53	8	37	6	43	10	34	7	00	12	26	6	49	14	18	8	06	16	25	8	39	18	38
20	7	51	8	41	7	41	10	37	7	59	12	29	7	49	14	22	9	07	16	30	9	40	18	42
21	8	48	8	45	8	40	10	41	8	58	12	33	8	49	14	26	10	08	16	34	10	41	18	46
22	9	46	8	49	9	38	10	45	9	58	12	37	9	50	14	30	11	09	16	38	11	42	18	51
23	10	43	8	53	10	36	10	48	10	57	12	40	10	50	14	34	12	10	16	43	12	44	18	55
24	11	41	8	56	11	34	10	52	11	56	12	44	11	50	14	37	13	11	16	47	13	45	19	00
25	12	38	9	00	12	33	10	56	12	55	12	48	12	50	14	41	14	12	16	51	14	46	19	04
26	13	36	9	04	13	31	10	59	13	55	12	51	13	51	14	45	15	13	16	56	15	47	19	08
27	14	33	9	08	14	29	11	03	14	54	12	55	14	51	14	49	16	14	17	00	16	48	19	13
28	15	31	9	12	15	28	11	06	15	54	12	58	15	51	14	53	17	15	17	04	17	49	19	17
29	16	28	9	16	16	26	11	10	16	53	13	02	16	52	14	57	18	16	17	09	18	51	19	22
30	17	26	9	20	17	24	11	14	17	53	13	06	17	52	15	01	19	17	17	13	19	52	19	26
31	18	24	9	23	18	23	11	17					18	53	15	06					20	53	19	30

A TABLE of the Right Aſcenſion and Declination of the principal fixed Stars, adapted for the Year 1726.

Conſtellations within the Zodiack.

Conſtellat.	The Parts of the Conſtellations wherein the Stars are placed.		R. Aſc. H.	R. Aſc. M.	Declin. °	Declin. ′	Denomin.	Magnit.
Aries.	The bright Star of *Aries.* ——	α	1	52	22	08	N.	2
Taurus.	The *Bull's Eye.* ALDEBARAN.	α	4	20	15	55	N	1
	In the Northern *Horn.* ——	β	5	09	28	20	N	2
	In the Southern *Horn.* ——	ζ	5	21	20	56	N	3
Gemini.	In the Foot of the Southern *Twin.*	γ	6	22	16	36	N.	2
	In the Knee of the Northern *Twin.*	ε	6	27	25	22	N.	3
	CASTOR. —— ——	α	7	17	32	27	N.	1
	POLLUX. ——	β	7	28	28	40	N.	2
Leo.	In the *Lyon's* Head. ——	μ	9	37	27	16	N.	3
	The *Lyon's Heart.* REGULUS.	α	9	54	13	17	N	1
	In the *Lyon's* Neck. —	γ	10	05	21	13	N.	2
	In the *Lyon's* Tail. DENEB. —	β	11	35	16	06	N.	1
Virgo.	SPICA VIRGINIS. ——	α	13	11	9	44	S.	1
Scorpio.	In the *Scorpion's* Head. —	β	15	50	19	01	S.	2
	Cor Scorpii. ANTARES. ——	α	16	13	25	47	S.	1
	In the Tip of the Tail. —	λ	17	15	36	51	S.	2
Sagittarius.	In the Southern part of the *Bow.*	ε	18	06	34	29	S.	2
	In the Shoulder of *Sagittarius.*	σ	18	38	26	35	S.	3
	In the Head of *Sagittarius.* —	π	18	53	21	26	S.	3

Conſtellations on the Northern Side of the Zodiack.

Andromeda.	In the end of the *Chain.* ——	ο	22	49	40	52	N.	3
	In the Head of *Andromeda.* —	α	23	54	27	34	N.	2
	In the *Girdle.* MIRACH. —	β	0	54	34	09	N.	2
	In the Foot. ALMAACH. —	γ	1	47	40	59	N.	2
Perſeus.	In the Shoulder of *Perſeus.* —	γ	2	45	52	25	N.	3
	Caput Meduſæ. ALGOL. ——	β	2	50	39	53	N.	2
	In the Side of *Perſeus.* ALGENIB.	α	3	05	48	51	N.	2
	In the Southern Foot. —— ——	ζ	3	37	31	03	N.	3
	In the Southern Knee. ——	ε	3	39	39	12	N.	3

An Eſtimate of the Meridian Altitudes in whole Deg. (where th same does not exceed 40 Deg.) in every 10 Deg. of Lat. from 6c Deg. North, to 60 Deg. South.

N. 60°	N. 50°	N. 40°	N. 30°	N. 20°	N. 10°	00°	S. 10°	S. 20°	S. 30°	S. 40°	S. 50°	S. 60°
- -	- -	- -	- -	- -	- -	- -	- -	- -	38	28	18	8
- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	34	24	14
- -	- -	- -	- -	- -	- -	- -	- -	- -	32	22	12	
- -	- -	- -	- -	- -	- -	- -	- -	- -	39	29	19	9
- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	34	24	14
- -	- -	- -	- -	- -	- -	- -	- -	- -	35	25	15	5
- -	- -	- -	- -	- -	- -	- -	- -	38	28	18	8	
- -	- -	- -	- -	- -	- -	- -	- -	41	31	21	11	
- -	- -	- -	- -	- -	- -	- -	- -	- -	33	23	13	
- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	37	27	17
- -	- -	- -	- -	- -	- -	- -	- -	- -	39	29	19	9
- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	34	24	14
20	30	40										
11	21	31	41									
4	14	24	34									
- -	3	13	23	33								
- -	6	16	26	36								
- -	13	23	33									
9	19	29	39									

11	- -	- -	- -	- -	- -	- -	- -	39	29	19	9	
- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	32	22	12
- -	- -	- -	- -	- -	- -	- -	- -	- -	36	26	16	6
11	- -	- -	- -	- -	- -	- -	- -	39	29	19	9	
22	12	- -	- -	- -	- -	38	28	18	8			
10	- -	- -	- -	- -	- -	- -	40	30	20	10		
19	9	- -	- -	- -	- -	41	31	21	11			
- -	- -	- -	- -	- -	- -	- -	- -	39	29	19	9	
9	- -	- -	- -	- -	- -	- -	- -	31	21	11		

A TABLE *of the Right Aſcenſion and Declination of the principal fixed Stars, adapted for the Year* 1726.

Conſtellations on the Northern Side of the Zodiack.

Conſtellat.	The Parts of the Conſtellations wherein the Stars are placed.		R. *Aſc.* H.	R. *Aſc.* M.	*Declin.* °	*Declin.* ′	*Denomin.*	*Magnit.*
Auriga.	In the preceding Should. CAPELLA.	α	4	56	45	41	N.	1
	In the following Shoulder.	β	5	39	44	53	N.	2
Boötes.	ARCTURUS.	α	14	03	20	39	N.	1
	In the preceding Shoulder.	γ	14	21	39	31	N.	3
	MIRACH.	ε	14	33	28	15	N.	3
	In the Head.	β	14	52	41	30	N.	3
	In the following Shoulder.	δ	15	04	34	22	N.	3
Coron. Bor.	In the *Northern Crown.*	α	15	23	27	39	N.	2
Aquila.	In the *Eagle's* Tail.	ε	18	47	14	43	N.	3
	ALTAIR.	α	19	37	8	10	N.	1
Pegaſus.	SCHEAT.	β	22	50	26	35	N.	2
	MARKAB.	α	22	51	13	43	N.	2
	ALGENIB.	γ	23	59	13	39	N.	2
Caſſiopea.	In the *Chair.*	β	23	55	57	39	N.	3
	SCHEDAR.	α	0	25	55	02	N.	3
	In the *Leg.*	ε	1	35	62	19	N.	3
Urſa Major.	In the fore Foot.	κ	8	43	48	14	N.	3
	In the following of the fore Legs.	θ	9	12	52	55	N.	3
	The firſt and Southermoſt in the □.	β	10	45	57	50	N.	2
	DUBHE.	α	10	46	63	13	N.	2
	The third in the □.	γ	11	39	55	13	N.	2
	In the Root of the Tail.	δ	12	02	58	35	N.	2
	The firſt in the Tail. ALIATH.	ε	12	42	57	30	N.	2
	The ſecond in the Tail.	ζ	13	13	56	23	N.	2
	The laſt in the Tail. BENETNASCH.	η	13	27	50	42	N.	2
	COR CAROLI.		12	43	39	49	N.	2
Hercules.	In the preceding Shoulder.	β	16	18	22	07	N.	3
	In the Side.	ζ	16	31	32	08	N.	3
	In the Head. RAS ALGETHI.	α	17	02	14	43	N.	3
	In the following Shoulder.	δ	17	04	25	11	N.	3
	In the following Knee.	θ	17	47	37	19	N.	3
Lyra.	LYRA.	α	18	28	38	33	N.	1

An Eſtimate of the Meridian Altitudes in whole Deg. (where the ſame does not exceed 40 Deg.) in every 10 Deg. of Lat. from 60 Deg. North, to 60 Deg. South.

N. 60°	N. 50°	N. 40°	N. 30°	N. 20°	N. 10°	00°	S. 10°	S. 20°	S. 30°	S. 40°	S. 50°	S. 60°
15	5	--	--	--	--	--	35	25	15	5		
15	5	--	--	--	--	--	35	25	15	5		
--	--	--	--	--	--	--	--	--	39	29	19	9
9	--	--	--	--	--	--	--	31	21	11		
--	--	--	--	--	--	--	--	--	32	22	12	
11	--	--	--	--	--	--	39	29	19	9		
--	--	--	--	--	--	--	--	36	26	16	6	
--	--	--	--	--	--	--	--	--	32	22	12	
--	--	--	--	--	--	--	--	--	--	35	25	15
--	--	--	--	--	--	--	--	--	--	--	32	22
--	--	--	--	--	--	--	--	--	33	23	13	
--	--	--	--	--	--	--	--	--	--	36	26	16
--	--	--	--	--	--	--	--	--	--	36	26	16
28	18	8	--	--	--	32	22	12	--			
25	15	5	--	--	--	35	25	15	5			
32	22	12	--	--	38	28	18	8	--			
18	8	--	--	--	--	--	32	22	12			
23	13	--	--	--	--	37	27	17	7			
28	18	8	--	--	--	32	22	12				
33	23	13	--	--	37	27	17	7				
25	15	5	--	--	--	35	25	15	5			
28	18	8	--	--	--	32	22	12				
27	17	7	--	--	--	33	23	13				
26	16	6	--	--	--	34	24	14	4			
21	11	--	--	--	--	--	29	19	9			
10	--	--	--	--	--	--	--	30	20	10		
--	--	--	--	--	--	--	--	--	38	28	18	8
--	--	--	--	--	--	--	--	38	28	18	8	
--	--	--	--	--	--	--	--	--	--	35	25	15
--	--	--	--	--	--	--	--	--	35	25	15	5
--	--	--	--	--	--	--	--	33	23	13		
--	--	--	--	--	--	--	--	32	22	12		

A TABLE of the Right Aſcenſion and Declination of the principal fixed Stars, adapted for the Year 1726.

Conſtellations on the Northern Side of the Zodiack.

Conſtellat.	The Parts of the Conſtellations wherein the Stars are placed.		R. Aſc. H.	R. Aſc. M.	Declin. °	Declin. ′	Denomin.	Magnit.
Cygnus.	In the *Eagle*'s Bill.	β	19	20	27	24	N.	3
	In the *Northern Wing.*	δ	19	36	44	29	N.	3
	In the *Breaſt.*	γ	20	12	39	23	N.	3
	In the *Tail.*	α	20	32	44	19	N.	2
	In the *Southern Wing.*	ε	20	35	32	57	N.	3
	Between the Feet of *Pegaſus.*	μ	21	32	27	30	N.	3
Cepheus.	In the preceding *Shoulder.*	α	21	12	61	27	N.	3
	In the *Girdle.*	β	21	25	69	22	N.	3
	In the following *Foot.*	γ	23	28	76	07	N.	3
Urſa Minor.	The *Pole-Star.*	α	0	38	87	51	N.	2
	KOCHAB.	β	14	52	75	17	N.	3
Draco.		α	13	57	65	42	N.	3
		θ	15	57	59	17	N.	3
		ζ	17	08	66	04	N.	3
		γ	17	50	51	33	N.	2
		δ	19	13	67	11	N.	3

Conſtellations on the Southern Side of the Zodiack.

Orion.	REGEL.	β	5	01	8	33	S.	1
	Betelgeuſe.	α	5	40	7	19	N.	1
	COLUMBA NOACHI.	υ	5	30	34	16	S.	2
		υ	5	41	35	54	S.	2
Canis Maj.	In the hind Foot.	ζ	6	10	29	57	S.	2
	SYRIUS.	α	6	33	16	21	S.	1
	In the Tail.	π	7	13	28	46	S.	2
Canis Min.	PROCYON.	α	7	25	5	55	N.	1
Hydra.	COR HYDRÆ.	α	9	14	7	29	S.	2
		ξ	11	19	30	18	S.	3
	The laſt but one in the Tail.	γ	13	04	21	43	S.	3
Centaurus.	In the following Shoulder.	θ	13	51	34	58	S.	2
Piſcis Notius.	FOMALHAUT.	α	22	42	31	03	S.	1
Cruzero.	The Southern Foot of the *Croſs.*	ζ	12	12	61	31	S.	2
	The Northermoſt of the four.	ε	12	17	55	30	S.	2
	The Southermoſt of the four.	ξ	12	33	58	06	S.	2

An Estimate of the Meridian Altitudes in whole Deg. (where the same does not exceed 40 Deg.) in every 10 Deg. of Lat. from 60 Deg. North, to 60 Deg. South.

N. 60°	N. 50°	N. 40°	N. 30°	N. 20°	N. 10°	00°	S. 10°	S. 20°	S. 30°	S. 40°	S. 50°	S. 60°
--	--	--	--	--	--	--	--	--	33	23	13	
14	4	--	--	--	--	--	36	26	16	6		
9	--	--	--	--	--	--	--	31	21	11		
14	4	--	--	--	--	--	36	26	16	6		
--	--	--	--	--	--	--	--	37	27	17	7	
--	--	--	--	--	--	--	--	--	33	23	13	
31	21	11	--	--	39	29	19	9				
39	29	19	9	--	31	21	11					
--	36	26	16	34	24	14	4					
--	--	--	32/28	22/18	12/8							
--	35	25	15	35	25	15	5					
36	26	16	6	--	34	24	14	4				
29	19	9	--	--	--	31	21	11				
36	26	16	6	--	34	24	14	4				
21	11	--	--	--	--	39	29	19	9			
37	27	17	7	--	33	23	13					

22	32											
--	--	--	--	--	--	--	--	--	--	--	33	23
--	6	16	26	36								
--	4	14	24	34								
--	10	20	30	40								
14	24	34	--									
--	11	21	31									
36	--	--	--	--	--	--	--	--	--	--	34	24
23	33											
--	10	20	30									
8	18	28	38									
--	5	15	25	35								
--	9	19	29	39								
--	--	--	--	9	19	29	39	--	--	11	21	31
--	--	--	5	15	25	35	--	--	--	5	15	25
--	--			12	22	32	--	--	--	8	18	28

A TABLE, *ſhewing the Right Aſcenſion of that Point of the Heavens, which comes to the Meridian at the reſpective Times herein expreſſed.*

Months, &c.		8h P.M. H.	8h P.M. M.	10h P.M. H.	10h P.M. M.	12h P.M H.	12h P.M M.	2h A.M. H.	2h A.M. M.	4h A.M. H.	4h A.M. M.
January	1	3	36	5	36	7	36	9	36	11	36
	16	4	39	6	39	8	39	10	39	12	39
February	1	5	44	7	44	9	44	11	44	13	44
	16	6	41	8	41	10	41	12	41	14	41
March	1	7	30	9	30	11	30	13	30	15	30
	16	8	24	10	24	12	24	14	24	16	24
April	1	9	22	11	22	13	22	15	22	17	22
	16	10	18	12	18	14	18	16	18	18	18
May	1	11	16	13	16	15	16	17	16	19	16
	16	12	15	14	15	16	15	18	15	20	15
June	1	13	21	15	21	17	21	19	21	21	21
	16	14	23	16	23	18	23	20	23	22	23
July	1	15	25	17	25	19	25	21	25	23	25
	16	16	25	18	25	20	25	22	25	00	25
Auguſt	1	17	27	19	27	21	27	23	27	1	27
	16	18	23	20	23	22	23	00	23	2	23
September	1	19	21	21	21	23	21	01	21	3	21
	16	20	15	22	15	00	15	2	15	4	15
October	1	21	10	23	10	1	10	3	10	5	10
	16	22	06	00	06	2	06	4	06	6	06
November	1	23	10	01	10	3	10	5	10	7	10
	16	00	12	2	12	4	12	6	12	8	12
December	1	1	18	3	18	5	18	7	18	9	18
	16	2	24	4	24	6	24	8	24	10	24

The Uſe of the preceding Tables has been already ſhewed from Page 167 to 172.

FINIS.

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